

BIAXIAL FLEXURE AND AXIAL LOAD INTERACTION IN SHORT RECTANGULAR REINFORCED CONCRETE COLUMNS

D. G. Row* and T. Paulay**

1. INTRODUCTION

It was only recently that structural designers in New Zealand became aware of the need to consider earthquake actions generated concurrently with respect to the principal axes of multistorey buildings. It became evident that reinforced concrete columns adequately designed for separate participation in two frames, which are expected to resist seismic loads at right angles to each other, may in fact be subjected to pre-mature hinging when beams, framing into such columns from two directions, develop their full capacity. To ensure a desirable response of reinforced concrete frames to large seismic disturbances it will be necessary to check out each column of the structure against such skew effects. A recent study, in which the elasto-plastic response of a prototype structure to the concurrent North-South and East-West components of the 1940 El-Centro Earthquake was examined, showed this clearly⁽¹⁾. The first step in this is to assess the strength of a reinforced concrete column section, preferably with a minimum of computational effort.

The strength equations of a reinforced concrete section, subjected to biaxial flexure and axial load, can be simply developed for a given neutral axis position. However, the analysis and design of column sections is difficult because a lengthy trial and adjustment procedure is necessary to find the inclination and depth of the neutral axis.

A very convenient way to design or analyse reinforced concrete columns, subject to biaxial bending, is from interaction diagrams which make up the column's complete failure surface. Hence, interaction diagrams were developed, and are presented here, in terms of suitable dimensionless parameters, with respect to a few distinct directions of loading. By utilizing a simple interpolation method the complete failure surface can effectively be created from these particular interaction diagrams, shown in the Appendix.

2. EXISTING TECHNIQUES

Although column design charts are readily available for uniaxial loading with various reinforcement distributions, the behaviour of such sections under biaxial loading has not been abundantly documented.

In the past investigators attempted to

* Post Graduate Student, University of California, Berkeley, U.S.A.

** Reader in Civil Engineering, University of Canterbury, Christchurch.

simplify the problem of biaxial design and analysis by generalising the failure surface with some mathematical formulation. Usually these attempts relied on the knowledge of both uniaxial interaction relationships and in some cases a particular skew interaction relationship.

A significant contribution to the design and analysis for biaxial actions was made by Weber⁽²⁾. He produced charts for square columns reinforced symmetrically by 4, 8, 12 and 16 bars with a yield strength of 60 Ksi, and loaded along a diagonal. These charts have been reproduced in a handbook issued by the American Concrete Institute (ACI)⁽³⁾. These interaction charts for diagonal loading combined with the existing uniaxial interaction charts⁽³⁾, provide a useful basis for design or analysis of square columns when Weber's⁽²⁾ simple interpolation method is utilised. Examples of the use of his charts and his interpolation method have been published^(2,3,4). It will be shown later in this paper that a simple modification of the parameters in Weber's charts make them applicable to rectangular columns of all aspect ratios for loading along a diagonal.

Design charts for rectangular sections containing 16 or more bars subject to compression and biaxial bending were published by Brettle and Warner⁽⁵⁾. Their charts had to be produced for particular aspect ratios and hence the number of charts required to cover all cases of rectangular columns would be very great.

A detailed study of biaxial flexure and compression was also recently published by the European Concrete Committee^(6,7).

3. THE DEVELOPMENT OF INTERACTION RELATIONSHIPS

3.1 Assumptions Used

These charts presented in this paper rest on the following assumptions:

(a) The applicable conditions of equilibrium and compatibility of linearly varying strains have been satisfied.

(b) The maximum strain at the extreme compression fibre at ultimate strength has been limited to 0.003.

(c) Young's modulus, E_s , for steel has been taken as 29,000 Ksi. The stress-strain relationship of the steel has been assumed to be perfectly bilinear; elastic up to the specified yield strength, f_y , and equal to f_y for greater strains. For compression steel placed in the region of the equivalent Whitney stress block, the steel compressive stress has been taken as $(f_s - 0.85 f'_c)$ or $(f_y - 0.85 f'_c)$ to account for

the displaced concrete. Here f'_c is the cylinder crushing strength of the concrete. To keep the equations independent of f'_c , these compression steel stresses have been taken as $(f_s - 0.85 \times 4)$ or $(f_y - 0.85 \times 4)$ Ksi.

(d) The tensile strength of the concrete has been neglected. The compressive stress has been assumed to vary parabolically up to a value of $0.85 f'_c$ at a strain of 0.002, and to remain at $0.85 f'_c$ up to a maximum strain of 0.003.

(e) The total reinforcement has been assumed to be divided equally among the four faces of the column, and uniformly distributed along each face. For the purpose of computation the steel was divided into ten discrete units per face.

(f) The concrete section has been divided into a grid of 400 discrete rectangular elements, each with dimensions of $1/20$ of the section width by $1/20$ of the section depth. The stress over each element has been assumed to be constant and equal to the value determined by the strain at its centroid.

Based on these assumptions the appropriate strain compatibility and equilibrium equations can be developed so as to suit computer processing. This is shown in the Appendix.

3.2 Specification of Loading Direction

Interpolation between charts for bending on a diagonal and about a principal axis appears to introduce rather large errors. Therefore, it is desirable to produce charts for intermediate loading directions. Charts cannot be produced for certain angles of loading on rectangular sections as this would imply a varying sense of bending as the aspect ratio, h/b varies. Thus the ratio h/b must be included in the parameter describing the sense of bending.

The loading direction can be expressed in terms of

$$\frac{M_{uy}}{M_{ux}} = K \frac{b}{h}$$

which, after substitution gives

$$\frac{m_y}{m_x} = K \quad \dots (1)$$

and this is independent of the aspect ratio. When $K = 1$, the failure conditions are such that the load is always along the diagonal; $K = 0$ implies bending about the x axis only; and $K = \infty$ implies bending about the y axis only; see Fig. 2. For a section in which the ratio of the distance between the centroids of the reinforcement in opposing faces to the overall dimensions of the section, f and g , as defined in Fig. 1, are constant, the failure surface is expressed adequately by a number of charts with K values varying from 0 to 1. For when biaxial bending is considered over the section, the axis about which the larger specific moment occurs, is arbitrarily termed the x axis. When $f \neq g$ this procedure cannot be followed. In these cases charts must be prepared for loading directions specified with K values ranging from

0 to ∞ .

For the case of $f = g$, charts were produced for $K = 0.0, 0.268, 0.577, 1.0$, see charts A1, A2, A3 in the Appendix. These values of K refer respectively to specific angles of loading, $\theta = \tan^{-1}K, 0^\circ, 15^\circ, 30^\circ$ and 45° , to the negative y axis. When $f \neq g$, charts for $K = 0.325, 0.727, 1.376, 3.078$ corresponding to specific angles $18^\circ, 36^\circ, 54^\circ$ and 72° were produced, see charts B1, B2, B3 in the Appendix. As bending about the x axis for these cases is independent of f , the $\theta = 0^\circ$ diagrams are the same as those of the A charts with the corresponding g values. As bending about the y axis is independent of g , the $\theta = 90^\circ$ diagrams are the $\theta = 0^\circ$ diagrams of the A charts with appropriate f values.

