

## SMART SEMI-ACTIVE MR DAMPER TO CONTROL THE STRUCTURAL RESPONSE

Nikoo K. Hazaveh<sup>1</sup>, J. Geoffrey Chase<sup>2</sup>, Geoffrey W. Rodgers<sup>3</sup>  
and Stefano Pampanin<sup>4</sup>

(Submitted March 2015; Reviewed July 2015; Accepted October 2015)

### ABSTRACT

One advanced means of protecting structures against earthquake ground motions is the use of semi-active devices to customise and limit structural response. Thus, the design, modelling and analysis of different semi-active control devices have received increasing research attention. This study presents a method to determine optimal control forces for magneto-rheological (MR) dampers, using three algorithms: a discrete wavelet transform (DWT), a linear quadratic regulator (LQR), and a clipped-optimal control algorithm. DWT is used to obtain the local energy distribution of the motivation over the frequency bands to modify conventional LQR. The clipped-optimal control algorithm is used to get the MR damper control force to approach the desired optimal force obtained from modified LQR. A Bouc-Wen phenomenological model is utilized to capture the observed nonlinear behaviour of MR dampers. Time history analysis for a single degree of freedom (SDOF) with periods of  $T=0.2-5.0$  sec is utilized to compare the impact of using classic and modified LQR in controlling the MR damper force under 20 design level earthquakes of the SAC (SEAOC-ATC-CUREE) project. Performance is assessed by comparing the maximum displacement ( $S_d$ ), total base shear ( $F_b$ ) and the controller energy. This study shows the proposed modified LQR is more effective at reducing displacement response than conventional LQR. The modified LQR method reduces the median value of uncontrolled  $S_d$  by approximately 40% to 88%, over all periods to 5.0 seconds. Moreover, the modified LQR uses about 45% less energy than conventional LQR. Overall, these results indicate the robustness of the proposed method to mitigate structural response and damage using MR devices.

### INTRODUCTION

Large, modern structures demand greater protection from external excitations, such as strong winds and severe earthquakes. One of the solutions to reduce loss of life and damage due to natural hazards is using supplemental control devices to reduce response. These structural control systems can be classified as active, passive or semi-active.

Active systems are designed to monitor the structural response at a given time, process this information and, generate internal forces to modify the structural behaviour to achieve a reduced structural response. However, active systems are complex, expensive, and require excessive impractical energy input to apply large forces. In contrast, an effective, robust and economical solution is using passive control systems that do not require an external power source. However, they must be exactly tuned, usually to structural design parameters, and cannot be modified during the seismic response of the structure. Therefore, these passive devices are not adaptive to changes in structural properties.

An interesting and appealing improvement of passive control is given by semi-active control systems that require only a small external power source for operation (e.g. a battery). Semi-active devices use small changes to manipulate or change larger reaction forces. Thus, they cannot destabilize the structure because they do not input energy to the system and just absorb or store vibratory energy [1]. Because of this low dependence on external power sources and the removal of instability concerns, semi-active systems have become an attractive potential solution.

Among many proposed semi-active devices Magneto-Rheological (MR) and Electro-Rheological (ER) dampers can achieve high-level, adaptive performance. However, MR dampers require lower voltage than ER dampers. MR dampers generate controllable damping forces using MR fluids composed of tiny magnetized particles in a mineral liquid, such as silicon oil. When a magnetic field is applied to this liquid, particle chains form and the fluid becomes a semi-solid that exhibits plastic behaviour and delivers greater resistive force than the fluid alone.

Although MR dampers are promising, their major drawback lies in their inherent non-linear behaviour and the complexities of modelling their dynamic response. There are two types of MR damper dynamic models: non-parametric and parametric. Non-parametric models include neural network-based models [2] and fuzzy logic-based models [3]. The Bingham model [4], non-linear hysteretic bi-viscous model [5], hyperbolic tangent model [6] and Bouc-Wen hysteresis model [7] are some of the parametric models that have been used to model the behaviour of MR dampers.

To characterize the behaviour of a MR fluid damper, Spencer *et al.* [8] introduced the Bouc-Wen model [7]. This model predicts the force-displacement and force-velocity behaviour well, and results capture experimental data [8]. However, the simple Bouc-Wen Model cannot capture the force roll-off when the acceleration and velocity have opposite signs and the magnitude of the velocities is small, to which Spencer *et al.* [8] proposed a modified version of the Bouc-Wen model.

<sup>1</sup> Corresponding Author, PhD student, University of Canterbury, Christchurch, [nikoo.hazaveh@pg.canterbury.ac.nz](mailto:nikoo.hazaveh@pg.canterbury.ac.nz) (Member)

<sup>2</sup> Professor, University of Canterbury, Christchurch (Member)

<sup>3</sup> Senior Lecturer, University of Canterbury, Christchurch (Member)

<sup>4</sup> Professor, University of Canterbury, Christchurch (Member)

Using an appropriate control algorithm is also very important to achieve the desired control performance with MR devices. The most widespread method used is the Linear-quadratic regulator (LQR), and the related Linear-quadratic-Gaussian (LQG) [9]. LQR is used widely, but suffers from its inability to explicitly account for an unknown excitation and linearity. To simulate realistic circumstances, the excitation must be known prior to determining the optimal control force to achieve more reliable solutions.

