

ASPECTS OF THE ANALYSIS OF FRAME-PANEL INTERACTION

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Synopsis

Some of the aspects involved in modelling frame-panel interaction by computer methods are discussed. These include the different types of infill and their strength and failure properties, the forces of interaction, and methods for handling material nonlinearity. The use of the finite element method to implement the analysis is described and examples are presented to illustrate the application of the method.

1.0. Introduction

The first reported attempt to analyse the stresses set up in an infill panel when this interacts with the surrounding frame was carried out in 1948 by Polyakov⁽¹⁾ using an approximate method based on the theory of elasticity. The method used a stress function to express the distribution of stresses around the boundary, the distribution being based on his experimental observations. Polyakov considered the panel to have failed when the maximum theoretical shearing stress in the infill panel reached the corresponding strength of the brickwork. In all his tests, the initial failure was caused by cracking around the perimeter of the infill, thus allowing the frame and infill to separate except near the compression corners. From this, Polyakov suggested that an infilled frame could be analysed satisfactorily by assuming the infill to act as a diagonal bracing strut.

Holmes^(2,3) considered the infill to act as a diagonal strut and used this as a means of predicting the strength and deformation of an infilled frame. He suggested that the strut should be taken to have the same thickness and modulus of elasticity as the infill, while the width should be taken as one-third the diagonal length of the infill. An elastic analysis of the resulting equivalent structure would predict the actual deformation. The strength of the infilled frame could then be predicted by assuming the infill to fail at a predetermined average diagonal strain that would depend on the type of infill material.

The method of analysis whereby the infill is replaced by an equivalent strut has been developed considerably by Stafford Smith⁽⁴⁻¹⁰⁾. As against the constant width for the equivalent strut assumed by Holmes, Stafford Smith has shown that the width depends on several factors. For both square and rectangular infilled frames

loaded laterally, theoretical solutions could be predicted approximately by statically analysing the equivalent pin-jointed frame in which the infills were replaced by equivalent diagonal struts. The effective width of the equivalent strut was found to be influenced by the relative stiffness of the column and infill, the length-to-height ratio of the infill, and, for multistorey frames, the height of the point on the structure at which the stiffness is to be predicted. The beam stiffness appeared to have little influence on the overall stiffness. The force distribution along the length of contact between the frame and the infill was assumed to have a triangular distribution and theoretical values for the width of the diagonal strut were derived on the basis of this assumption. A comparison of theoretical and experimental values for the width of this equivalent strut showed a marked discrepancy which Stafford Smith put down to the particular frame-panel force distribution that had been chosen. The experimental values were suggested to be the more reliable for use.

Several other workers have also sought to find ways of satisfactorily analysing the strength and stiffness of infilled frames. On the basis of tests they carried out, Benjamin and Williams^(11,12) came to the conclusion that the results from a lattice analogy analysis were no more accurate than those given by a simpler strength of materials analysis on account of the variables involved. Satchanski⁽¹³⁾ assumed the contact forces between the frame and the infill to be replaced by 30 redundant reactions and then determined a function that could represent the boundary stress distribution equivalent to the derived reactions. Polyakov⁽¹⁴⁾ in a further analysis, represented the panel stresses by polynomial functions. Also, a type of finite element analysis has been reported by Karamanski⁽¹⁵⁾.

Mallick and Severn⁽¹⁶⁾ carried out finite element analyses using the simplest possible rectangular elements to determine the lateral stiffness of infilled frames. By this means, they determined the points of separation between the frame and the infill, as well as the stress distribution in the contact intervals, as an integral part of the solution. The stress components σ_{xx} , σ_{yy} , σ_{xy} were expressed in terms of the Airey stress function, ϕ , which in turn was expressed as a particular polynomial function of x , and y . The edge displacements for the above were taken to be linear functions of the nodal displacements. Separation between the frame and the infill was taken into account in the analysis and the theoretical results were shown to agree remarkably well with the experimental ones, especially for square panels.

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In dealing with multistorey infilled frames, Mallick and Severn proposed an approach based upon the idea of a "shear structure" that would be less time consuming than a full finite element approach. It was assumed that there would be no rotation of a horizontal section at each floor level and the relative displacement between the floors would be horizontal. In this case, the stiffness of each storey depends only on the relative displacement of the two floor levels of that storey. Each storey was treated as a beam element acting only in shear, and it was assumed that one shear displacement at each floor level would be sufficient to define the deformed structure. The experimental load/deflection curve of a single, laterally loaded infilled frame was used to derive the stiffness for each storey.

The only dynamic analysis of infilled frames that has been reported to date is that described by Mallick and Severn in a further paper(17). They investigated the dynamic characteristics with regard to damping natural frequencies and mode shapes.

Recently, Liauw⁽¹⁸⁾ presented some experimental and theoretical results for the elastic behaviour of infilled frames under racking load. The theoretical approach made use of a stress function, expressed in the form of a Fourier series, to derive the stresses and deformations in the infill and frame. The theoretical stress distribution in the infill appears to agree reasonably well with the results from a photo-elastic model.

As part of a study of reinforced concrete frames and panels, Franklin⁽¹⁹⁾ investigated the application of the finite element method to the analysis of frame-panel interaction. The panels were represented by linear strain quadrilateral finite elements. Special frame elements were used and these allowed for the cracking and yielding that takes place in reinforced concrete beams and columns. Two model reinforced concrete frames were analysed to study their change in behaviour when infill panels were added.

The brief review of existing methods for assessing the strength and stiffness of infilled frames, which has been given above, suggests that further investigation is necessary in order to more closely model the behaviour of infilled frames of different rectangular shapes, infilled with either concrete or masonry, and subjected to both horizontal and vertical loads. In addition, other factors such as slip between the frame and infill, variations in contact length and contact stresses between frame and infill, tensile cracking, and crushing and shear of the infill material should be automatically treated by the computational method.

This paper discusses these and other factors as they affect the study of frame-panel interaction where the analytical model comprises

- (1) triangular finite elements to represent two dimensional panels and walls,
- (2) deformable beam elements for the surrounding frame, and
- (3) special rigid tie link elements that connect the frame elements to adjacent wall elements.

Tensile cracking and compressive failures in the panel elements are included so that the assemblage of elements could be used to perform a nonlinear analysis. Nonlinear material properties could also be incorporated into the method. Small displacement theory is assumed throughout the analysis.

2.0. Structural Idealisation

This is the process of formulating a discrete element model of the structural system wherein a finite number of degrees of freedom are defined. This discrete system is then taken as a substitute for the real system with its infinite number of degrees of freedom. From this idealisation there are obtained a finite number of simultaneous algebraic equations relating externally applied loads and structure displacements which can be solved by standard methods of matrix algebra.

For the idealisation of beam or column members, the substitute members are one-dimensional elements whose properties are assumed to be concentrated along the centroidal axis of the member and are a function of distance along the member. Joints between members are concentrated at the points where the centroidal lines intersect and inter-element continuity of displacements are enforced at these points.

For a continuum-type system, the continuous system is replaced by a finite number of discrete elements or regions connected together at a finite number of points (as against the infinite number in the real continuum). However, the displacement shapes of these discrete elements are selected so that while the interconnection of the regions or elements occurs at a relatively few points, the boundaries remain in contact at all times.

The final stage in the idealisation of the real structure by finite elements is to transform the internal and external forces into a statically equivalent set of forces concentrated at the appropriate nodal points. The real structure that is to be analysed is then represented by the set of discrete finite elements together with the set of nodal forces that represent the applied loads.

2.1 Panel Elements

Since the panels are subject to in-plane forces only, the simplest triangular finite element was chosen for the panel idealisation (fig.1 (a)). A triangular element was chosen in preference to a quadrilateral finite element as it meant that the panel need not be idealised by a uniform mesh and smaller elements could be used in the regions where stress concentrations were expected to occur.

2.2 Beam elements

For the present analysis, the beams were idealised by simple linear elements in the conventional manner (fig.1 (b)). Thus they were taken to be of constant section and elastic properties throughout their length. As a result the normal slope deflection equations could be utilised to establish their stiffness properties.

2.3 Panel-to-frame Connection

The special connections between the infill panels and the surrounding frame members were provided by discrete element tie links (fig. 1 (c)). These enabled two adjacent nodes to be held together or released according to the displacements of the node-pair, thus representing the interaction between the frame and the elements around the edge of the panel. For this study, the tie links were capable of transferring compressive and bond forces but incapable of transferring tensile forces. The bond forces were expressed in terms of an equivalent friction force based on the compressive force in the tielink and a coefficient of friction.