3.3 Technique of Solution

Interaction surfaces were developed by translating the neutral axis across the section. Initially the neutral axis is placed in a position such that the failure is strongly dominated by compression and the component moments are calculated. This process continues until a neutral axis position is found when the section fails in pure flexure. One problem in developing an interaction diagram for loading about an axis, which is not an axis of symmetry, arises from the fact that the direction of the neutral axis is unknown. The sequence of solution used in chart development was made as follows:

- The values of f, g and f_y were specified.
- A direction of loading was chosen, i.e. K was defined.
- A value of ρ_{Tm} was specified.
- K_y (see Fig. 1) was set and K_x was adjusted until the ratio of specific moments, m_y/m_x , complied with the loading direction. This set of conditions was recorded and K_y decreased and the step repeated.
- When $P_u/f'_c b h$ became negative ρ_{Tm} was incremented and step (d) was repeated.
- When the required range of ρ_{Tm} had been exhausted, a different loading direction, K , was specified and steps (c), (d) and (e) were repeated.

Six particular charts, which should cover the majority of columns designed in practice, are shown in the Appendix. There the "specific" axial load is plotted against the resultant "specific" moment m_θ , where

$$m_\theta = \sqrt{m_x^2 + m_y^2} = \frac{M_{ux}}{f'_c b h^2} \sqrt{1+K^2} = \frac{M_{uy}}{f'_c h b^2} \sqrt{1+\left(\frac{1}{K}\right)^2} \quad \dots (2)$$

3.4 Modification of Weber's Parameters

In producing interaction diagrams for bending along the diagonal of symmetrically reinforced square columns, Weber² plotted $P_u/f'_c h^2$ against $P_u e/f'_c h^3$. If these parameters are replaced by $P_u/f'_c b h$ and m_θ respectively, his charts become applicable to rectangular columns of any aspect ratio. For diagonal bending $K = 1$,

and hence

$$m_{\theta} = \frac{M_{ux}}{f'_c b h^2} \sqrt{2} = \frac{M_{uy}}{f'_c h b^2} \sqrt{2}$$

In the case of a square column $b=h$, therefore

$$m_{\theta} = \frac{M_{ux}}{f'_c h^3} \sqrt{2} = \frac{P_u e_x}{f'_c h^3} \sqrt{2} = \frac{P_u e}{f'_c h^3}$$

Hence in the special case of a square column, the general parameter, m_{θ} , reduces to that used by Weber. Also the interpolation method used by Weber must be modified. Instead of interpolating with respect to the true angle of loading, interpolation must be carried out with respect to the specific angle of loading, $\theta = \tan^{-1}K$. As Weber's charts have been produced only for bending about the diagonal, interpolation must be carried out between the results for a uniaxial loading, $K = 0$, and the particular diagonal loading case for which $K = 1.0$. This can lead to rather large and unconservative errors in reinforcement assessment.

3.5 Sections with Axial Tension

Interaction diagrams have been prepared for axially compressive loads only, for simplicity and convenience of chart presentation. However, a conservative approximation of a sections interaction relation under axial tension can be made by assuming a linear relationship between the section's pure tensile strength to its moment capacity at zero axial load. For low values of $\rho_T m$ this approximation will introduce negligible error on the safe side.

3.6 Accuracy of Interpolation

An analysis of errors, introduced through the proposed interpolation method for design and analysis, was made⁽¹⁾ and it was found that the method is very adequate in determining required reinforcement or assessing section strengths. The only limitation on chart use is the applicability of the basic assumptions.

For sections whose steel placement and distribution agrees with the prepared charts, maximum errors of approximately 3% in steel assessment, of 1% in determination of moment capacity, could be expected, as a result of interpolation, over extremely local loading directions for sections reinforced so that $\rho_T m$ is greater than 0.2.

4. EXAMPLES OF DESIGN AND ANALYSIS

Example 1

Given are the following dimensional and strength properties and loading:

$$\begin{aligned} h &= 20" & f'_c &= 4 \text{ Ksi} & P'_u &= 224 \text{ Kips} \\ b &= 16" & & & M'_{u1} &= 3095 \text{ Kip-in.} \\ f &= g = 0.8 & f_y &= 60 \text{ Ksi} & M'_{u2} &= 990 \text{ Kip-in.} \end{aligned}$$

Also see Fig. 3.

To comply with current code requirements

for tied columns a capacity reduction $\phi = 0.7$ must be introduced, therefore design for

$$P_u = 320 \text{ Kips}$$

$$M_{u1} = 4421 \text{ Kip-in.}$$

$$M_{u2} = 1415 \text{ Kip-in.}$$

$$\text{Specific moment about axis 1: } m_1 = 4421/4 \times 16 \times 20^2 = 0.1727$$

$$\text{Specific moment about axis 2: } m_2 = 1415/4 \times 20 \times 16^2 = 0.0691$$

as $m_1 > m_2$, axis 1 is taken as x axis

$$K = 0.0691/0.1727 = 0.40 \text{ and } \theta = \tan^{-1}K = 21.8^\circ$$

$$m_{\theta} = 0.1727 (1 + (0.4)^2)^{\frac{1}{2}} = 0.1860$$

$$P_u/f'_c b h = 320/4 \times 16 \times 20 = 0.250$$

From chart A2 in Appendix B;

quadrant, $K = 0.268 (\theta = 15^\circ)$ gives $\rho_T m = 0.477$ and

quadrant, $K = 0.577 (\theta = 30^\circ)$ gives $\rho_T m = 0.554$

Linearly interpolating for $\theta = 21.8^\circ$ yields

$$\rho_T m = 0.477 + \left(\frac{21.8-15}{30-15}\right) \times (0.554-0.477) = 0.512$$

$$\text{As } m = f_y/0.85 f'_c = 60/0.85 \times 4 = 17.65$$

$$\rho_T = 0.512/17.65 = 0.029 \text{ and therefore}$$

$$A_T = 0.029 \times 16 \times 20 = 9.29 \text{ in}^2$$

12-No.8 = 9.42 in² i.e. 4 bars in each face of the column.

Example 2

Data given:

$$\begin{aligned} h &= 26" & f'_c &= 4 \text{ Ksi} & & 16\text{-No. 8 bars are} \\ & & & & & \text{distributed evenly} \\ b &= 23" & f_y &= 60 \text{ Ksi} & & \text{among four faces} \end{aligned}$$

Also see Fig. 4.

$$f = 18/23 = 0.783 \approx 0.8$$

$$g = 21/26 = 0.807 \approx 0.8$$

(An interpolation for $f = 0.78$ may be made using charts A2 and B1 if necessary.)