In addition, the effect of the specific earthquakes has been accounted for in only a few studies [10, 11]. For example Panarillo *et al.* [12] introduced a method based on updating weighting matrices from a database of earthquakes. Nonetheless, in these studies, offline databases were still required. Basu *et al.* [13] proposed time-varying LQR controller by updating the weighting matrices online. However, this method did not apply in the specific semi-active or active systems. After that Amini *et al.* [14] proposed a Particle swarm optimization (PSO)-based-LQR control to design the controller by updating the weighting matrices for an active tuned mass damper (ATMD). However, that method is fairly complicated and it was used only for ATMD. Therefore, there is a need to use a simple and robust control method to achieve the desirable force of the semi-active MR damper.

In this article, the modified Bouc-Wen model is used with a clipped-optimal control algorithm based on a modified LQR method to find the optimal MR control force. The proposed method could be more robust, as it has the ability to vary its properties according to the external load to more effectively control the structure.

### MODIFIED BOUC-WEN MODEL

The modified Bouc-Wen model for a MR device is shown in Figure 1 where the nonlinear force,  $F$ , of is defined [15]:

$$F = \alpha z + c_0 (\dot{x} - \dot{y}) + k_0 (x - y) + k_1 (x - x_0) \quad (1)$$

$$= c_1 \dot{y} + k_1 (x - x_0)$$

Where  $\alpha$  is Bouc-Wen model parameter related to the MR material yield stress and  $z$  is the hysteretic displacement defined:

$$\dot{z} = -\gamma |\dot{x} - \dot{y}| z |z|^{n-1} - \beta (\dot{x} - \dot{y}) |z|^n + A (\dot{x} - \dot{y}) \quad (2)$$

where  $\dot{y}$  is defined:

$$\dot{y} = \frac{1}{c_0 + c_1} \{ \alpha z + c_0 \dot{x} + k_0 (x - y) \} \quad (3)$$

where  $c_0$  is the viscous damping parameter at high velocities;  $c_1$  is the viscous damping parameter for the force roll-off at low velocities;  $k_0$  controls the stiffness at large velocities;  $k_1$  represents the accumulator stiffness;  $x_0$  is the initial displacement of the spring stiffness  $k_0$ .  $\gamma$ ,  $\beta$  and  $A$  are adjustable shape parameters of the hysteresis loops, that capture the linearity in the unloading and the transition between pre-yielding and post-yielding regions.

Optimal performance for MR damper control systems is gained by varying the applied voltage to the current driver according to measured feedback at any moment. Thus, to determine a comprehensive model that is valid for fluctuating magnetic fields, the parameters  $\alpha$ ,  $c_0$ ,  $c_1$  and  $k_0$  in Equations 1-3 are defined as a linear function of the efficient voltage  $u$ :

$$\alpha(u) = a_a + a_b u \quad (4)$$

$$k_0(u) = k_{0a} + a_b u \quad (5)$$

$$c_0(u) = c_{0a} + c_{0b} u \quad (6)$$

$$c_1(u) = c_{1a} + c_{1b} u \quad (7)$$

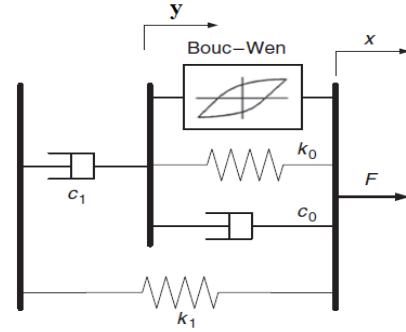


Figure 1: Modified Bouc-Wen model for a MR device [8].

where the dynamics involved in the MR fluid reaching rheological equilibrium are accounted for through the first order filter [8].

$$\dot{u} = -\eta(u - v) \quad (8)$$

where  $v$  is the voltage applied to the current driver and  $\eta$  is the time constant of the first-order filter. Optimal values of a total of fourteen parameters ( $c_{0a}$ ,  $c_{0b}$ ,  $k_0$ ,  $c_{1a}$ ,  $c_{1b}$ ,  $k_1$ ,  $x_0$ ,  $a_a$ ,  $a_b$ ,  $\gamma$ ,  $\beta$ ,  $n$ ,  $\eta$  and  $A$ ) must be determined for the prototype MR damper. The accuracy of this modelling approach was validated for a device with 3kN design force in [8].

### INTEGRATED STRUCTURE-MR DAMPER SYSTEM

When  $n$ -degree-of-freedom (N-DOF) systems with  $r$  MR dampers are subjected to external excitation and control forces, the governing equations of motion are written:

$$M \ddot{q}(t) + C \dot{q}(t) + Kq(t) = L f_c(t) + H f_e(t) \quad (9)$$

where  $M$ ,  $C$ , and  $K$  are the mass, damping, and stiffness matrices of the structure without dampers, respectively. If  $q(t)$  is  $n \times 1$  vector of the relative displacement with respect to the ground, then the mass matrix  $M$  can be considered diagonal. The damping matrix  $C$  takes a form similar to  $K$ .