The "no tension" tie link used in this study is suitable for the case where the infill material is only slightly bonded on to an enclosing steel frame, as could be the case with a weak concrete or masonry infill. For the case where the infill is reinforced and tied to a frame of steel or concrete, only a small modification is required to enable the tielink to transfer a tensile force based on the tensile strength of the reinforcing.

3.0 Direct Stiffness Method of Analysis

3.1 Beam and Tielink elements

The stiffnesses of these elements were determined by means of the normal slope deflection equations.

The tielink elements were removed from the structure once they were found to be carrying tensile forces. However, should the adjacent nodes later move back towards each other, then the tielinks could be introduced again. For the analysis the tielinks were given a large cross-sectional area representing the edge contact area between the frame and the elements on each side of the node, but the moments of inertia were set at zero to ensure that the tie elements transferred no bending moment.

3.2 Finite elements

For the present analysis, the panel was simulated by the simplest possible two dimensional finite element. This is the CONSTANT STRAIN TRIANGLE or CST element which has a total of six degrees of freedom made up of two degrees of freedom at each of the three corner nodes (see fig. 1 (a)).

The displacement shapes of the element must be such that the displacements along the boundary of the element between the nodes must match the displacements of the adjoining element and the displacement shape of the element edge must be defined solely by the nodes on that edge. Thus for the constant strain triangle with a node at each corner, the displaced shape of the edge must be linear. The number of independent displacement shapes must not exceed the number of degrees of freedom of the element. Included among these shapes there must be a rigid body displacement; if there is not, then should the element be given a rigid body motion, it would develop internal strains and stresses as a result. This obviously does not occur in practice.

Details of the CST element and the derivation of the element stiffness matrix have been given previously by Clough⁽²⁰⁾.

3.3 Analysis of the complete structure

Once the individual element stiffness matrices have been formed, the next step in the analysis is to assemble them in such a manner that they give the stiffness matrix $[K]$ for the whole structure. Each six by six element stiffness matrix $[k_e]$ may be partitioned into nine two by two submatrices and added into the total stiffness matrix for the structure. The resulting total stiffness matrix is banded and symmetric.

Also required is the applied load vector $\{P\}$ associated with displacements for the whole structure. The individual loads may be (a) due to point loads, (b) moments applied to the joints, or (c) fixed end effects due to loads applied along a beam member.

This then gives the matrix equation

$$[K] \cdot \{r\} = \{P\}$$

where $[K]$ is the total stiffness matrix for the structure with boundary conditions applied,

$\{P\}$ is the vector of joint loads,
 $\{r\}$ is the vector of joint displacements.

Since $[K]$ is symmetric positive definite, it is well conditioned for equation solving by direct decomposition methods. Thus the displacements $\{r\}$ can be found directly.

With the displacements (and the rotations where beam elements are used) of every joint known, it is then a simple process to go back and, after selecting the displacements that correspond to each node, to compute the internal forces in the element or beam.

4.0 Non Linear Structural Analysis

It is well known that non linear structural behaviour occurs as a result of one or more of several factors. In particular, the effects of nonlinear material properties and geometric nonlinearity of a structure have been studied by many research workers in a variety of contexts. Also, in the study of frame-panel interaction, nonlinearity in structural behaviour has been shown to arise from failure of the bond between frame and panel, crushing at the ends of the loaded diagonal, and cracking at the centre of the panel⁽¹⁶⁾.

4.1 Material Nonlinearity

The nonlinear behaviour of infilled frame structures is usually a result of the properties of the material. For instance, while concrete might be described adequately as a linearly elastic material over the range of stresses used in design, it is not likely to be described so when subjected to earthquake forces. For these conditions it is probable that stresses and deformations over part of the structure at least may rise to levels at which the material behaviour can only be described as nonlinear (fig. 2).

Two methods that have been developed for carrying out elastic-plastic analyses using

finite elements are the method of initial strains and the tangent modulus method. These have been summarised by Marcel⁽²¹⁾. The method of initial strains modifies the equilibrium equations for the structure so that the elastic relations can be used throughout on the L.H.S. of the equations. The R.H.S. of the set of equations is modified to compensate for the fact that the plastic strains do not cause any change in the stresses. This means that the work involved in solving the equilibrium equations at each step is greatly reduced as the stiffness matrix need only be inverted or decomposed at the first step.

The tangent modulus method is based on the linearity of the incremental laws of plasticity and approaches the problem in a piecewise linear fashion. For each increment of load a new set of coefficients is obtained for the equilibrium equations. Thus the equilibrium equations must be solved at each load step.

A further approach that is used often is the secant modulus method⁽¹⁹⁾. This is similar to the tangent modulus method in application except that for each increment, the modulus of elasticity is taken as the secant to the stress-strain curve for the particular increment of stress and strain.

Even though the inclusion of nonlinear material properties in the analysis is probably necessary in order to obtain a high correlation between the theoretical analysis and the experimental model results, it was felt wiser to use a simplified tangent modulus approach initially.

4.2 Geometric Nonlinearity

In certain situations, nonlinear structural behaviour may arise from geometric effects. This means that when the deflections of the structure become sufficiently large then the equilibrium of the structure can no longer be represented by linear equations. This comes about from two causes. Firstly, some terms in the strain-displacement relations can no longer be neglected due to the large displacements. Secondly, additional internal forces are developed due to the deflected shape of the structure and must be included in the force-displacement equations. While it is well known that force amplifications occur in beam-column members, it was decided for the sake of simplicity not to include large deflection effects for the present analysis. Thus the analysis is based solely on small deflection theory.

4.3 Member Interaction

Changes in overall structural behaviour can also be caused by changes in the interaction between the infill panel elements and the frame, as well as between the separate panel elements themselves. These changes influence the overall stiffness of the structure and may cause a nonlinear response to the applied loading even though the structural elements remain linearly elastic and the deformations of the structure remain small.

The effects of cracking in the infill panel and the breakdown of bond between the infill panel and the frame eventually lead to failure in the real structure. The effect of shearing

along the mortar planes is an additional factor in the case of masonry infills. In the analytical model, changes in the total stiffness matrix will arise as the tielinks between frame and panel elements fail and as the tensile stress induced in a finite element exceeds the tensile strength of the infill material. At this stage the element is considered to be destroyed and is removed from the system.

5.0. Material Properties

5.1 Concrete Infill

The stress-strain curve for the concrete infill was assumed to be parabolic with a maximum stress, σ_c , occurring at a strain, ϵ_c , as shown in fig. 2. Thus the stress, σ corresponding to a strain ϵ , can be written as

$$\frac{\sigma}{\sigma_c} = \frac{2\epsilon}{\epsilon_c} - \left(\frac{\epsilon}{\epsilon_c}\right)^2$$

From this expression, the tangent modulus

$$E_t = \frac{2\sigma_c}{\epsilon_c} \left(1 - \frac{\epsilon}{\epsilon_c}\right)$$

can be written as

$$E_t = \frac{2\sigma}{\epsilon_c} \sqrt{1 - \frac{\sigma}{\sigma_c}}$$

and this was the form used in this study with σ being taken as the maximum absolute value of the two principal stresses.

Strictly, the above expression for the tangent modulus only applies in the case of uniaxial stress, whereas the elements of the infill panel are subjected to biaxial stress. Thus the correct stress-strain relation is

$$\{\sigma\} = [C] \{\epsilon\}$$

$$\text{where } [C] = \frac{1}{1 - \nu_1 \nu_2} \begin{bmatrix} E_1 & \nu_1 E_2 & 0 \\ \nu_2 E_1 & E_2 & 0 \\ 0 & 0 & G_{12}(1 - \nu_1 \nu_2) \end{bmatrix}$$

and E_i are the moduli of elasticity for the material in the orthogonal directions, ν_i are the Poisson's ratios, and

$$G_{12} = \frac{E_1(1 - \nu_2) + E_2(1 - \nu_1)}{4(1 - \nu_1 \nu_2)}$$

In order to make use of this biaxial stress-strain relation, the values of the E_i and ν_i for all combinations of stress must be known. Also, symmetry of the matrix C requires that $\nu_2 E_1 = \nu_1 E_2$. This means that as one of the moduli of elasticity varies, the Poisson's ratios must also change.

Kupfer, Hilsdorf and Rusch⁽²²⁾ have presented some of this information as a result of their investigations, though their study is still continuing. They present stress-strain

curves for concrete under biaxial compression, biaxial tension and combined tension and compression. It is noticeable that the biaxial strength is a function of the strength of the concrete. Kupfer et al found that for stresses below the elastic limit, the modulus of elasticity and Poisson's ratio were independent of the stress ratio. They report that they are still studying the failure mechanism and a universal failure criterion for concrete.