Find the design moment capacities M_{ux} and M_{uy} when a design axial compression of 218 Kips is to be considered as shown.

$$P_u = 218/0.7 = 311.4 \text{ Kips}$$

$$A_T = 12.57 \text{ in}^2 \quad \rho_T = 12.57/26 \times 23 = 0.0210$$

$$m = 60/4 \times 0.85 = 17.65 \quad \rho_T m = 0.371$$

$$M_{uy}/M_{ux} = \tan 30^\circ = 0.5774$$

$$m_y/m_x = \frac{26}{23} \times 0.5774 = 0.6527 = K$$

$$\text{hence } \theta = \tan^{-1} K = 33.13^\circ$$

$$P_u/f'_c b h = 311.4/4 \times 23 \times 26 = 0.13$$

From chart A2 in Appendix B.

Quadrant, $K = 0.577 (\theta = 30^\circ)$ gives $m_\theta = 0.143$ and

Quadrant, $K = 1.0 (\theta = 45^\circ)$ gives $m_\theta = 0.138$

Interpolating for $\theta = 33.13^\circ$ gives

$$m_\theta = 0.143 - \frac{3.13}{15} \times 0.005 = 0.142$$

$$\text{now } m_\theta = \frac{M_{ux}}{f'_c b h^2} (1 + K^2)^{\frac{1}{2}} = \frac{M_{uy}}{f'_c h b^2} (1 + (\frac{1}{K})^2)^{\frac{1}{2}}$$

and hence

$$M_{ux} = \frac{0.142 \times 4 \times 23 \times 26^2}{(1 + (0.6527)^2)^{\frac{1}{2}}} = 7395 \text{ Kip-in.}$$

$$M_{uy} = \frac{0.142 \times 4 \times 26 \times 23^2}{(1 + (1/0.6527)^2)^{\frac{1}{2}}} = 4270 \text{ Kip-in.}$$

The design capacities are therefore :

$$M_{ux} = 0.7 \times 7395 = 5176 \text{ Kip-in.}$$

$$M_{uy} = 0.7 \times 4270 = 2989 \text{ Kip-in.}$$

Example 3

Data given: (also see Fig. 5).

$$\begin{aligned} h = 40'' & \quad f'_c = 3.5 \text{ Ksi} & \quad P'_u = 425 \text{ Kips} \\ b = 14'' & \quad f_y = 60 \text{ Ksi} & \quad M'_{ux} = 3534 \text{ Kip-in.} \\ & \quad f = 0.7 & \quad M'_{uy} = 2474 \text{ Kip-in.} \\ & \quad g = 0.9 \end{aligned}$$

Axis 1 becomes the x axis, to comply with the notation of chart B2 in the Appendix. Introducing $\phi = 0.7$, the design loads become,

$$P_u = 608 \text{ Kips}$$

$$M_{ux} = 5049 \text{ Kip-in}$$

$$M_{uy} = 3534 \text{ Kip-in}$$

$$m_x = 5049/3.5 \times 14 \times 40^2 = 0.0644$$

$$m_y = 3534/3.5 \times 14^2 \times 40 = 0.1288$$

$$m_y/m_x = K = 2.0 \quad \theta = \tan^{-1} 2.0 = 63.6^\circ$$

$$m_\theta = 0.0644 \sqrt{1 + 2^2} = 0.144$$

$$P_u/f'_c b h = 608/3.5 \times 40 \times 14 = 0.31$$

From chart B2 in Appendix B

Quadrant, $K = 1.376 (\theta = 54^\circ)$ gives $\rho_{Tm} = 0.358$ and

Quadrant, $K = 3.078 (\theta = 72^\circ)$ gives $\rho_{Tm} = 0.327$

linearly interpolating for $\theta = 63.6^\circ$ gives

$$\rho_{Tm} = 0.358 - \frac{(63.6 - 54)}{(72 - 54)} \times 0.031 = 0.342$$

With $m = 60/0.85 \times 3.5 = 20.17$

$$\rho_T = 0.0169 \text{ and } A_T = 9.5 \text{ in}^2$$

Provide 16-No.7 = 9.62 in²; see Fig. 5.

The charts are equally suitable when a yield stress a little different from 60 Ksi is specified. With $f_y = 55$ Ksi the interaction curves could hardly be distinguished from those given in the charts. The above problem would be solved as follows:

$$m = 55/0.85 \times 3.5 = 18.49$$

$$\text{hence } A_T = \frac{0.342 \times 14 \times 40}{18.49} = \frac{60}{55} \times 9.5 = 10.36 \text{ m}^2$$

5. ACKNOWLEDGEMENT

The material presented here is part of the report prepared by the first author for his degree of Master of Engineering, under the guidance of the second author. The computations were processed and plotted in the Computer Centre of the University of Canterbury. The assistance of Mr. W. McLelland, Senior Technician in the Department of Civil Engineering, in preparing the charts is gratefully acknowledged.

6. REFERENCES

1. Row, D.G., "The Effects of Skew Seismic Response on Reinforced Concrete Frames", Master of Engineering Report, University of Canterbury, Department of Civil Engineering, 1973, p.113.
2. Weber, D.C., "Ultimate Strength Design Charts for Columns with Biaxial Bending", ACI Journal, Proceedings V.63, No. 11, Nov. 1966, pp.1205-1230.
3. A.C.I. Committee 340, "Ultimate Strength Design Handbook", Volume 2, Columns, ACI Special Publication No. 17A, 1972.
4. Park, R. and Paulay, T., "Ultimate Strength Design of Reinforced Concrete Structures", Seminar Publication for the University of Canterbury, 1969.
5. Brettell, H.J., Warner, R.F., "Ultimate Strength Design of Rectangular Reinforced Concrete Sections in Compression and Biaxial Bending", Civ. Eng. Trans. I.E. Aust, April 1968.
6. European Concrete Committee, "Bulletin d'Information", No. 82, April 1972.
7. European Concrete Committee, "Bulletin d'Information", No. 83, April 1972.

7. APPENDIX

(A) STRAIN COMPATIBILITY AND EQUILIBRIUM EQUATIONS

(a) General Strain Equation.

The strain at any point on the column

section can be expressed in terms of the neutral axis position, the ultimate strain at the extreme compression fibre, and the co-ordinates of the point with respect to the rectilinear axes shown in Fig. 1.

