MOMENT REDISTRIBUTION IN CONTINUOUS BEAMS OF EARTHQUAKE RESISTANT MULTISTOREY REINFORCED CONCRETE FRAMES

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ABSTRACT

To allow a more uniform and efficient distribution of load resistance in reinforced concrete frames that may be subjected to seismic excitation well beyond the range of elastic response, the concepts of moment redistribution, well known from the plastic design of steel structures, are re-examined. The aims of moment redistribution, with particular reference to earthquake loading, are stated and relevant terms are defined. To allow advantageous but statically admissible adjustments of moments, derived from an elastic analysis, to be made, the requirements of equilibrium and lateral load resistance are assembled into a set of simple rules. Certain limits for redistribution, which appear to be reasonable in the light of existing provisions for structural ductility, and yet are not too restrictive, are proposed. A technique, involving simple graphic manipulations well suited for design office use, is suggested and with the aid of an example its application is shown in detail.

1. INTRODUCTION

In the design of continuous beams of earthquake resistant reinforced concrete ductile frames the bending moments are usually derived from an elastic frame analysis. From the combination of factored gravity loads and code prescribed lateral loads, simulating earthquake effects, bending moment envelopes are constructed and are subsequently used for the determination of sectional strength properties.

The superposition of different loading cases usually results in maximum (negative) moment demands at the supports, the (positive) moments, near the centre of the spans, being considerably smaller. These uneven moment demands often do not permit the full strength utilisation of the structural depth in prismatic members. Moreover the disproportionate steel demand may result in congestion of reinforcement at column-beam junctions, which inherently present construction difficulties.

The allocation of reinforcement to the bottom and the top of a beam at either side of a column, in accordance with the elastic moment envelope, may be such that excessive moments may be generated in that column when, due to seismic motions, large displacements result in plastic hinges at these locations. Actual earthquake induced moment input from beams into columns may be considerably in excess of what is required for the code specified lateral loading. If columns are to be protected against unexpected yielding they have to be proportioned accordingly so that they can satisfactorily meet this excess strength demand. Undesirable increase of potential beam strengths, to be fully developed only during large seismic excitations, can result in cases when the beam moment demands, obtained from the superimposed results of elastic analyses, are different at the two faces of an interior column. Designers are often reluctant to change the top beam reinforcement at these sections which are very close to each other. Consequently an unnecessary flexural capacity increase results which, in accordance with the philosophy of capacity design(1), will need to be followed up by providing matching column capacity.

The following sections set out the principles of moment redistribution which, when used with certain restrictions, will usually alleviate these problems and will result in a more even and economical steel demand. The basic concepts used will be familiar to all designers. The advantages of the results are well known, particularly in the design of continuous steel beams and industrial frames.

2. THE AIMS OF MOMENT REDISTRIBUTION

In the efficient design of reinforced concrete continuous beams of frames there are three aims that the designer should attempt to achieve:

2.1 - Reduce the absolute maximum moment, usually in the negative moment region, and compensate for this by increasing the moments in the non-critical (usually positive) moment regions. Thus a better distribution of strength demand is achieved, particularly along prismatic members.

2.2 - Equalise the critical moment demands in beams at either side of an interior column. This will obviate the necessity of having to terminate and anchor beam bars at interior beam-column joints where a congestion of reinforcement commonly presents construction difficulties.

2.3 - Fully utilise the potential positive moment capacity of beam sections at column faces, where this must be at least one half of the negative moment capacity at the same section(2). This aim can be extended and combined with that listed in 2.1 so as to

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approach equal positive and negative moment demands at the same section. It would allow the full utilisation of sections with equal top and bottom flexural steel content.

3. DEFINITIONS

In order to enable simple rules to be established, certain definitions, to be used subsequently, need to be made. The purpose of these rules is to ensure that the requirements of equilibrium are satisfied at all stages and that the specified minimum strength for lateral loading is maintained. The definitions are illustrated with reference to an example structure shown in Fig. 1.