While the use of a uniaxial stress-strain relation simplified the analysis in the initial stages of development, it is intended to investigate the usefulness of a simplified biaxial stress relation based on the work of Kupfer, Hilsdorf and Rusch. The major disadvantage of using biaxial stress-strain curves lies in the need to continually refer to a set of curves from which the modulus of elasticity will need to be interpolated since the ratio of the principal stresses will seldom correspond to an experimental curve. In addition, it will be preferable to use an incremental secant modulus approach when using a biaxial stress relation rather than the tangent modulus method used in the study reported herein.

5.2 Masonry Infill

While the factors that affect the strength of masonry have been known for some time, it is only recently that the results of systematic research into these have become generally available. The main parameters are:

- (1) the biaxial strengths of the masonry,
- (2) the failure criterion for the masonry under the triaxial state of stress,
- (3) the uniaxial compressive strength of the mortar,
- (4) the behaviour of the mortar under a state of triaxial compression,
- (5) the bond shear strength between the mortar and masonry, and
- (6) the coefficient of friction between the masonry and mortar.

For the case of uniaxial compression, Hilsdorf⁽²³⁾ and Rao⁽²⁴⁾ have presented some useful results. Hilsdorf describes the failure criterion for brick masonry in the form shown in fig. 3. The failure criterion for the brickwork (line A) can be expressed as

$$\sigma_x = f_{bt}^1 \left[1 - \frac{\sigma_y}{f_b^1} \right]$$

where σ_x, σ_y = stresses in orthogonal directions

f_b^1 = uniaxial compressive strength of brick,

and f_{bt}^1 = biaxial tensile strength.

From his experimental work, Rao obtained typical stress-strain curves that show the tangent modulus for brick prisms to increase with increasing stress before reaching a constant value. As the yield stress is approached, the modulus decreases with further increase in stress. This change appears to be caused by the modulus being dependent initially on the properties of both brick and mortar but as the

stress increases the tangent modulus is more nearly that of the brick alone.

Sinha and Hendry⁽²⁵⁾ found that for a wall acted on by a racking load, the effective rigidity and shearing modulus decrease non-linearly with increase in racking load. In addition, these properties and the shear strength were found to increase with precompression on the wall. They show that the shear strength of the brickwork can be computed from the expression

$$f_s = f_{bs} + \mu \cdot f_n$$

where f_{bs} = bond shear strength of the brickwork,

μ = coefficient of friction at brick/mortar interface, and

f_n = normal compressive stress.

Providing suitable values can be obtained experimentally for the various parameters governing the strength of masonry, it should be possible to include these in further finite element analyses. This will be an improvement of the assumption, often made in analyses to date, that masonry can be "described" as a weak concrete-like material.

6.0. Interaction Studies

For this study, the panel was taken to be similar to one of those tested by Stafford Smith⁽⁸⁾, being 12" x 8" x $\frac{3}{4}$ " surrounded by a frame of $\frac{3}{4}$ " x $\frac{3}{4}$ " section. In this way it was possible to compare theoretical results with some experimental results. The finite element idealisation of the frame, panel and tielinks is shown in fig. 4. A flowchart for the computer program used is given in the Appendix.

6.1 Choice of Mesh Layout

The mesh chosen to idealise the panel was basically a 4 x 4 grid comprising 32 triangular elements. However, since the ends of the compression diagonal of the panel were known to be subjected to a greater compressive stress than the remainder of the panel, it was decided to use smaller elements in these regions than elsewhere. This produced an idealisation using 36 elements. Also utilised were five tielinks on each side of the panel and on the top. This appears to give a reasonable representation of the interaction between the frame and the panel.

While the use of a finer mesh should provide an even better representation of the interaction and ultimate behaviour of the structure, this would be at the expense of increased computing time - as well as requiring more input data. Other studies are currently being undertaken to investigate the comparative stiffness of various panel idealisations so that the increased stiffness that results from using fewer finite elements can be allowed for. This should then make it possible to analyse multi-storey structures using a reasonable idealisation, at an acceptable cost.

6.2 Details of the Analysis

Several aspects of this study of frame-panel interaction are worthy of comment.

(a) Loading. In the usual type of frame and panel structure, the horizontal loads induced in the structure by earthquake forces are generally spread, to some extent at least, over the length of the panel. However, for the purposes of this study the horizontal load was taken as a concentrated force applied to one of the corner nodes. While this produces a stress concentration in the infill, it does not appear to be very significant as the beam force and displacement were almost constant along the length. While the use of a single point load fitted Stafford Smith's experimental test set-up, it also simplified the computer programming though this was not a factor that had a bearing on the choice of load application.

(b) Crushing and cracking. The magnitudes of the principal stresses in the finite elements were checked after each iteration to see if they exceeded either the tensile or compressive strength of the infill material. An output message was printed in either case. If the tensile strength of the material was exceeded, then a flag was set in the program to cause the particular element to be ignored in subsequent iterations. If the compressive stress exceeded the crushing strength, no special action was taken. Usually, the resulting reduction in the tangent modulus was sufficient to produce a redistribution of stress that kept the principal compressive stress at about the crushing level. The progress of cracking and crushing in the finite elements is given in Tables 1 and 2.

(c) Ultimate load. The ultimate load producing "failure" of the infill was considered to be reached when

- (1) the total stiffness of the structure fell to less than 10^{-7} of the initial stiffness,
 - (2) one or more nodes became detached from the structure and were free to undergo large displacement, or
 - (3) when the maximum nodal displacement during any iteration after the first exceed the maximum nodal displacement for the previous iteration by a factor of more than 2.5.
- The program terminated automatically if any one of these three criteria were fulfilled.

(d) Initial scaling of the horizontal load. The initial load on the frame-panel comprised a unit horizontal load and (possibly) a constant vertical load. The horizontal load was then incremented over successive load stages. This gave rise to curve A for the load-displacement diagram shown in fig. 5, with 19 load stages being required to reach the "failure" load.

In order to reduce the number of load stages (and hence speed up the analysis), it was decided to try some form of scaling. Felippa⁽²⁶⁾ reports using a method whereby the variable load was applied first. This load was then scaled so that the stress in the most highly stressed element just reached yield. Any constant load was then applied to the structure and a further analysis undertaken. Increments of load were measured in terms of the load causing first yield. However, this method of scaling was considered to be unsatisfactory for the study of frame-panel interaction.

Instead, it was decided to apply both the constant vertical load and a unit horizontal load to the structure. The horizontal load was then scaled so that the maximum stress in any

element was just less than the crushing strength of the infill. Since three iterations were always carried out for the first load stage, three alternatives for scaling were investigated. These were:

- (1) scale on only the first iteration,
- (2) scale on both of the first two iterations, and
- (3) scale on all three iterations.

The resulting load-displacement diagrams are shown as curves B, C and D in fig. 5. Curves A and B follow the shape of the experimental results⁽¹⁶⁾, while the ultimate loads of curves C and D are closer to the experimental failure loads⁽⁸⁾. Scaling methods 1 and 2 have been used for most of the work to date.

6.3 Effect of Vertical Loading

The load-displacement curves for a frame-panel with differing vertical loads are shown in fig. 6. Here the vertical load is expressed nondimensionally in terms of $f_c^1 l t$ where f_c^1 is the uniaxial compressive strength of the infill, l is the length, and t is the infill thickness. As the vertical load increases, the stress-strain curve becomes more curved and the elastic limit for the frame-panel is reached under a reducing horizontal load.

The relation between vertical load and horizontal strength is shown in fig. 7, where theoretical results are compared with experimental results. While the theoretical results do not appear to agree very closely with the experimental results when vertical load is present, it should be noted that Stafford Smith's experimental results for this shape of panel do not follow the same pattern as for square and longer rectangular panels. In addition, Mallick and Severn found that square panels gave a better correlation between theoretical and experimental results than did rectangular panels. Another factor contributing to the discrepancy is that the finite element idealisation makes the panel appear stiffer than it really is. The panel could be made more flexible either by using more elements or by suitably modifying the element stiffnesses. A separate study is currently being carried out to investigate the latter.

Franklin⁽¹⁹⁾ reports that for his analyses the initial horizontal load was made as large as possible without causing cracking or crushing of the infill. This trial and error form of scaling was carried out to reduce computation time. The present study shows that the effect of the scaling of the horizontal load depends on the magnitude of the horizontal load applied in the first load stage. This effect can be seen in fig. 6 where the dashed curve gives the results for cases where the magnitude of the initial horizontal load was two kips.