Consideration of the similar triangles of the strain diagram of Fig. 1 yields

$$\frac{\epsilon(x,y)}{K_y h - y - h/2 - (b/2 - x) \cot \phi} = \frac{\epsilon_u}{K_y h} \quad \dots (A1)$$

$$\text{but } \cot \phi = \frac{K_y h}{K_x b}$$

therefore

$$\epsilon(x,y) = 0.003 \left(1 - \frac{1}{2K_y} - \frac{1}{2K_x} - \frac{y}{K_y h} + \frac{x}{K_x b} \right) \quad \dots (A2)$$

(b) Concrete Actions.

The concrete stress at any point can be expressed as:-

$$f_c = \alpha_i f'_c$$

where α_i is a function of the strain at the i th concrete element. Summing over all the discrete concrete elements, yields the total concrete force, P_c , taken positive for compression.

$$P_c = \sum_{i=1}^{400} \alpha_i f'_c \frac{bh}{400} \quad \dots (A3)$$

The contribution of each element in bending about the x and y axis can be found directly by multiplying the element forces by the appropriate y and x co-ordinates respectively, i.e.

$$M_{cx} = - \sum_{i=1}^{400} \alpha_i f'_c \frac{bh}{400} y_i \quad \dots (A4)$$

$$M_{cy} = \sum_{i=1}^{400} \alpha_i f'_c \frac{bh}{400} x_i \quad \dots (A5)$$

(c) Steel Actions.

If the steel stress at the j th discrete steel element is f_{sj} , and the area of each element is $A_j = A_T/40$, where A_T is the total area of reinforcement, then the total steel force is:

$$P_s = \frac{A_T}{40} \sum_{j=1}^{40} f_{sj} \quad \dots (A6)$$

The steel moments are simply

$$M_{sx} = - \frac{A_T}{40} \sum_{j=1}^{40} f_{sj} y_j \quad \dots (A7)$$

$$M_{sy} = \frac{A_T}{40} \sum_{j=1}^{40} f_{sj} x_j \quad \dots (A8)$$

the sign being determined by the right hand screw rule.

(d) Dimensionless Forms.

At this stage of the formulation the following dimensionless parameters are introduced.

$$X = x/b; Y = y/h; \rho_T = A_T/bh; m = f_y/0.85 f'_c$$

Eq. (A2) becomes

$$\epsilon(X,Y) = 0.003 \left(1 - \frac{1}{2K_y} - \frac{1}{2K_x} - \frac{Y}{K_y} - \frac{X}{K_x} \right) \quad \dots (A9)$$

The "specific" column axial load, $P_u/f'_c bh$, is defined by

$$\frac{P_u}{f'_c bh} = \frac{P_c + P_s}{f'_c bh} = \frac{1}{400} \sum_{i=1}^{400} \alpha_i + \frac{0.85 \rho_T m}{40 f_y} \sum_{j=1}^{40} f_{sj} \quad \dots (A10)$$

The "specific" moment about the x axis, m_x , is given by:

$$m_x = \frac{M_{ux}}{f'_c bh^2} = \frac{M_{cx} + M_{sx}}{f'_c bh^2} = - \frac{1}{400} \sum_{i=1}^{400} \alpha_i Y_i - \frac{0.85 \rho_T m}{40 f_y} \sum_{j=1}^{40} f_{sj} Y_j \quad \dots (A11)$$

Similarly the "specific" moment about the y axis, m_y becomes

$$m_y = \frac{M_{uy}}{f'_c hb^2} = \frac{M_{cy} + M_{sy}}{f'_c hb^2} = \frac{1}{400} \sum_{i=1}^{400} \alpha_i X_i + \frac{0.85 \rho_T m}{40 f_y} \sum_{j=1}^{40} f_{sj} X_j \quad \dots (A12)$$

The right hand sides of Eqs. (A9), (A10), (A11) and (A12) are independent of b, h and f'_c and only depend on $\rho_T m, f_y$ and the prescribed ϵ_c strain conditions.

(B) DESIGN CHARTS

The following six charts have been prepared for the following material properties:

Strength of concrete $f'_c \leq 4000$ psi (28 N/mm²)

Yield strength of reinforcement $f_y = 60$ Ksi (414 N/mm²)

The variable for the six cases is the distance between the layers of reinforcing bars relative to overall dimensions of the column sections as shown in the upper right hand corner of each chart.

CHART A1	$f = g = 0.7$
CHART A2	$f = g = 0.8$
CHART A3	$f = g = 0.9$
CHART B1	$f = 0.7, g = 0.8$
CHART B2	$f = 0.7, g = 0.9$
CHART B3	$f = 0.8, g = 0.9$