3.1 - The Base Structure results from the removal at convenient locations of an adequate number of static redundancies from the real structure so as to make it statically determinate. A base structure of the frame shown in Fig. 1(a) is that given in Fig. 1(b).

3.2 - M^o defines the bending moment resulting from gravity load only on the base structure. An example is given in Fig. 1(c), where for convenience the magnitudes at midspans are recorded in moment units.

3.3 - M^i defines the moments due to continuity, when gravity load only is applied to the real structure. This is shown for the example beam in Fig. 1(d). The two moments, M^o and M^i, just defined are simply the commonly known components of the total bending moment obtained with an elastic analysis for the original structure, shown in Fig. 1(a). It is evident that the total moment for gravity load is $M^g = M^o + M^i$ as given in Fig. 4(e).

3.4 - ΔM^i is the moment introduced to the continuous beam at a column-beam joint by a column. These are the moment increments at A, B and C, specially labelled in Fig. 1(d). ΔM^i is a component of M^i.

3.5 - M^E defines the moments at any point along the beam introduced by earthquake induced shear forces in the columns at the code specified load level. For example a set of shear forces acting from the left to the right at the top column hinges at A, B and C will produce the moments shown by the full lines in Fig. 2(a). Similarly a reversed load will give the dashed line bending moment pattern. To identify more easily the direction of earthquake action with the resulting bending moments, the symbols $\vec{M}^E$ and $\overset{\leftarrow}{M}^E$ are used.

3.6 - ΔM^E is the moment applied to the continuous beam at a column-beam joint by the column. These are the moment increments at A, B and C, specially labelled in Fig. 2(b). The arrow indicates the causative earthquake shear forces acting at the tops of the upper columns.

4. THE REQUIREMENTS OF EQUILIBRIUM

For the purpose of gravity load analysis of multistorey frames it is customary to consider isolated subframes only. Such is shown in Fig. 1(a). This subframe is then subjected to gravity load only so that it carries no external horizontal forces. However, because in the analysis no lateral sway is assumed to take place while the subframe is subjected to the design gravity load, the horizontal reactions at the top and bottom column hinges will normally not add up to zero. Consequently an additional force, labelled X in Fig. 1(a), will be required to prevent lateral movement (sway) of the beam and hence to satisfy the horizontal force equilibrium for the frame subjected to vertical loading only. In typical frames the horizontal force X is small, particularly in comparison with the horizontal earthquake induced forces. For convenience it may be assumed that the horizontal forces induced by gravity load, as shown in Fig. 1(a), are maintained throughout the subsequent loading of the frame.

To maintain equilibrium at all stages of the proposed moment redistribution it is essential to satisfy the following criteria:

4.1 - $\Sigma \Delta M^i$ = constant, because the small restraining force X, shown in Fig. 1(a), is assumed to be maintained throughout the combined loadings. If in the process of moment redistribution any one of the moment increments, ΔM^i, is changed, a compensating change must be made in the other moment increments along the beam. For example ΔM_B^i may be eliminated in the example of Fig. 1(d) but then ΔM_A^i must be increased or ΔM_C^i decreased by an equal amount so as to satisfy the above equilibrium criterion. This statically admissible procedure involves moment redistribution between columns and in the beam. The quantities in question are usually relatively small. In the example chosen $\Sigma \Delta M^i = 0$.

4.2 - The statically indeterminate moments due to gravity load, M^i, for any span of the beam shown in Fig. 1(d), may be freely altered. To observe the laws of equilibrium, however, the bending moment between two ends of any span must for obvious reasons vary linearly. This involves moment redistribution in the beam only and results in consequent changes in the vertical reactions.

4.3 - The earthquake induced shear across the columns above, $\Sigma V'$, and below the floor, ΣV, represents the lateral load demand on the subframe and it can be expressed for the frame of Fig. 2 in terms of beam moments as follows:

$$\sum_{A}^{C} \left(h_1 V' + h_2 V \right) = \sum_{A}^{C} \Delta M^E$$

Therefore the lateral load resistance of the subframe is not affected, and consequently the requirements of equilibrium are not violated, if individual moment increments associated with a particular direction of the lateral loading, such as $\Delta \vec{M}^E$, are altered, provided that the sum of the increments remains the same. Such a change means redistribution of moments between columns and in the beam.