Fig. 8 shows typical principal stress distributions in infill panels with and without vertical load. The extent of cracking in the infill at failure under several combinations of horizontal and vertical load is shown in fig. 9. In general these agree with experimental tests in that diagonal cracking takes place under a horizontal load while vertical cracking occurs when vertical load is applied. The progress of cracking and crushing in the infill is given in Tables 1 and 2 and this corresponds

with the description given by Stafford Smith (10).

7.0. Final Remarks

The frame-panel interaction study described herein shows that computer analyses of the problem can provide useful results. Some further development of the computer program is possible and this is being undertaken. Providing that the behaviour of the infill material can be described in reasonably simple terms, any type of infill could be dealt with.

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Appendix

Schematic Flowchart for Frame-Panel Interaction Computer Program

The notes that follow relate to the lettered boxes in the accompanying flowchart.

- (A) Read and print control parameters, beam and element data. Set $I = 0$.
- (B) For tielink member, set moment of inertia = 0. Calculate stiffness matrix for beam and tielink members using slope deflection equations.
- (C) Check whether finite element is normal, cracked or crushed. Calculate elastic properties. Calculate stiffness matrix for element.
- (D) Assemble total stiffness matrix for the structure. Apply boundary conditions. Check stiffness matrix for degeneracy and set termination flag if matrix is degenerate. Invert stiffness matrix and solve for nodal displacements.
- (E) Print load stage information. Print incremental nodal displacements.
- (F) Check whether tielink element is already present. If it is, and the tielink strength has been exceeded, remove the tielink from the system. If the tielink element is not already present and the distance between the two nodes is now less than originally, add tielink to system.
- (G) Determine principal stresses. Print message if element has either crushed or cracked.
- (H) Carry out scaling if required.
- (I) Print member and element forces.
- (J) Is a further iteration to be carried out for this load stage?
- (K) Add incremental displacement to total displacement. Increment horizontal load.
- (L) Print member forces and total nodal displacements to date.
- (M) Is a further iteration to be carried out, or is a further load stage to be undertaken?
- (N) Print out nodal forces. Print out load-displacement history for reference point.

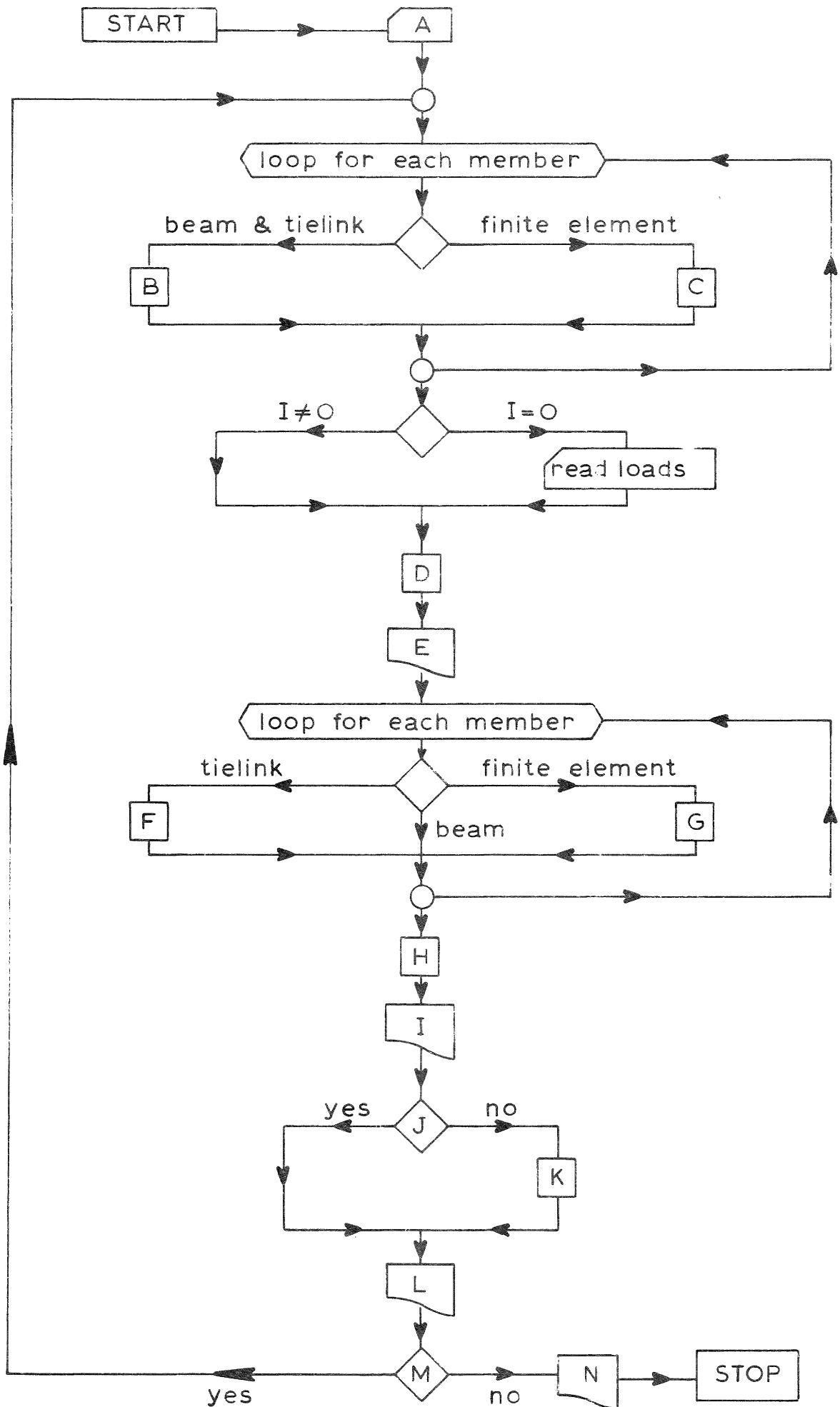


TABLE 1: The development of cracking and crushing in the panel infill (load scaling on first iteration only).

Vertical Load (kips)	Horizontal Load (kips)	Load Stage	Iteration Cycle	Number of each element which becomes cracked	Number of each element which becomes crushed
0	4.25 5.25 5.75 6.0 6.25 6.50 6.75	2 6 8 9 10 11 12	1 1 1 2 2 3 3 1 2 3	28 21 2 20 13,30 general cracking	1 35 36 2
4.875	5.25 5.50 5.75	10 11 12	2 3 1 2 3	2 3 9, 35 general cracking	35 1
9.60	5.25 5.50 5.75	12 13 14	3 2 3 1 2 3	2 9 1 general cracking and crushing	2 1 35
14.38	5.0 5.25 5.50	13 14 15	3 2 3 2 3	2 general cracking and crushing	2 1 35
19.17	5.0 5.25	14 15	2 3 2	2 8,10,16,24 general cracking and crushing	2 1
23.945	5.0	15	2 3	general cracking and crushing	8

TABLE 2: The development of cracking and crushing in the panel infill (Scaling on first two iterations).

Vertical Load (kips)	Horizontal Load (kips)	Load Stage	Iteration Cycle	Number of each element which becomes cracked	Number of each element which becomes crushed
0	5.90	2	3		35
	6.00	4	2		2, 36
			3		1
	6.25 6.5	5 6	2 2	2 general cracking	8
4.875	4.75	1	3		2
	5.50	4	1	2	
	5.75	5	2 1	3 general cracking	1
9.60	7.25	2	1	28	
			2		1,2,35
	7.50	3	3 3	2 general cracking and crushing	36
14.38	6.00	2	2		36
			3		1
	6.25	3	3		2
	6.50 6.75	4 5	3 3	2 general cracking and crushing	
19.17	4.00	1	3		36
	5.50	7	3		2,35
	5.75	8	2		1
	6.00	9	3	2 general cracking and crushing	
23.945	5.50	3	2		35
	6.25	6	2		2
			3		1
	6.50 6.75	7 8	2 2	2 general crushing and cracking	3, 4
28.76	5.50	2	1	general cracking	