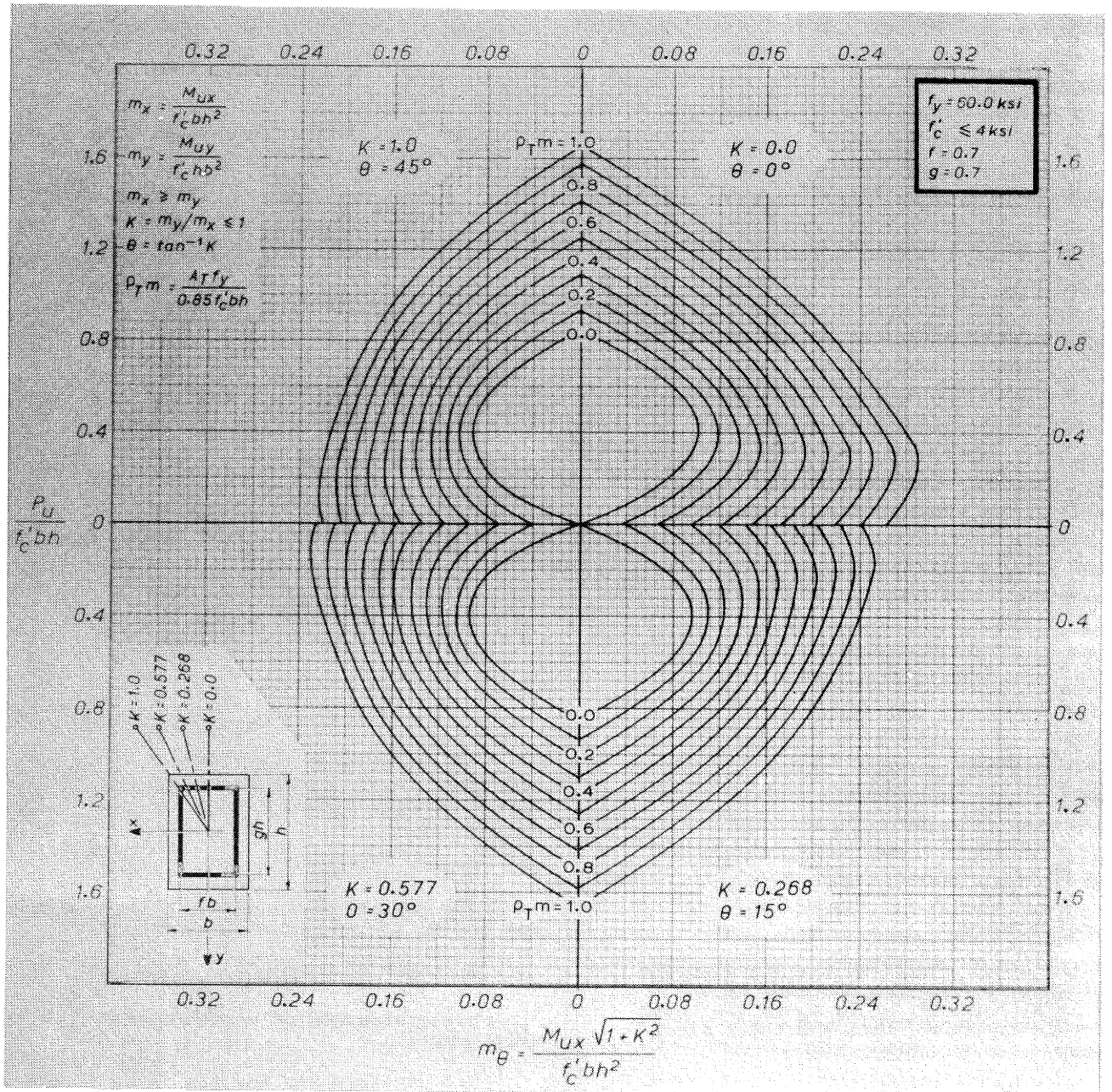


CHART A1 $f = g = 0.7$

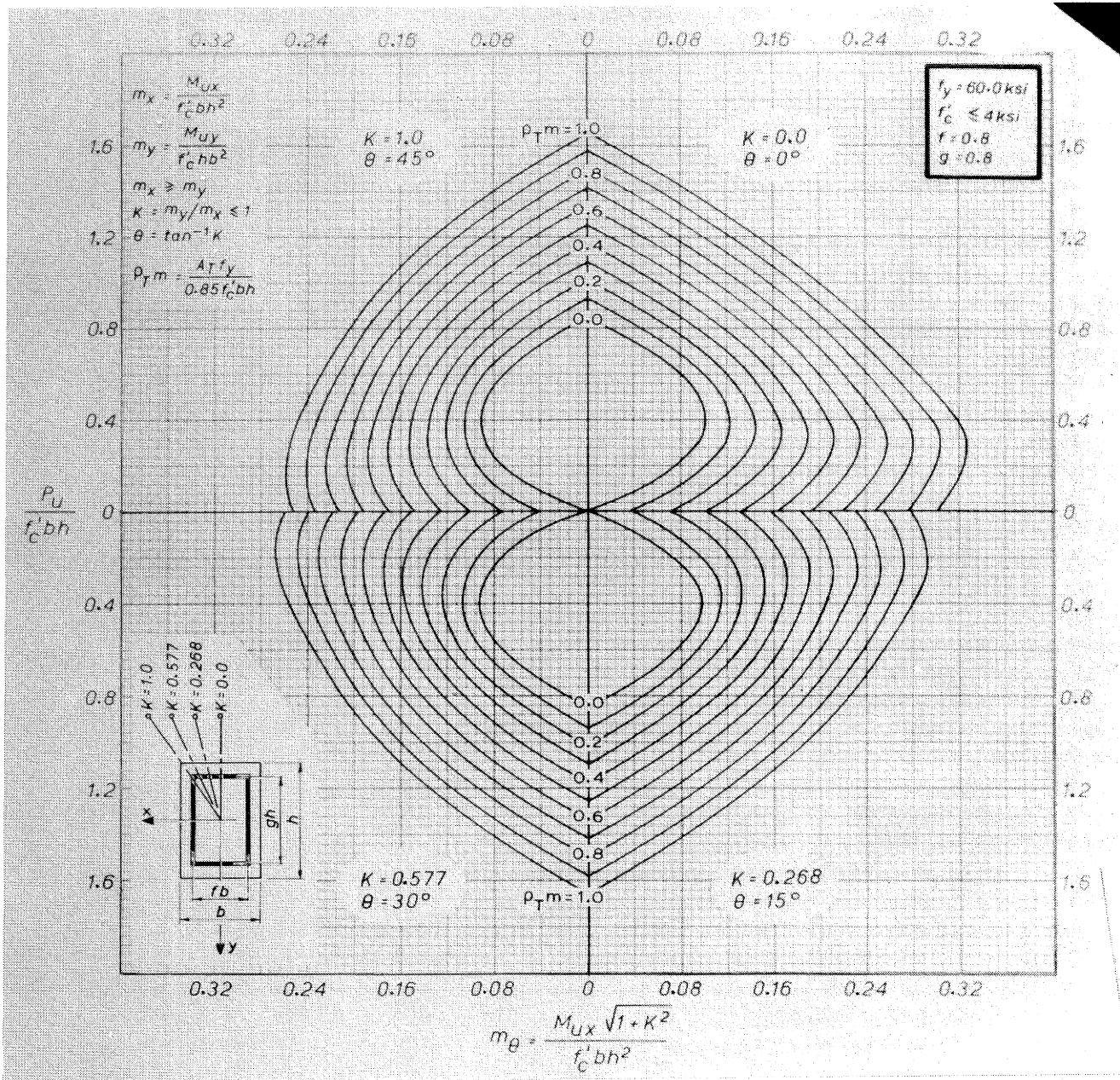