The relative redistribution of the two components of the moment increment ΔM_B^E at B, shown in Fig. 2(b), between the two adjacent beams may be freely altered without affecting the resistance of the subframe.

This involves moment redistribution in the beam only.

Moment redistribution between columns means redistribution of the shear forces, such as V' and V shown in Fig. 2(a), between individual columns.

5. THE LIMITS OF MOMENT REDISTRIBUTION

Moment redistribution within the subframe, combined with sustained load carrying capacity, is only possible if adequate rotational ductility is available. One of the most important aims of our present seismic design philosophy is to provide for ample ductility which may be called upon to develop during very large earthquakes. For reasons well known, the rotational ductility is primarily to be built into the beams of the frames. The ductility of columns is relied upon only in extreme lateral load conditions. Consequently the proposed redistribution process relies entirely on moment redistribution in beams. The redistribution of moments or shear forces between individual columns must also rely entirely on hinge rotations within the beams. No inelastic deformations of any kind are expected to occur within any of the columns while the moment redistribution takes place.

Moment redistribution in a frame may alter slightly the ultimate ductility demand of potential plastic beam hinges by increasing the demand in some hinges and at the same time decreasing it in others. The total ductility demand for the structure as a whole remains unaltered. It may be said that the average ductility demand in the localities of potential energy dissipation does not change, and that, with limited redistribution during the design process, the deviation from the average ductility demand during a very large earthquake will be very small.

It is therefore proposed here that in beams of ductile earthquake resistant frames the magnitude of moment to be redistributed, ΔM, should be limited as follows:

5.1 - In any span of a beam ΔM should not exceed at any point 30% of the absolute maximum moment derived for that span from elastic analyses for any combination of earthquake and factored gravity load.

5.2 - Moment redistribution between columns should not change the maximum value of the combined end moments in any column, derived from elastic analyses for any of the load combinations referred to in 5.1 by more than ±15%. This limitation is satisfied if the redistribution of shear forces between columns is limited to ±15% of the shear force acting on the column in question.

The latter limit may appear excessive. However, it must be remembered that errors introduced in lateral load analyses and lateral load definitions are of larger order. Moreover, such column moment changes may be considered to be adequately absorbed by the overstrength, dynamic magnification or capacity reduction factors, which are subsequently used in arriving at the desired ideal column flexural strength.

6. A TECHNIQUE OF MOMENT REDISTRIBUTION AND AN ILLUSTRATIVE EXAMPLE

When moments are being redistributed the designer will usually manipulate with bending moment diagrams rather than use an arithmetical process to achieve the aims, as set out in section 2, without violating the requirements of equilibrium or exceeding the specified limits of redistribution. Traditionally moment diagrams of various shapes are plotted on a horizontal base line. The curves are then raised or lowered to optimise intercepts with the base line. In a trial and error graphic procedure, which is required in the case of the subframe shown in Fig. 1, it is much easier to adjust straight lines only. Hence it is suggested that the curves, resulting from gravity loading only, be used as immovable base lines. It will be noted that the moments M^i and M^E, defined in section 3 and shown in Figs. 1(d) and 2(b), may be changed within the limits of the laws of equilibrium and that they consist of straight lines only. Therefore they are best suited for convenient graphic manipulation. Accordingly a moment redistribution technique is outlined in the following steps, each stage being illustrated with the aid of the example structure given in Fig. 1.

To illustrate structural behaviour associated with a particular step of moment redistribution, first the more conventional form of bending moment patterns will be used in combination with a simple arithmetical control, set out in tabular form. This is best done by assigning numerical values for the moment ordinates of the example structure.