The control force vector,  $f_c(t)$ , is of size  $l \times 1$ , and  $f_e(t)$  is the external dynamic force vector of dimension  $r \times 1$ ,  $L$  and  $H$  are  $n \times l$  and  $n \times r$  location matrices that define the locations of the control forces and the external excitations in the state space equations, respectively. A state-space representation of Eq. 9 can thus be written:

$$\{\dot{x}\} = [A]\{x\} + [B]\{f_c\} + [E]f_e \quad (10)$$

where:

$$\{x\} = \begin{bmatrix} q(t) \\ \dot{q}(t) \end{bmatrix} \quad (11)$$

and  $\{x\}$  is the state vector of dimension  $2n \times 1$ , and the state space matrices are defined:

$$A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix} \quad (12)$$

$$B = \begin{bmatrix} 0 \\ M^{-1}L \end{bmatrix} \quad (13)$$

$$E = \begin{bmatrix} 0 \\ M^{-1}H \end{bmatrix} \quad (14)$$

where these matrices are of dimension  $2n \times 2n$ ,  $2n \times 1$ , and  $2n \times r$  and define the system, control location, and external excitation

location matrices, respectively. The matrix “ $I$ ” in Equation 12 denotes the identity matrix of size  $n \times n$ . The matrices “ $0$ ” in Equations 12, 13 and 14 denote the zero matrices of size  $n \times n$ ,  $n \times l$  and  $n \times r$ , respectively. The LQR algorithm can determine the optimal control forces for the system with the aim of minimizing the cost function defined:

$$J = \int_0^{t_f} [\{x\}^T [Q] \{x\} + \{f_c\}^T [R] \{f_c\}] dt \quad (15)$$

where  $t_f$  is time duration of excitation, matrices  $Q$  and  $R$  are the response and control energy weighting matrices, and the optimal control force is defined:

$$f_{opt} = -R^{-1} B^T P x \quad (16)$$

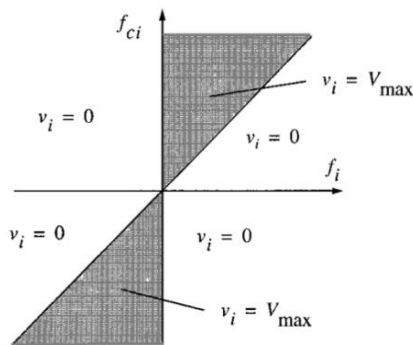
where  $P$  is the Riccati matrix solution and  $x$  is the state feedback of the system at each time step.

Semi-active control systems are typically highly non-linear. One algorithm that has been shown to be effective for use with the MR damper is a clipped-optimal control approach [16]. The clipped optimal semi-active controller is investigated here to show the effectiveness of the proposed modulating control law. In particular, it has been shown to be one of the most promising and applicable bang-bang (on-off) controllers that can more effectively mitigate structural response compared to other well-established decentralised and centralised methods [7, 17].

The clipped-optimal control approach is to design a linear optimal controller that calculates a vector of desired control forces based on the measured structural responses and the measured control force vector applied to the structure. If the magnitude of the force produced by the damper is smaller than the magnitude of the desired optimal force and the two forces have the same sign, the voltage applied to the current driver is increased to the maximum level so as to increase the force produced by the damper to match the desired control force. Otherwise, the commanded voltage is set to zero (Figure 2). The algorithm for selecting the command signal for the MR damper is defined:

$$v_i = V_{max} H(\{f_{ci} - f_i\} f_i) \quad (17)$$

in which  $V_{max}$  denotes the maximum voltage that is associated with the saturation of magnetic field in the MR damper and  $H$  is the Heaviside function.



**Figure 2: Graphical Representation of Algorithm for Selecting Command Signal [7].**

Although a variety of approaches may be used to design the optimal controller, LQR methods are advocated because of their successful application in previous studies. The approach to optimal control design is discussed in detail in [18].

## MODIFIED LQR METHOD

In this study, a real time discrete wavelet transform (DWT) controller is updated at regular time steps from the initial time ( $t_0$ ) until the current time ( $t_c$ ) to achieve the local energy distribution of the input over frequency bands. The time interval under consideration [ $t_0, t_c$ ] is sub-divided into time window bands. The time of  $i^{th}$  window is [ $t_{i-1}, t_i$ ] of which the signal can be decomposed into time frequency bands by wavelet transform (DWT). Through DWT with multi-resolution analysis (MRA) the exact decomposition of signals over time window bands are obtained in real time. The local energy content at different frequency bands over the considered time window are given by the MRA. It is obvious that the frequency that contains the maximum energy is the dominant frequency of that window. When the domain frequency of each window approaches to the natural frequency of the system, resonance occurs in the structure. This resonance causes high displacement response in the system.