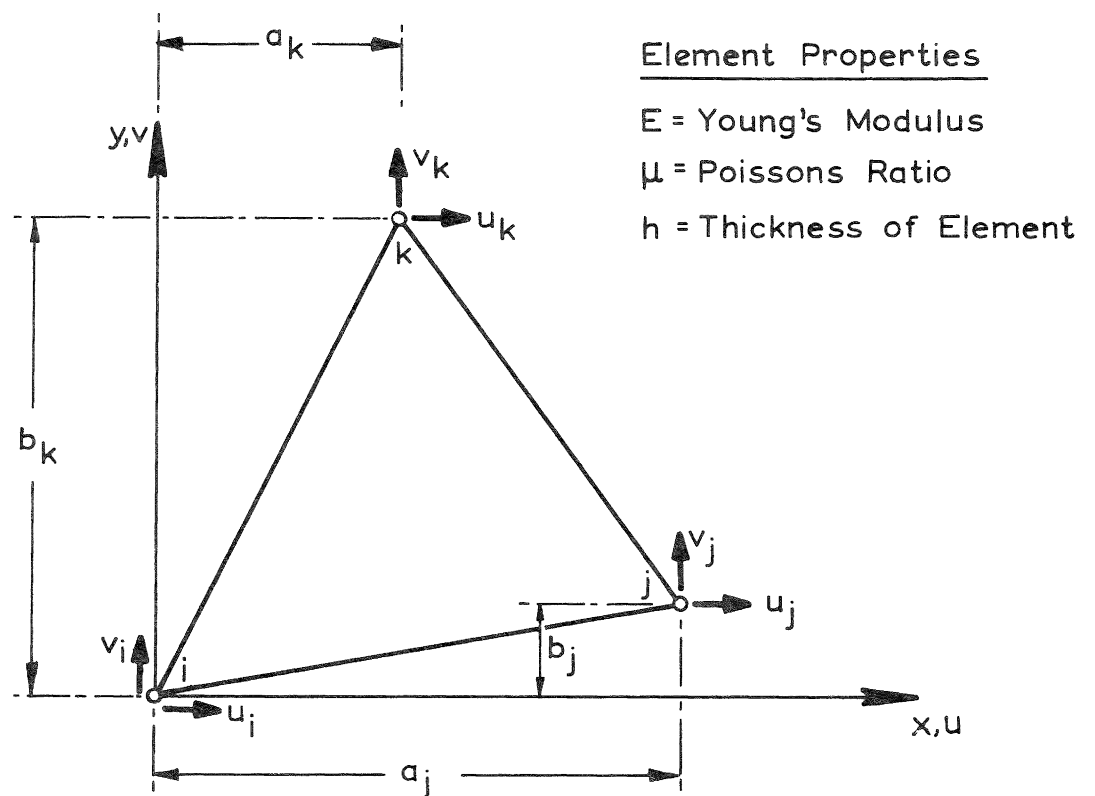
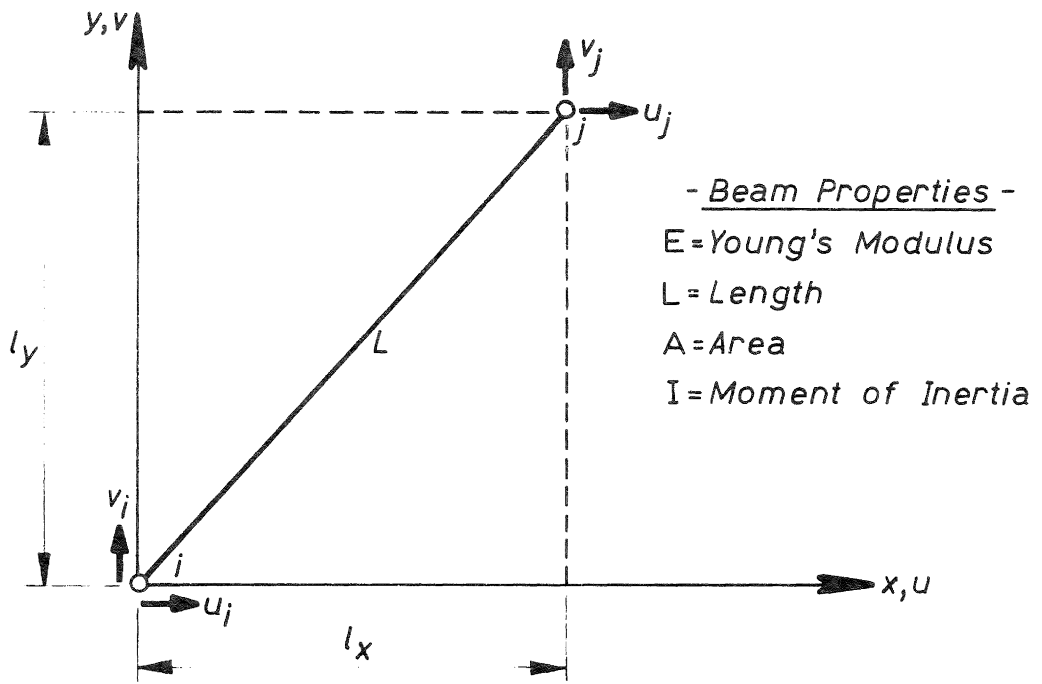
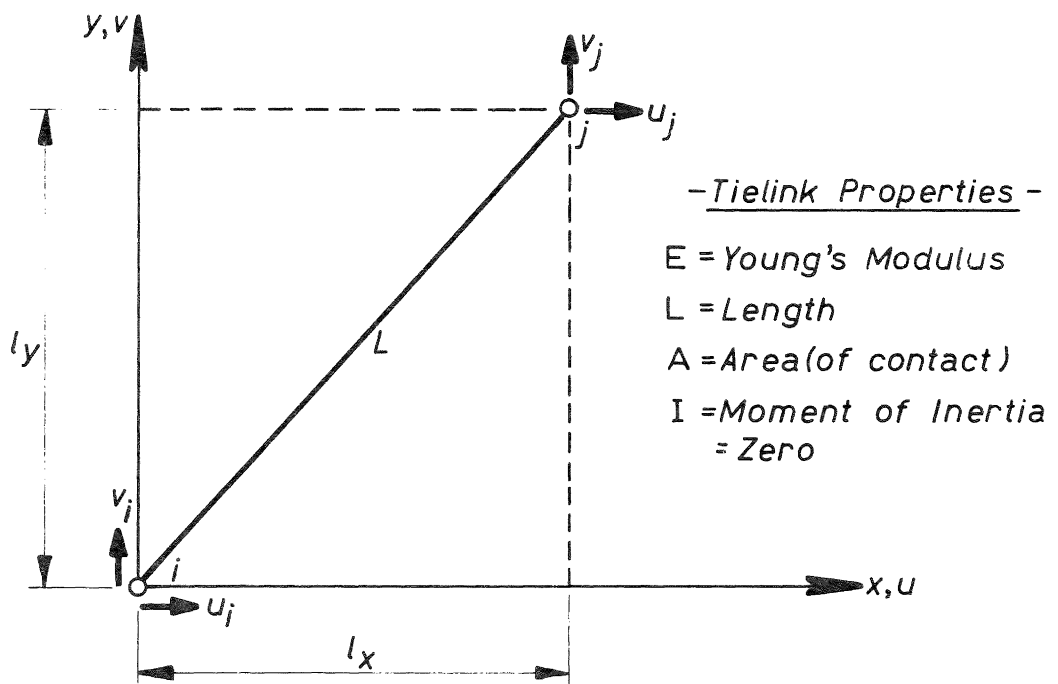


Fig.1a: Dimensions & degrees of freedom of the CONSTANT STRAIN TRIANGLE (CST) Finite Element.