CHART A2 $f = g = 0.8$

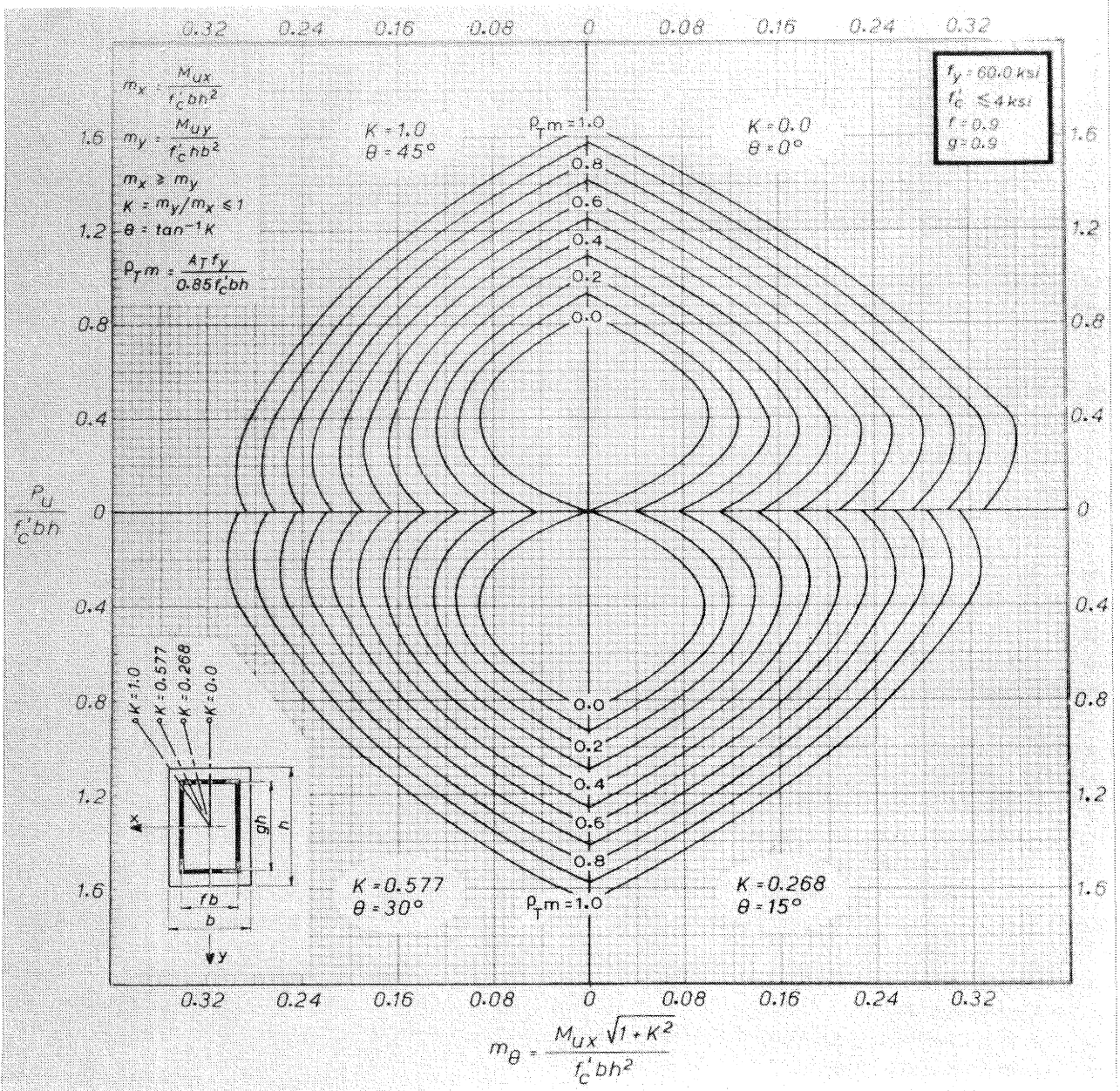


CHART A3 $f = g = 0.9$