To mitigate displacement response, high control force is needed, which can be obtained in design by decreasing the weighting matrix [ $R$ ]. The advantage of local optimal solution is that it has the ability to change the value of the matrix  $R$  on specific frequencies, in contrast to the classical LQR which is a global optimal solution. To achieve this, the control energy weighting matrices are updated for every time window by a scalar multiplier and can be defined as:

$$R = \delta [I] \quad (18)$$

where  $\delta$  is a scalar parameter used to scale the weighting matrix and is obtained based on the time-frequency analysis of a response state. Hence, the scalar parameter of the gain matrix can be defined:

$\delta \neq 1$  if the frequency of excitation is close to the natural frequency of the system,

$\delta = 1$  otherwise.

The value of the  $\delta$  has been proposed as less than one when resonance occurs. This makes it possible to change the weighting matrices for different frequency bands. The control energy weighting matrices are reduced when the structure has a significantly high value of displacement response. This reduction of weighting matrices sets off the lesser displacement without penalty. Therefore, the positive aspect of the proposed method is that the gain matrices are calculated adaptively by using the time-varying weighting matrices depending on the online response characteristics, instead of a *priori* (offline) choice of the weights as in the classical case [14].

Figure 3 shows the block diagram of the semi-active device and flowchart of the LQR method and modified LQR method.

## ANALYSIS

This paper investigates the relative effectiveness of the proposed LQR control method of the MR damper on seismic structural response. The model structure includes inherent structural viscous damping of 5% (Figure 4). The period of the SDOF is changed by modifying the stiffness, keeping a constant mass of 1000 kg. The MR damper has a force capacity of 3 kN and is installed to control the seismic responses of system. Optimal values for the Bouc-Wen phenomenological model parameters for this damper are given in Table 1 and the maximum input voltage for this damper is equal to 2.25V.