It is assumed that for the subframe shown in Fig. 1 the gravity moments have been evaluated for the load combination of $(D + 1.3L_R)$, where D represents dead load and L_R is the reduced live load(1). The critical ordinates of the moment components M^o and M^i are recorded in Figs. 1(c) and 1(d).

For the design load combination of $U = E + D + 1.3L_R$ the terminal beam moments recorded in Figs. 1 and 2 can be superimposed as follows:

(a) For $\vec{M}^E$ actions at locations

A : $M_{AB} = -20 + 100 = 80$ units

B : $M_{BA} = 60 + 110 = 170$ units

$M_{BC} = -80 + 80 = 0$ units

C : $M_{CB} = 40 + 80 = 120$ units

$\Sigma = 370$ units

Note that according to the sign convention used here moments applied to the ends of a beam in any span are considered positive if they have a clockwise rotating effect.

The gravity load moment pattern in the chosen example subframe is such that $\Sigma\Delta M^i = - 20 - 20 + 40 = 0$.

(b) For $\overleftarrow{M}^E$ actions the terminal beam moments are as follows. At locations

$$\begin{aligned} A : \quad M_{AB} &= -20 - 100 = -120 \text{ units} \\ B : \quad M_{BA} &= 60 - 110 = -50 \text{ units} \\ M_{BC} &= -80 - 80 = -160 \text{ units} \\ C : \quad M_{CB} &= 40 - 80 = -40 \text{ units} \\ \Sigma &= -370 \text{ units} \end{aligned}$$

It is seen that in both cases $\Sigma\Delta M^E$ = 370 units. This represents the strength requirement for lateral earthquake loading.

These terminal moments are combined with the M^o requirement (Fig. 1(c)) for gravity load, representing the results of the elastic analysis, and are shown by the thin full lines in Figs. 3(a) and 3(b).

To enable the moment redistribution to be followed and checked readily, the manipulations affecting terminal moments, are also recorded in Table 1.

In accordance with Section 5.1 the maximum moment that could be redistributed in this beam is for

Case (a) $\Delta M_{max} = 0.3 \times 170 = 51$ units (Fig. 3(a))

Case (b) $\Delta M_{max} = 0.3 \times 160 = 48$ units (Fig. 3(b))

The maximum moment that could be redistributed between columns is, in accordance with section 5.2 at

Column A $M_{max} = 0.15 \times 120 = 18$ units

Column B $M_{max} = 0.15\ (160 + 50) = 31.5 \simeq 32$ units

Column C $M_{max} = 0.15 \times 120 = 18$ units

These values are recorded in line (1) of Table 1 to remind us that they are not to be exceeded. The terminal moments computed previously are recorded in line (2) for Case (a) and in line (3) of Table 1 for Case (b).

A comparison of the bending moment diagrams in Fig. 3, that resulted from the elastic analyses, reveals that it would be desirable

(i) to reduce the value of the negative moments at B and to make them similar for both load cases to ensure identical top beam steel requirements at the central column.

(ii) to reduce the negative moments at the exterior columns and to make them similar to the positive moment demands at the same sections. This would approach the case when equal top and bottom beam steel might be used.

To achieve these aims the moment redistribution can be carried out in the following steps:

Step 1 : Reduce the negative moment at BA in case (a) from 170 to 122 units by redistributing 48 units to section BC. This is shown in line (4) of Table 1. The resulting moments for the beam are given by the dashed lines in Fig. 3(a).

Step 2 : Redistribute 38 moment units in case (b) to reduce the maximum moment of 160 units to 122 units at BC. This can be done by increasing the moment at BA to 88 units. This is done in line (5) and is shown in Fig. 3(b) by the dashed lines. Note that in both of the above cases moments in the beams have been redistributed from one side of column B to its other side.

Step 3 : For load case (a) the maximum moment at CB can be reduced by 18 units. If this moment is redistributed to Section BC it might result in excessive midspan moments in span B-C. For obvious reasons we would not want to increase the negative moment at B. Thus the best solution is to distribute ΔM = 18 units from column B to column A as shown in line (6). The resulting bending moments are shown by the heavy full lines in Fig. 3(a).