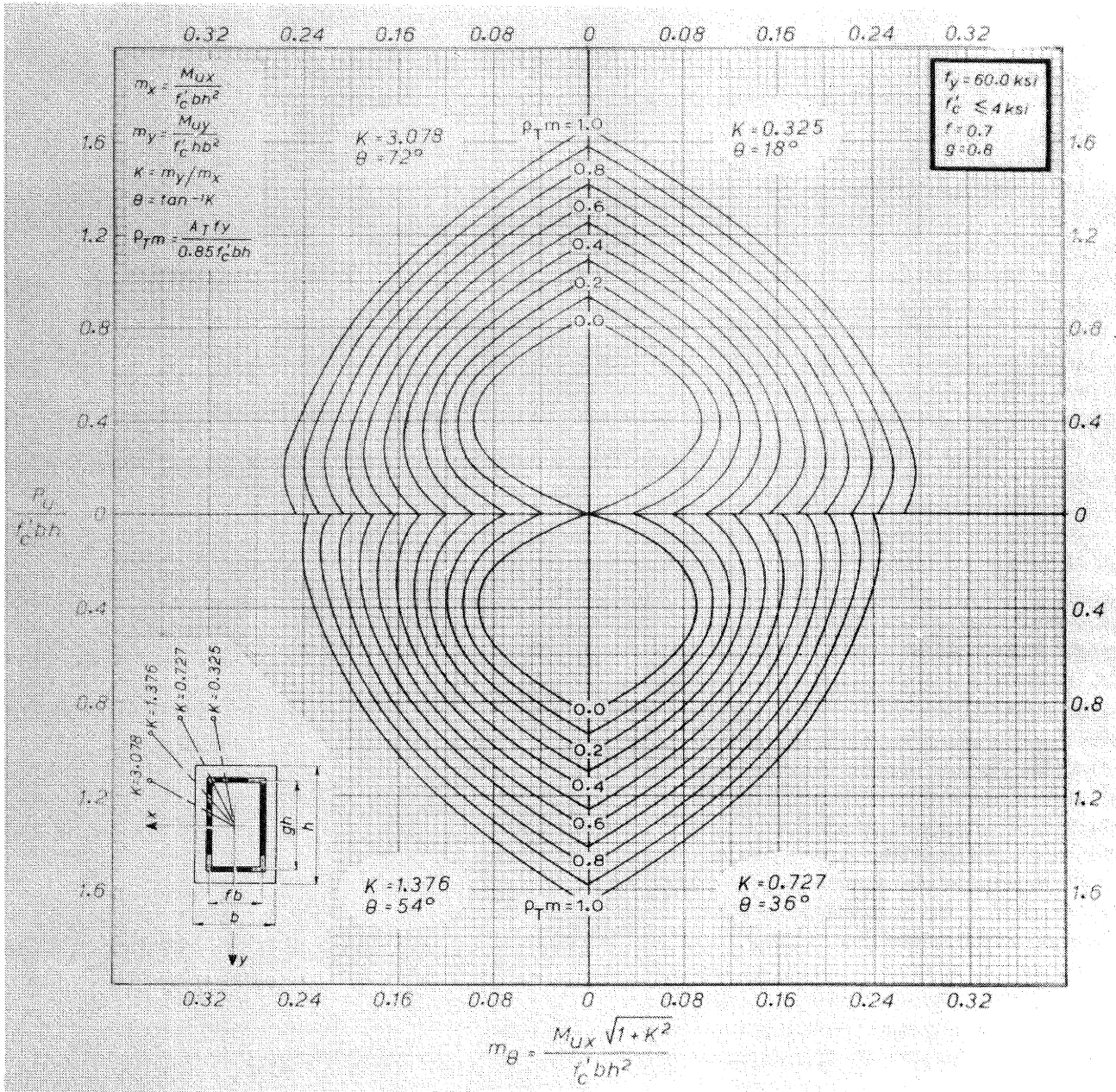
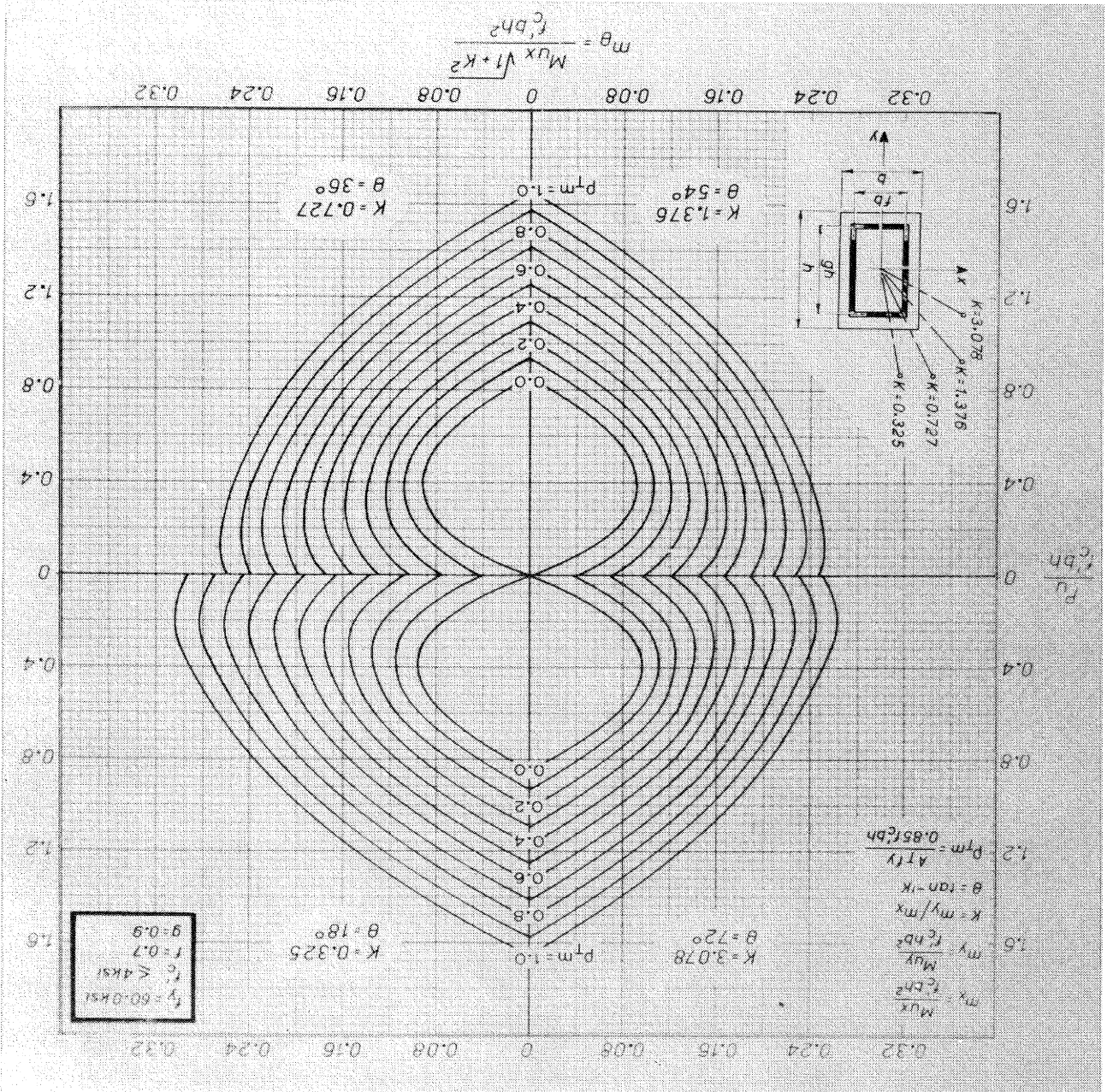