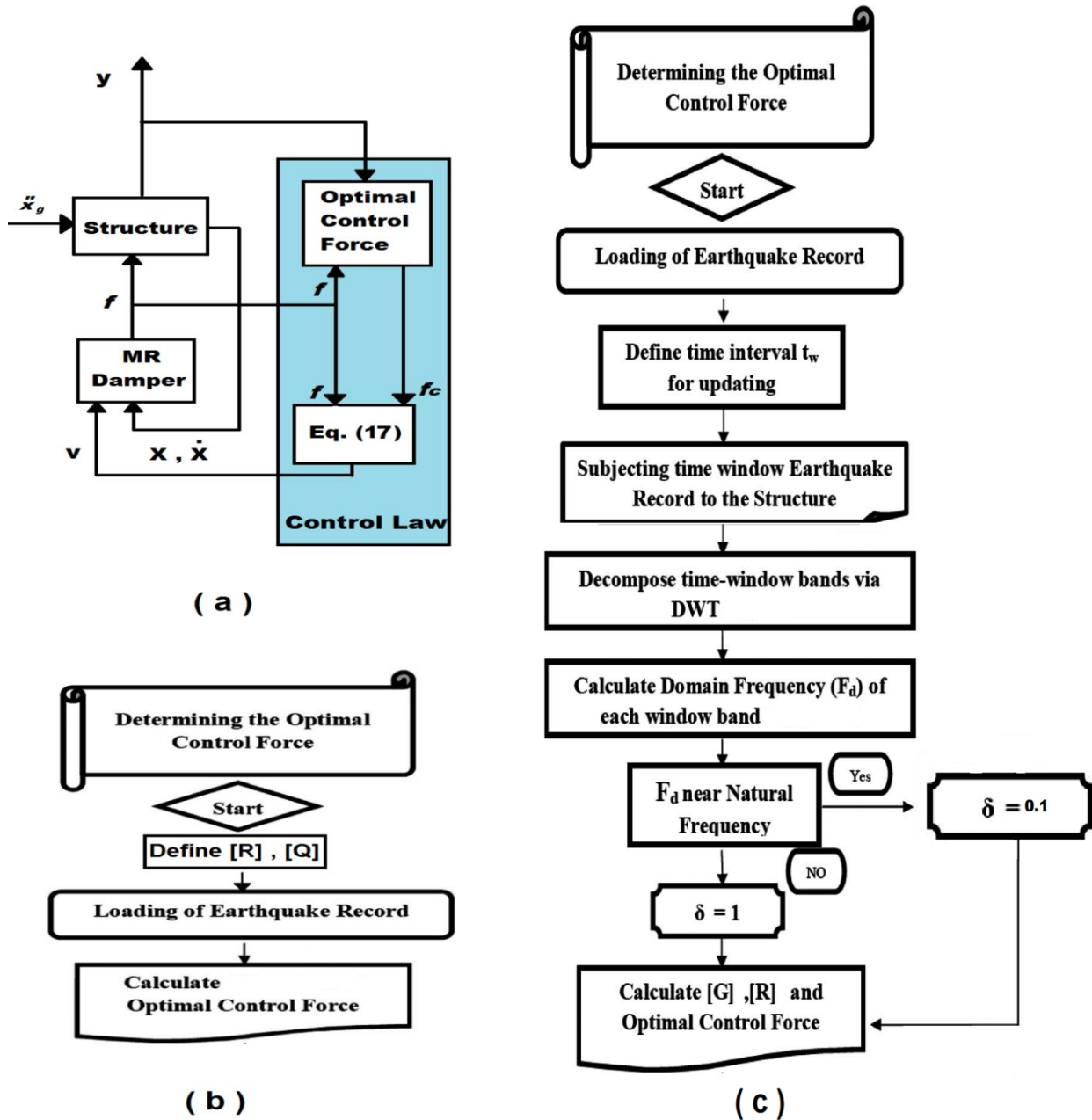


Figure 3: (a) Block diagram of semi-active control system; (b) Flowchart of the classical LQR method; (c) Flowchart of the modified LQR method.

Table 1: Parameters for 3kN MR damper model.

Parameter	Value	Parameter	Value
$C_{0a}$	21.0 N S cm <sup>-1</sup>	$\alpha_a$	140 N cm <sup>-1</sup>
$C_{0b}$	3.50 N S cm <sup>-1</sup> V <sup>-1</sup>	$\alpha_b$	695 N cm <sup>-1</sup> V <sup>-1</sup>
$K_0$	46.9 N cm <sup>-1</sup>	$\gamma$	363 cm <sup>-2</sup>
$C_{1a}$	283 N S cm <sup>-1</sup>	$\beta$	363 cm <sup>-2</sup>
$C_{1b}$	2.95 N S cm <sup>-1</sup> V <sup>-1</sup>	A	301
$K_1$	5.00 N cm <sup>-1</sup>	n	2
$X_0$	14.3 cm	$\eta$	190 s <sup>-1</sup>

The analyses utilize a 20 event earthquake suite from the SAC project [19], and the ground motions were scaled for equal probability of occurrence. As such, the PGA values do not necessarily match the original values in the raw recording. The suite, referred to as the medium suite, consists of design level events with probabilities of exceedance of 10% in 50 years in the Los Angeles region (Table 2). Response statistics can be generated from the results of this probabilistically scaled suite with an expected return period of 475 years. Figure 5 shows the standard acceleration response spectra of each ground motion from the medium suite of the SAC project and the median design spectra of those results.

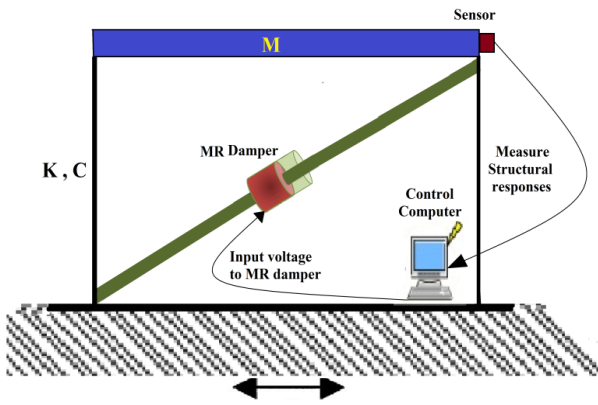


Figure 4: Schematic of a SDOF structural system fitted with the MR damper.

The parameter  $\delta$  used for scaling the weighting matrices is 0.1 when the central frequency of each window band is close to the natural frequency of the SDOF system, and for others frequencies is assumed to be 1.0. Hence, the weighting matrix component  $[R]$ , equals 0.1  $[I]$  for resonance frequency bands and for the rest of the frequency bands it is kept as  $[I]$ . In addition, the matrix  $Q$  is chosen as the identity for each band. Daubechies wavelets of order 4 (db4), is used as a mother wavelet to decompose the time history of acceleration for different window bands, to determine the frequency distribution of each band. The Daubechies wavelets have reasonably good localization in time and frequency to capture the effects of local frequency content in a time signal, and allow for fast decomposition using MRA. The signals recorded in real time are decomposed for each interval window, which is considered as 1.0 second for updating. The gain matrices are updated for each window by solving the Riccati equation. Therefore, the control forces and controlled responses are calculated. To illustrate the potential application of the proposed method, the response of the semi-active clipped optimal controller based on the modified LQR is compared with the conventional LQR and uncontrolled structures. MATLAB (The Mathworks, Natick, MA, USA) is utilized to calculate all computation.

Response spectra are produced for the suite of ground motions, in terms of structural displacement demand ( $S_d$ ) for a

period range of  $T = 0.2 - 5.0$  sec with increments of  $\Delta T = 0.2$  sec. Reductions achieved by the addition of semi-active viscous damping devices are represented by reduction factors (RFs), normalized to the uncontrolled, no device baseline case results. These multiplicative factors enable easy comparison of the two control laws. RFs less than 1.0 indicate reduction in the response metric, whilst RFs greater than 1.0 indicate an increase in response.