1(b) Beam Element



1(c) Tielink Element

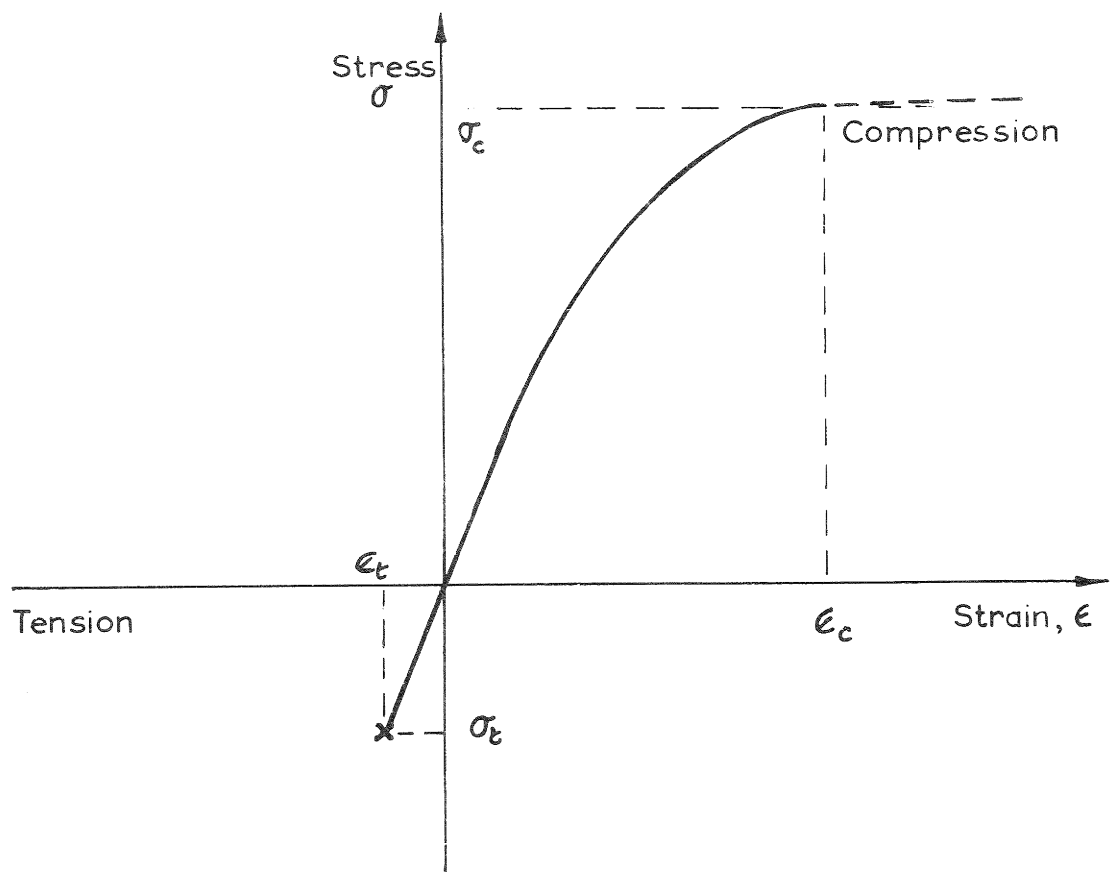


Fig.2 : The assumed stress-strain curve for the concrete in-fill

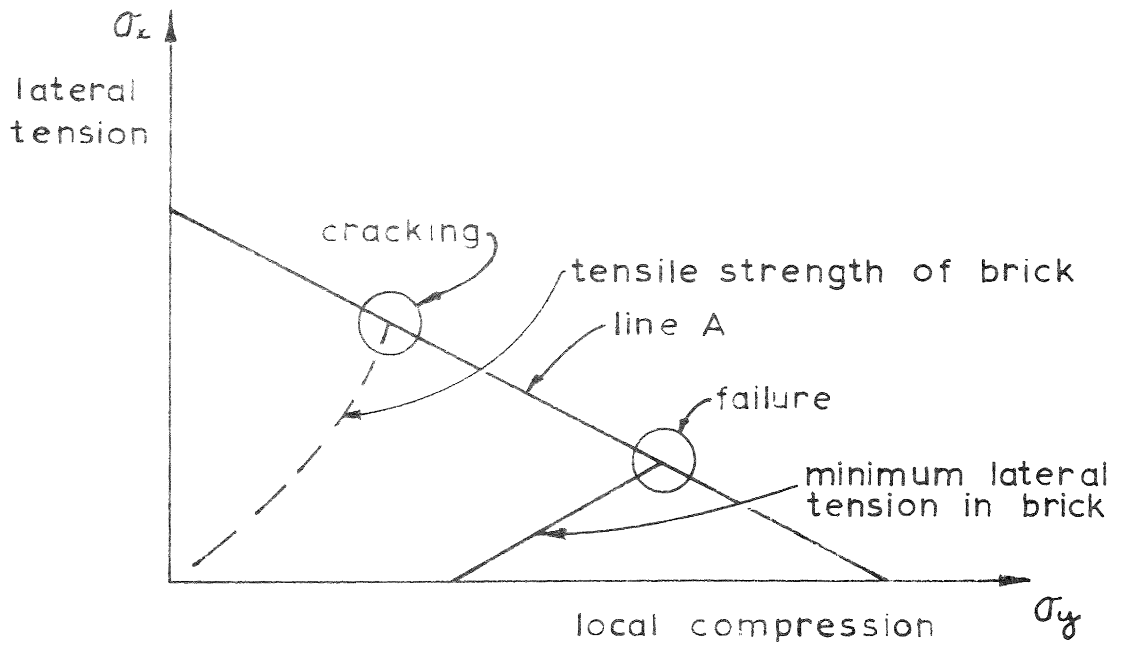


Fig.3 : Suggested failure criterion for brick masonry in uniaxial compression

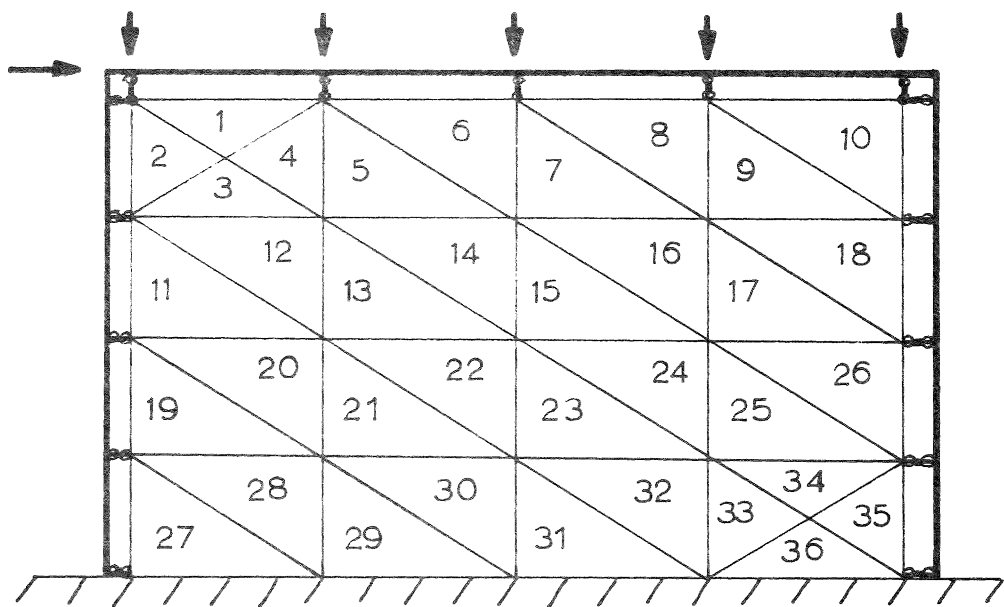


Fig.4 : Finite element idealisation of frame, tielinks and panel

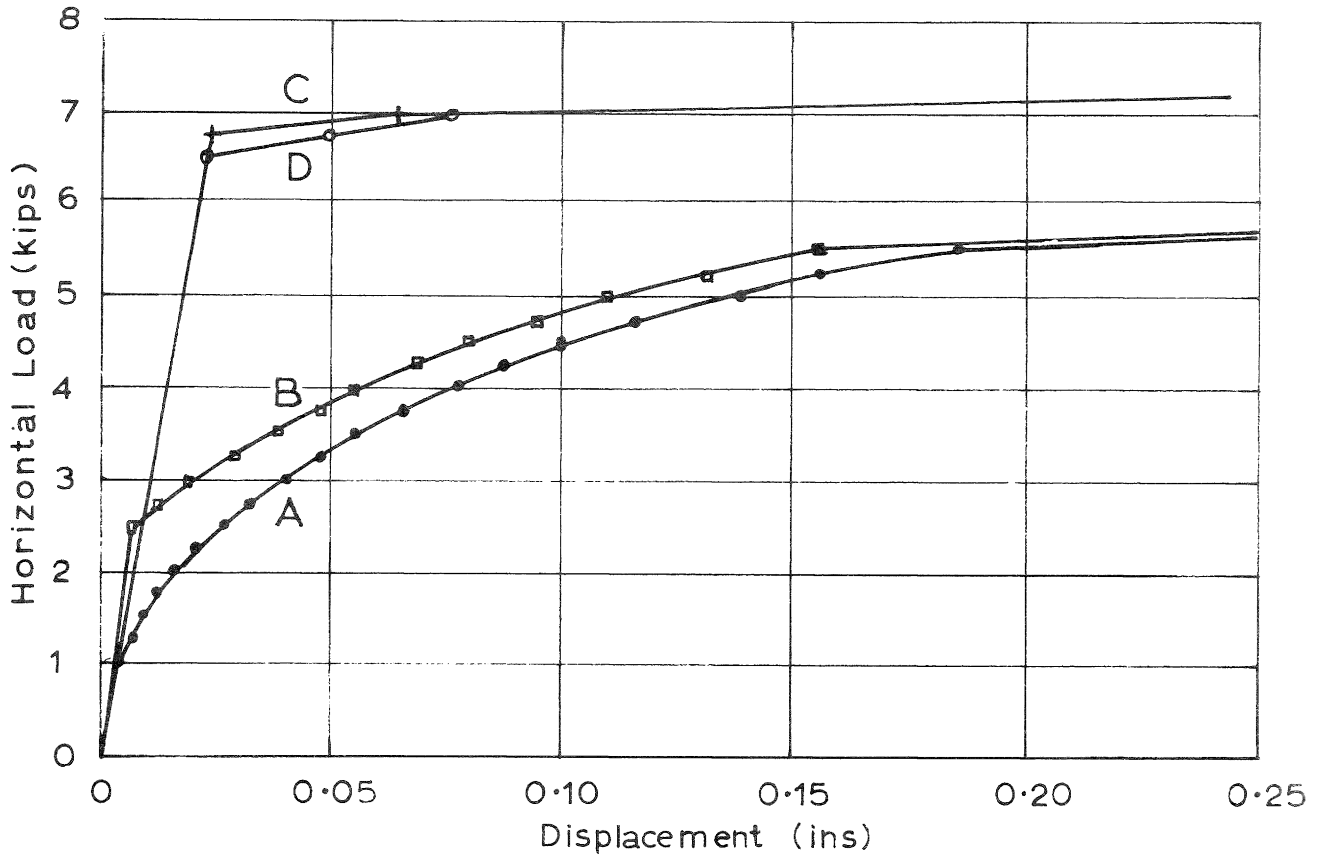


Fig. 5 : Load-displacement diagram showing the effect of different initial scaling procedure