CHART B1 $f = 0.7, g = 0.8$

CHART B2 $f = 0.7, g = 0.9$



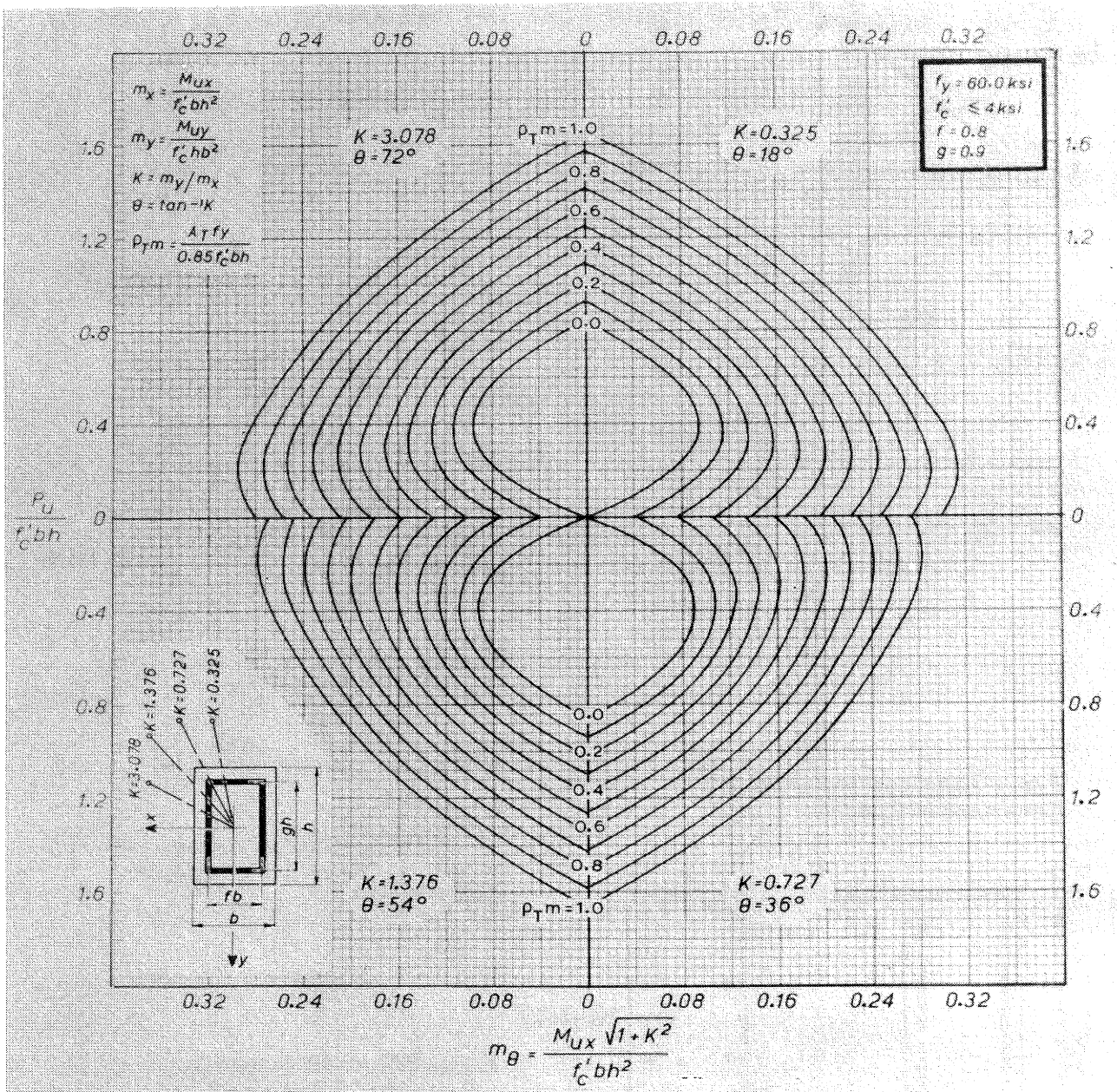


CHART B3 $f = 0.8, g = 0.9$

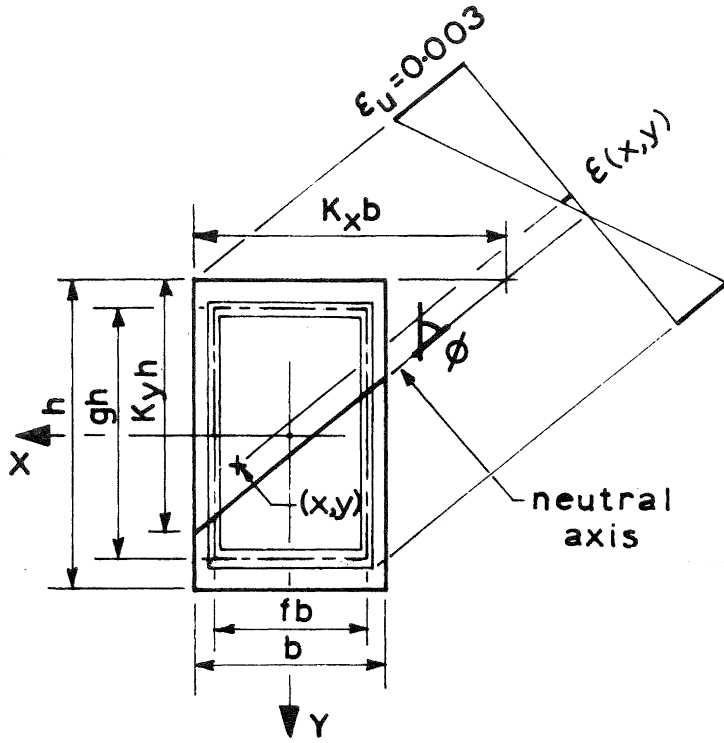


Figure 1 Section with Biaxial Flexure and Axial Load

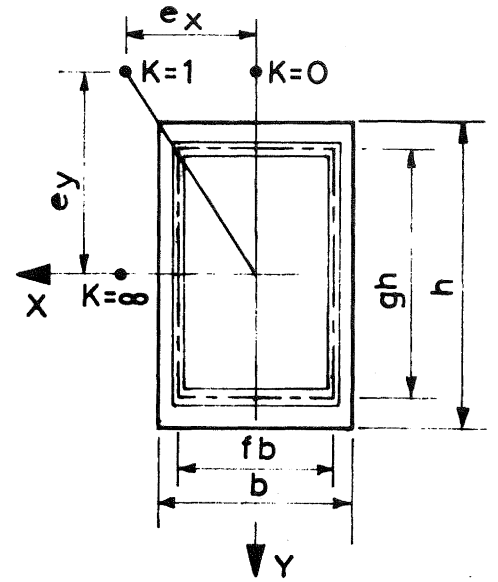


Figure 2 Loading Directions

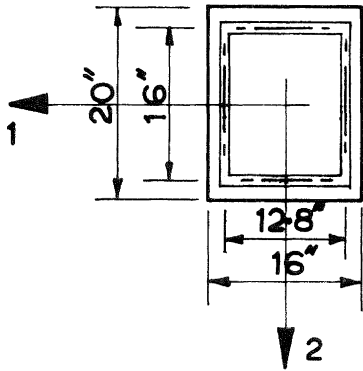


Figure 3 Sectional Properties Considered in Example 1

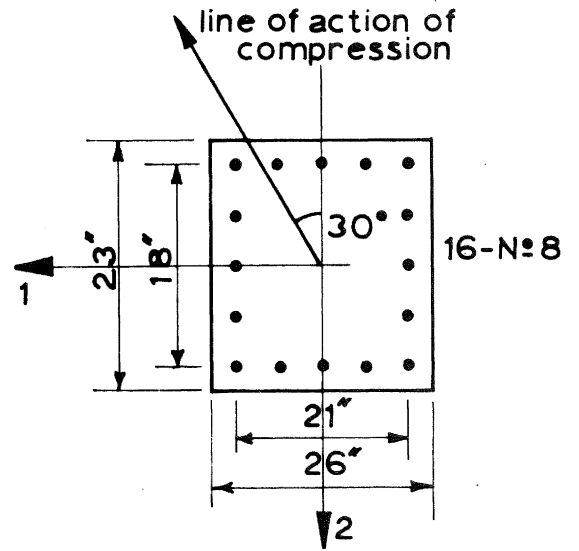


Figure 4 Sectional Properties Considered in Example 2

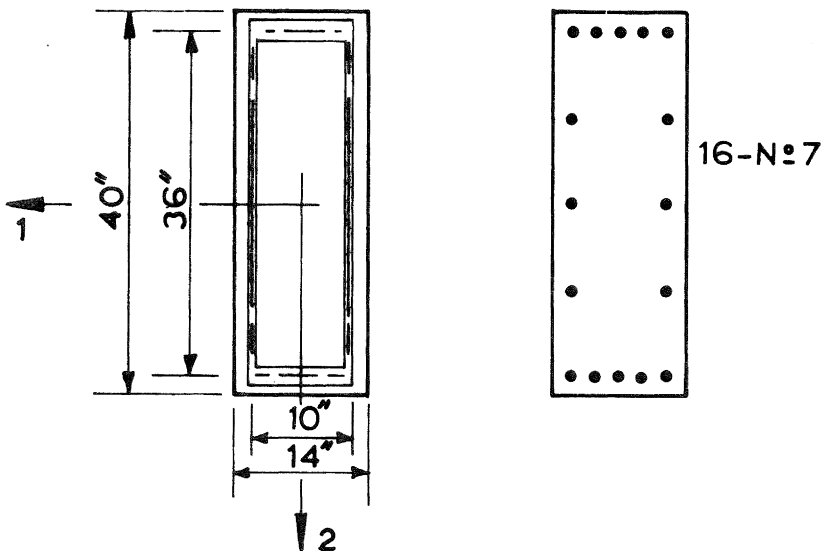


Figure 5 Sectional Properties Considered in Example 3