Step 4 : Similar considerations indicate the advantage of redistributing 18 moment units from column A to column C for load case (b). This is recorded in line (7) of Table 1 and by the full line moments in Fig. 3(b).

Step 5 : No further improvements appear to be warranted and hence the final moments are obtained in line (8) and (9) for the two loading cases respectively. As a check the sums of the terminal moments are computed to show that the lateral load requirements for a total of 370 moment units have not been inadvertently altered.

It is evident that to illustrate the distribution procedure a number of curves had to be drawn. The manipulations required are, however, greatly simplified if the M^o moments, shown in Fig. 1(c), are taken as the base and the linearly varying moments, due to beam terminal actions, are varied only. This procedure is illustrated in Fig. 4. To allow a direct graphical addition of moments, the sign conventions shown in Figs. 1(c) and 1(d) need to be followed. Accordingly all moments below the curved base line will cause tension in the top fibres of the beam and conversely moments above the base line will cause sagging in the beam.

Each step of the moment redistribution, carried out previously and recorded in Table 1, is shown by an appropriate straight line in Fig. 4(a). The circled numbers refer to the operation recorded in the identically numbered line of Table 1. The moments associated with $\overleftarrow{M}^E$ actions are shown by full lines and those due to $\overrightarrow{M}^E$ are in dashed lines. The end result is reproduced, for the sake of clarity only, in Fig. 4(b). A comparison with the steps involved in Fig. 3 shows the simplicity of the recommended distribution technique.

When earthquake actions are to be considered together with minimum gravity load effect, i.e. the combination (E + 0.9D), the advantages of moment redistribution can be readily established. The terminal moments due to gravity load only (0.9D) will be proportionally smaller than those obtained for the case of $(D + 1.3L_R)$, i.e. those shown in Fig. 1(d). Consequently somewhat smaller moment redistribution is involved in arriving at the terminal moments for the loading case (0.9D + E). However, these will be exactly the same as those shown in Fig. 4 because their magnitudes have been determined by earthquake load requirements, i.e. $\Sigma\Delta M = 370$ units. It is therefore permissible to add to Fig. 4(b) a new base line corresponding with the M^o moments for reduced dead load only (0.9D). These are shown by the heavy dashed lines in Fig. 4(b).

It is seen that the second load combination does not affect critical design quantities. It only alters the lengths over which the top beam reinforcement must be extended. This, however, is seldom a critical item because of code requirements(2) for the minimum top steel that must be carried through the entire span.

The positive and negative moment envelopes for all combinations of specified gravity and earthquake loadings are appropriately shaded in Fig. 4(b).

Using a similar but much simpler procedure the design moments for the case of gravity load only, i.e. $(1.4D + 1.7L_R)$, can be obtained. This is not shown here. For the purpose of this study terminal beam moments at column centre lines were considered. In the actual design process the beam moments at column faces would dictate the desirable magnitude of moment redistribution.

7. SUMMARY

A technique of moment redistribution for continuous beams of ductile earthquake resistant multistorey reinforced concrete frames was proposed. The purpose of moment redistribution is to reduce peak moment demands along a beam to equalise, if possible, moments at critical sections situated at either side of columns, and to make better use of the bottom beam reinforcement at column faces. More even distribution of reinforcement and easing of steel congestion at critical sections are additional benefits to be gained.

The proposal rests on the precept that in members of reinforced concrete frames, that are designed and detailed for ductility so as to meet the demands of the largest expected seismic motions, ample capacities for plastic deformations are available to allow moment redistributions of the order suggested to take place without affecting the overall structural response for energy dissipation. It is suggested that the plastic rotations associated with the proposed redistribution of moments are negligible in comparison with similar rotations that would occur as a consequence of a response corresponding with an overall displacement ductility factor of four.

The technique lends itself to graphic manipulations leading to the construction of design moment envelopes for various load combinations.

One of the major advantages of moment redistribution is that the moment demand on columns, to be mobilised during a very large seismic disturbance, is not unnecessarily increased beyond the basic strength requirements stipulated by the loading code(1).