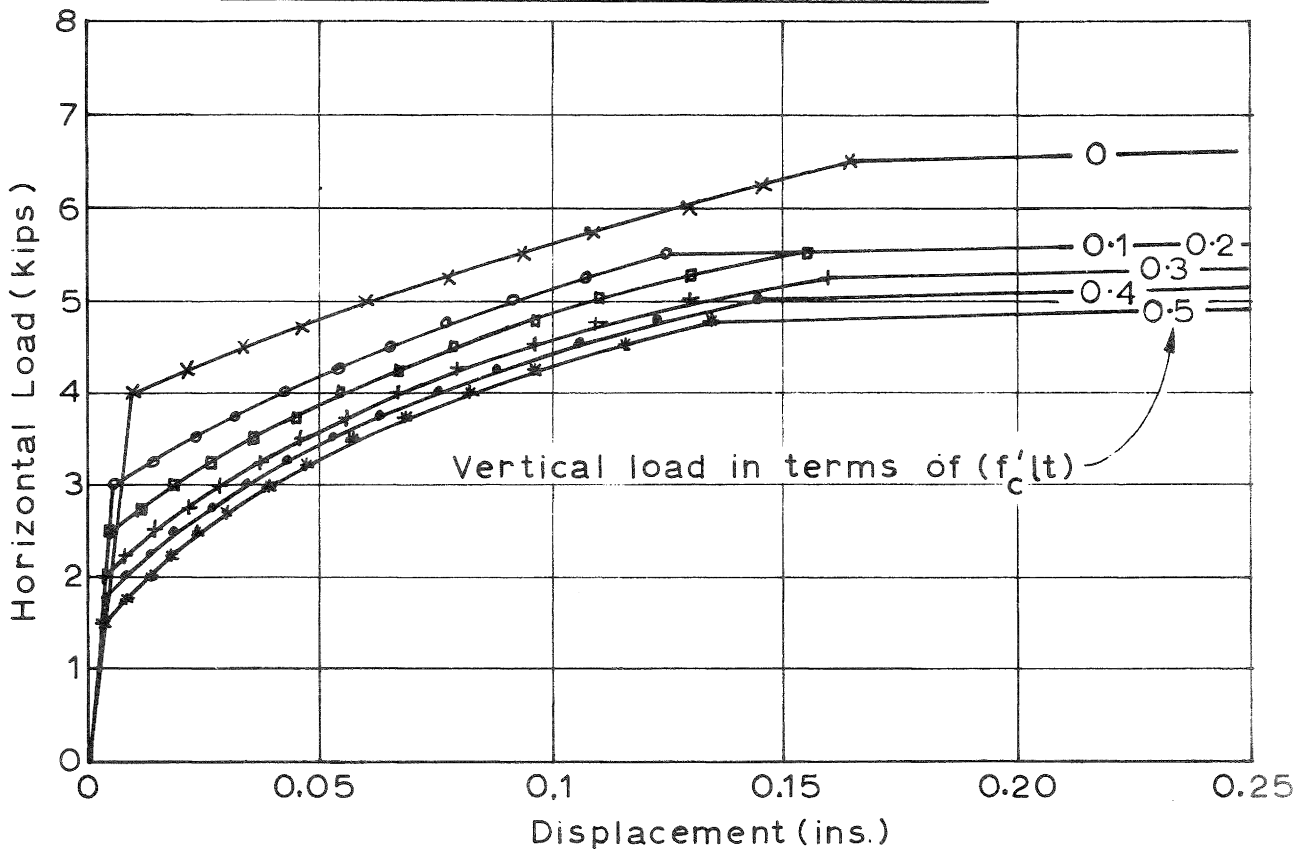


Fig. 6 : Load-displacement diagram showing the influence of vertical load

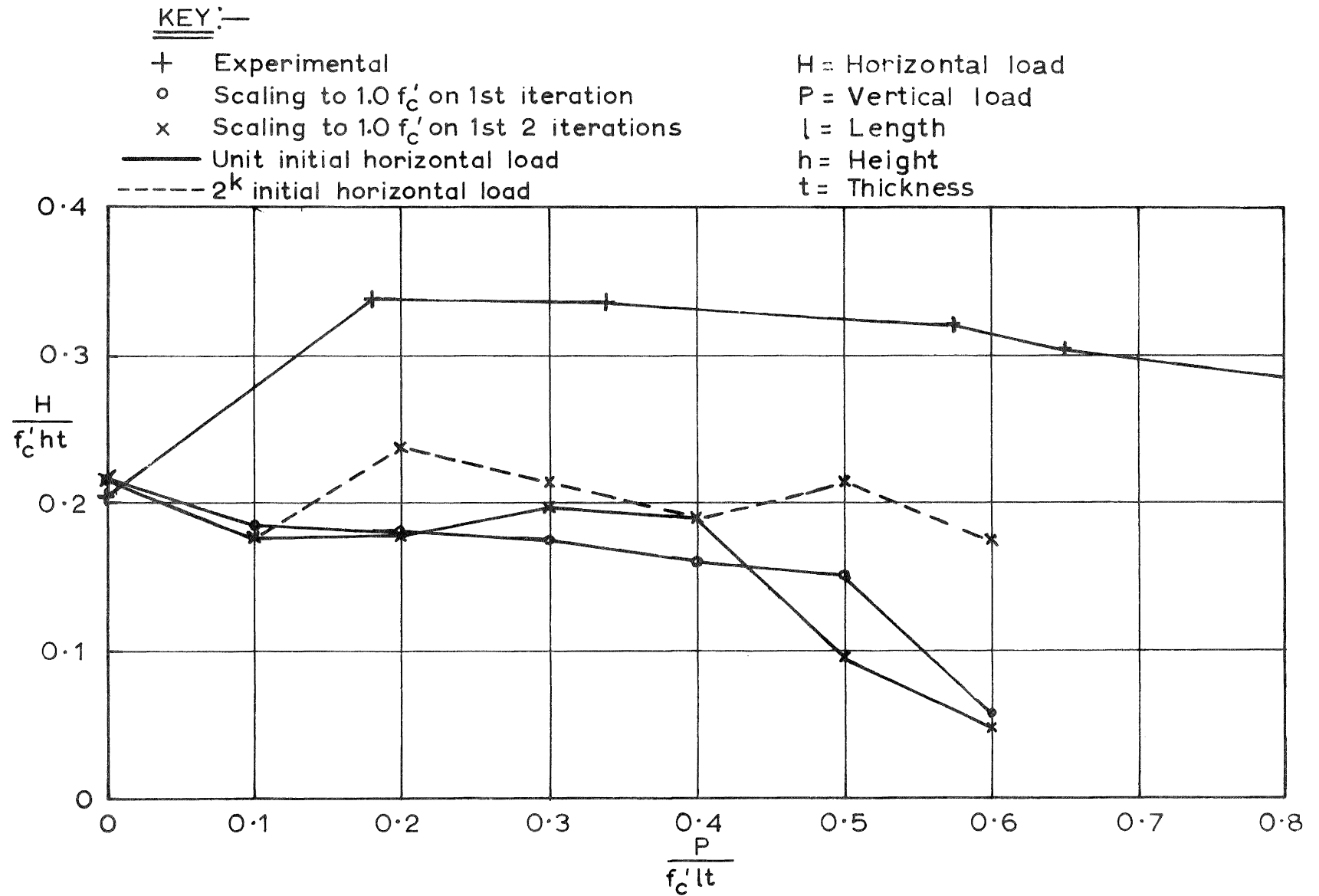
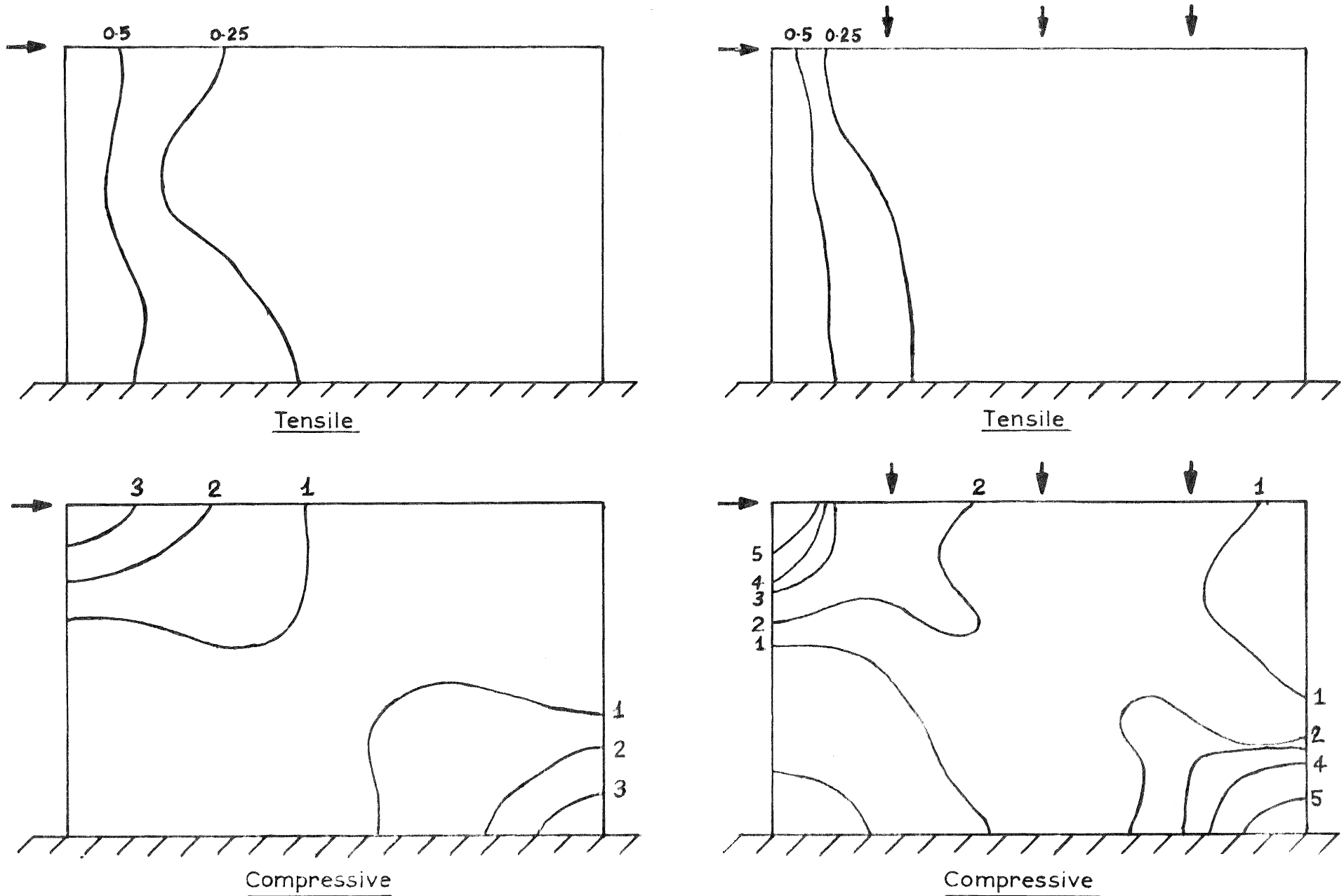


Fig.7 : Relation between vertical load & horizontal strength