The 5<sup>th</sup>, 25<sup>th</sup>, 50<sup>th</sup> (median), 75<sup>th</sup>, and 95<sup>th</sup> percentile results [20] of the response spectra over the suite of events capture the variation in response over the suite of design level ground motions. The percentiles define relative risks of exceedance. The median (50<sup>th</sup> percentile) is the middle (expected) result. The 95<sup>th</sup> percentile shows the largest RF likely to occur for ground motions with this likelihood of occurrence. These statistics provide the framework for a probabilistic or risk-based performance based design approach indicating the likely range of benefit ( $RF < 1.0$ ) or degradation ( $RF > 1.0$ ). This approach eliminates the likelihood of erroneous conclusions being drawn about the viability of a control law due to atypical performance for a single or few earthquakes, and should be robust to variabilities.

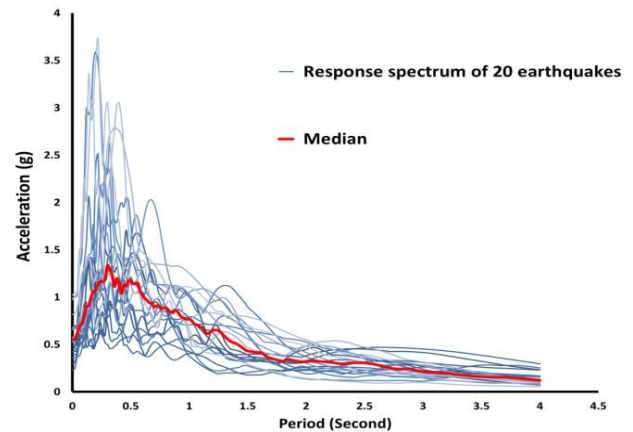


Figure 5: Acceleration response spectrum of 20 ground motions of medium suite of SAC project and median response of medium suit.

Table 2: Detail of selected Los Angeles ground motions with (probability of exceedance of 10% in 50 years).

SAC Name	Record	Earthquake Magnitude	Distance (Km)	Scale Factor	Duration (sec)	PGA (cm/sec <sup>2</sup> )
LA01	Imperial Valley, 1940, El Centro	6.9	10	2.01	39.38	452.03
LA02	Imperial Valley, 1940, El Centro	6.9	10	2.01	39.38	662.88
LA03	Imperial Valley, 1979, Array #05	6.5	4.1	1.01	39.38	386.04
LA04	Imperial Valley, 1979, Array #05	6.5	4.1	1.01	39.08	478.65
LA05	Imperial Valley, 1979, Array #06	6.5	1.2	0.84	39.08	295.69
LA06	Imperial Valley, 1979, Array #06	6.5	1.2	0.84	39.08	230.08
LA07	Landers, 1992, Barstow	7.3	36	3.2	79.98	412.98
LA08	Landers, 1992, Barstow	7.3	36	3.2	79.98	417.49
LA09	Landers, 1992, Yermo	7.3	25	2.17	79.98	509.7
LA10	Landers, 1992, Yermo	7.3	25	2.17	79.98	353.35
LA11	Loma Prieta, 1989, Gilroy	7	12	1.79	39.08	652.49
LA12	Loma Prieta, 1989, Gilroy	7	12	1.79	39.08	950.93
LA13	Northridge, 1994, Newhall	6.7	6.7	1.03	59.98	664.93
LA14	Northridge, 1994, Newhall	6.7	6.7	1.03	59.98	664.93
LA15	Northridge, 1994, Rinaldi RS	6.7	7.5	0.79	14.94	523.3
LA16	Northridge, 1994, Rinaldi RS	6.7	7.5	0.79	14.94	568.58
LA17	Northridge, 1994, Sylmar	6.7	6.4	0.99	59.98	558.43
LA18	Northridge, 1994, Sylmar	6.7	6.4	0.99	59.98	801.44
LA19	North Palm Springs, 1986	6	6.7	2.97	59.98	999.43
LA20	Northridge, 1994, Sylmar	6	6.7	2.97	59.98	967.61