8. ACKNOWLEDGEMENT

The preparation of these suggestions was stimulated by a series of discussions initiated by the New Zealand National Society for Earthquake Engineering in its endeavours to assist in the clarification of numerous issues related to a seismic structural design philosophy in New Zealand.

9. REFERENCES

(1) New Zealand Standard 4203:1976, Code of Practice for General Structural Design and Design Loadings for Buildings, Standards Association of New Zealand, 80pp.

(2) ACI Committee 318 "Building Code Requirements for Reinforced Concrete (ACI 318-71)", American Concrete Institute, Detroit, 1971, 78pp.

TABLE 1

MOMENT REDISTRIBUTION FOR SUBFRAME

(9)	- 102	- 88	- 122	- 58	$\Sigma = -370$
(7)	18			- 18	
(5)		- 38	38		
(3)	- 120	- 50	- 160	- 40	$\overleftarrow{M}^E$
	AB	BA	BC	CD	
(1)	(18)	(32)		(18)	ΔM_{max}
(2)	80	170	0	120	$\overrightarrow{M}^E$
(4)		- 48	48		
(6)	18			- 18	
(8)	88	122	48	102	$\Sigma = 370$

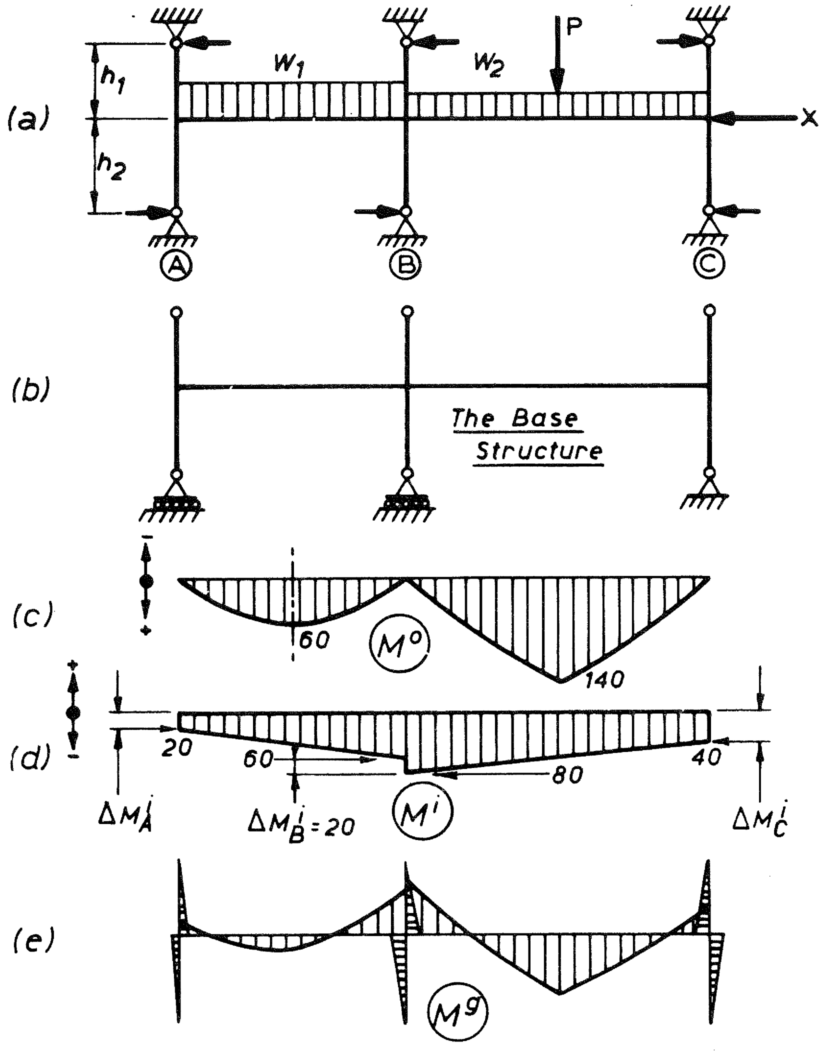

FIGURE 1: BENDING MOMENTS INDUCED IN THE REAL STRUCTURE AND ITS BASE STRUCTURE BY GRAVITY LOADING.