(a) Horizontal Loading Only

(b) Horizontal & Vertical Loading

Fig.8 : Typical principal stress distribution in in-fill panels

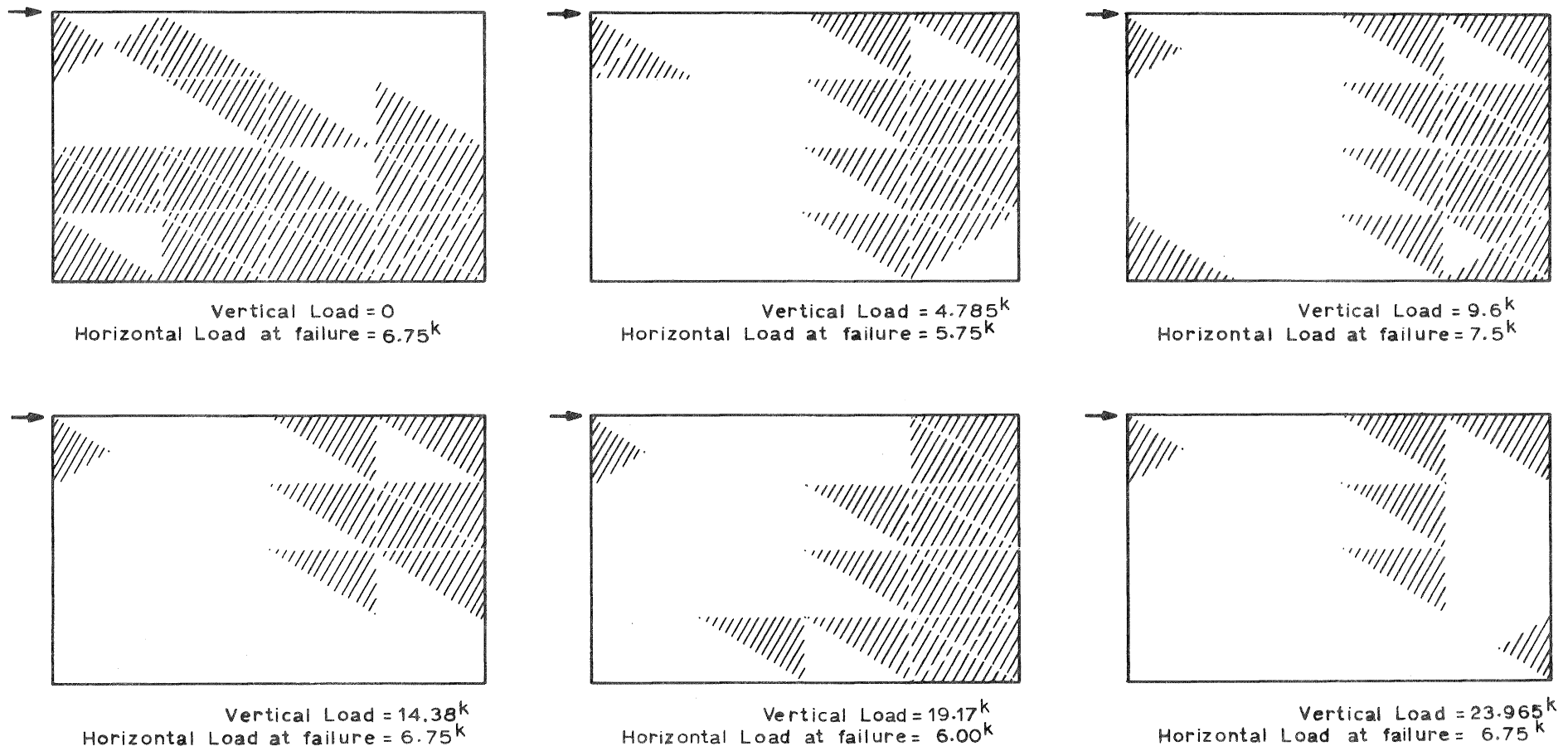


Fig.9 : Development of cracking for several combinations of vertical and horizontal load