**RESULTS AND DISCUSSION**

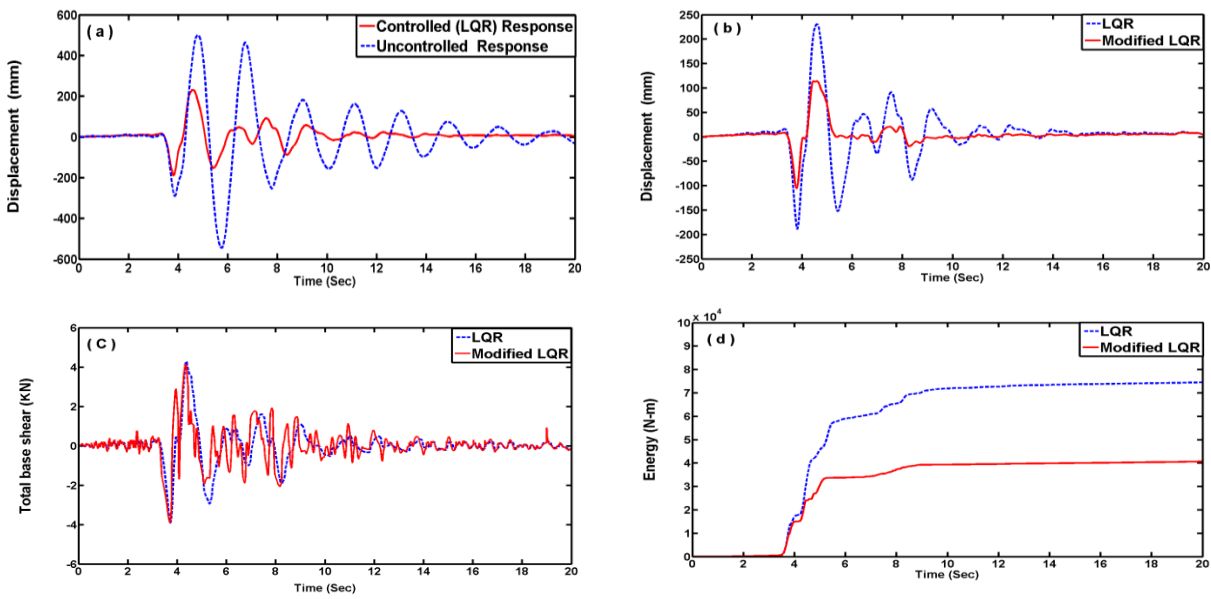
Figures 6–7 show the time history response of the uncontrolled structure with period of 2.0 seconds and the controlled response with a MR damper that utilized the classic LQR and modified LQR control method under Northridge (Sylmar, 1994- LA18) and Imperial Valley (El Centro, 1940-LA03) ground motions, respectively. To illustrate the potential application of the proposed method, the response of the structure, the total base shear and the energy corresponding to the controller for the proposed method are compared with the conventional LQR. In this study, the energy of control forces is computed using Eq.19 where  $F_c$  is the control force and  $\dot{u}$  is an n-dimensional velocity vector.

$$E_c = \int_0^t F_c^T \dot{u} dt \quad (19)$$

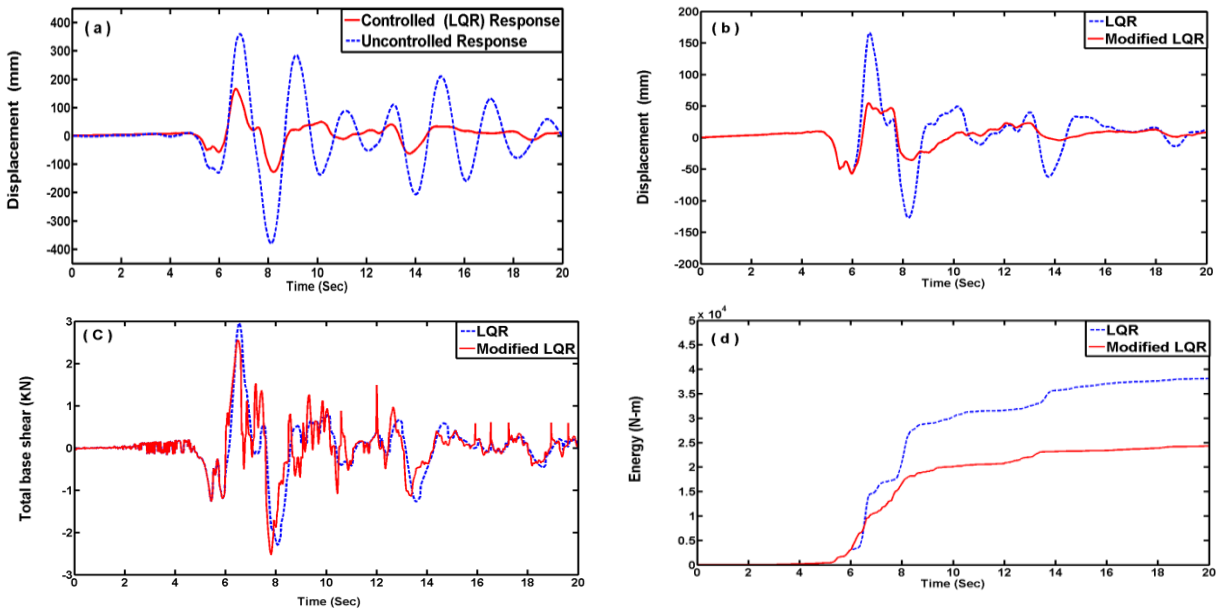
Figures 6–7 show that the displacements, as well as the total control energy demand are significantly reduced with using

the proposed method. Although the control force increases to counteract the undesirable response the total base shear does not change. The maximum displacements and energy demand of this structure under the entire 20 earthquakes in the medium suite of SAC with 2 control methods project are shown in Table 3 and Figure 8.

The results show that the proposed method could better control the response of structure with lower energy demand. The proposed method could improve the classic LQR method in control of MR damper by about 1%-68% with reducing energy demand of about 5%-59%. The advantage of the modified LQR is that it has the ability to change the value of control force on specific frequencies when undesirable response occurs, in contrast to the classical LQR. That means the modified LQR method is more robust than the classic LQR method to control of MR damper and structures.