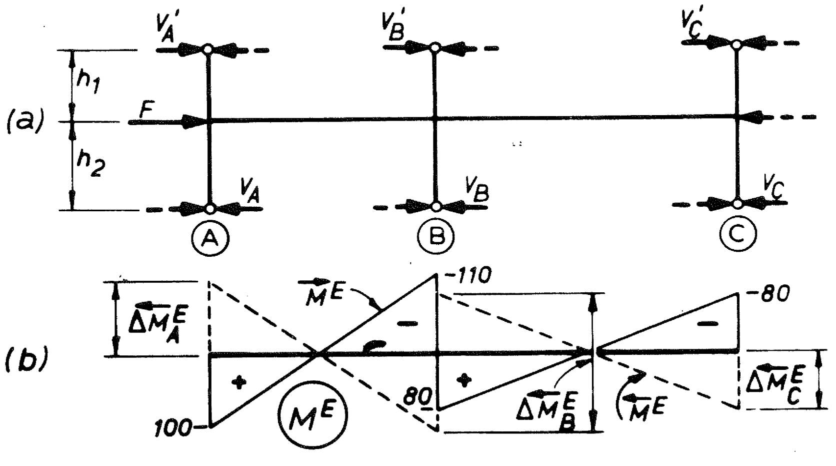

FIGURE 2: EARTHQUAKE LOADING ON THE SUBFRAME.

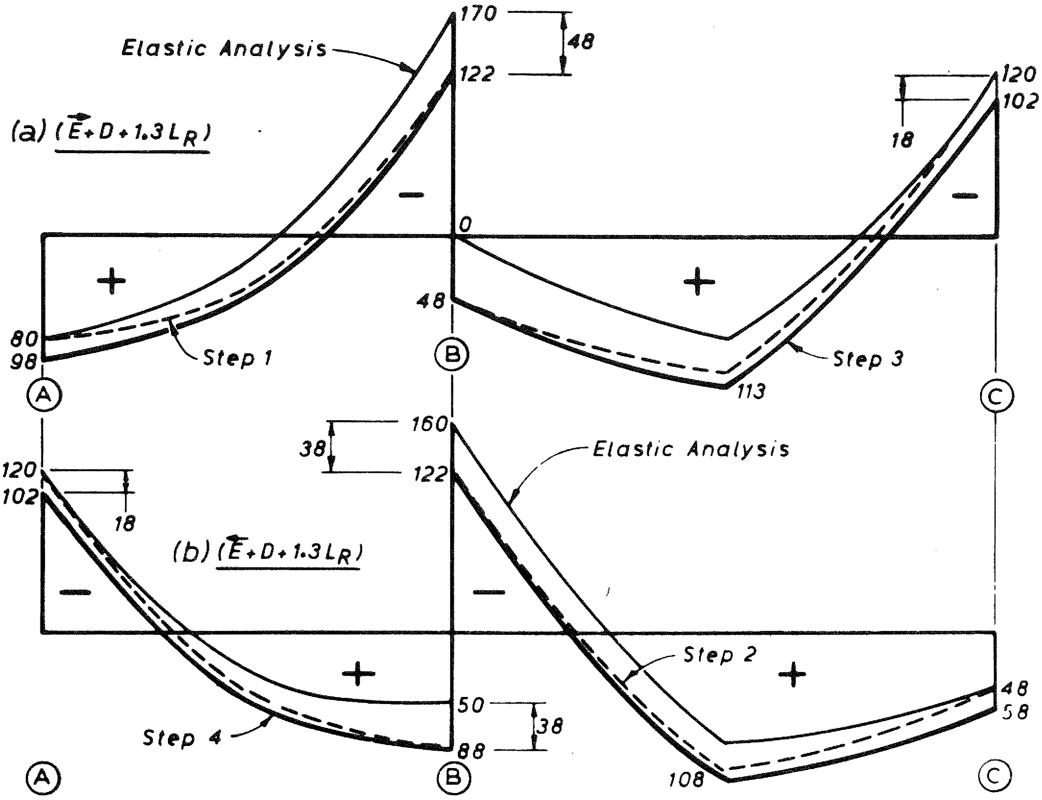

FIGURE 3: MOMENT REDISTRIBUTION FOR TWO LOADING CASES USING HORIZONTAL BASE LINES.

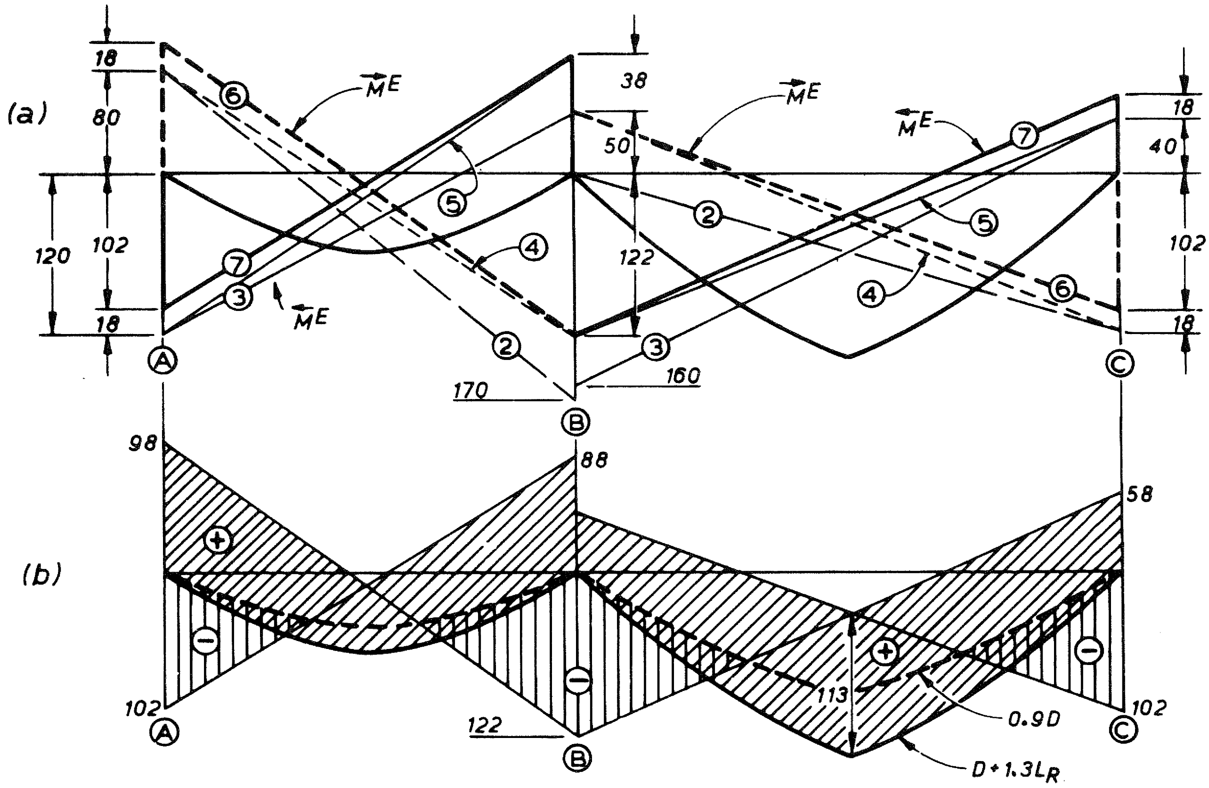

FIGURE 4: MOMENT REDISTRIBUTION FOR FOUR LOADING CASES USING CURVED BASE LINES.