**Figure 6: Result for the 1994 Northridge, Sylmar (LA18): (a) controlled and uncontrolled displacement, (b) controlled displacement, (c) control force, (d) energy demand.**



**Figure 7: Result for the 1940 Imperial Valley, El Centro (LA03): (a) controlled and uncontrolled displacement, (b) controlled displacement, (c) control force, (d) energy demand.**

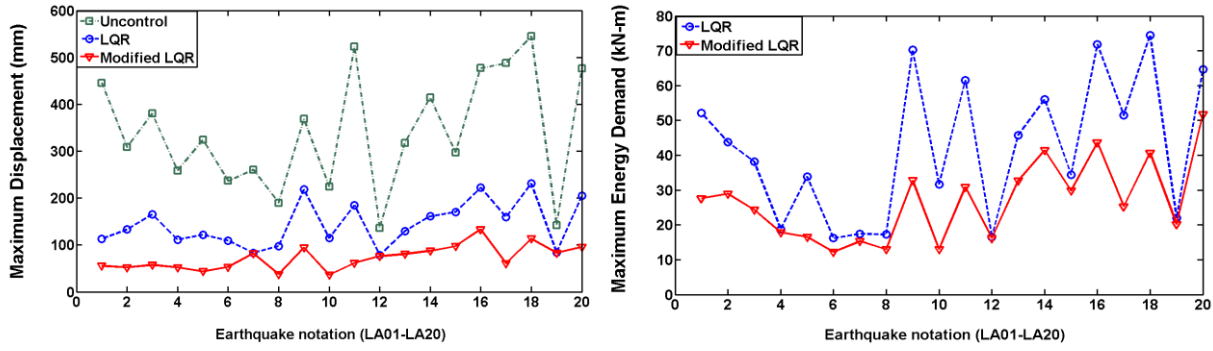


Figure 8: Maximum displacement and energy demand of structure with LQR and the modified.

Table 3: Maximum displacement and energy demand of structure with LQR and the modified LQR, and the percent difference between them.

Earthquake	uncontrolled Spectral Displacement, $S_d$ , (mm)	Controlled Spectral Displacement, $S_d$ , (mm)		Reduction of uncontrolled $S_d$ with Modified LQR $\frac{([2]-[4])}{[2]}$ %	Maximum energy demand (kN-m)		Percentage Reduction LQR → Modified LQR	
		LQR	Modified LQR		LQR	Modified LQR	$\frac{([3]-[4])}{[3]}$ %	$\frac{([7]-[6])}{[6]}$ %
[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]
LA01	445.72	113.16	55.58	87.53%	52.12	27.65	50.88%	46.94%
LA02	308.32	133.06	52.01	83.13%	43.77	28.96	60.91%	33.84%
LA03	380.06	165.72	57.41	84.89%	38.12	24.29	65.35%	36.28%
LA04	258.63	111.14	52.24	79.80%	18.75	17.87	53.00%	4.70%
LA05	324.45	122.00	43.64	86.55%	33.91	16.56	64.23%	51.16%
LA06	236.82	109.51	53.02	77.61%	16.20	12.2	51.58%	24.67%
LA07	260.37	82.86	82.24	68.41%	17.44	15.29	0.748%	12.36%
LA08	189.94	97.03	37.65	80.18%	17.27	12.93	61.20%	25.14%
LA09	368.55	218.30	94.32	74.41%	70.23	32.77	56.79%	53.34%
LA10	224.45	114.60	36.24	83.85%	31.71	13.04	68.37%	58.88%
LA11	523.37	184.48	61.51	88.25%	61.55	30.88	66.66%	49.83%
LA12	136.08	78.13	75.73	44.35%	16.48	16.14	3.065%	2.063%
LA13	317.51	129.70	80.58	74.62%	45.76	32.66	37.87%	28.63%
LA14	414.68	161.25	87.65	78.86%	55.99	41.45	45.64%	25.97%
LA15	297.52	170.52	97.41	67.26%	34.38	29.81	42.87%	13.30%
LA16	477.08	222.43	133.48	72.02%	71.88	43.7	39.99%	39.20%
LA17	487.78	159.19	60.36	87.63%	51.42	25.3	62.09%	50.79%
LA18	545.03	230.71	113.49	79.18%	74.48	40.66	50.81%	45.41%
LA19	142.11	84.16	83.24	41.43%	21.95	20.21	1.091%	7.927%
LA20	476.29	204.54	95.82	79.88%	64.67	51.75	53.15%	19.98%

Figure 9 compares the maximum energy of the MR damper and the maximum displacement of structures with periods of 0.2-5.0 seconds under 20 SAC earthquakes. The results indicate that the range of the maximum displacement of the LQR is about 90 -250 mm and the energy range is 1.0-12 kN-m. However, the range of displacement and energy are 20-140 mm and 1.0-5.8 kN-m when the proposed method is used, respectively. The required input energy of the MR damper

when using the proposed method is half of the amount of energy when using LQR. Moreover, the amount of input energy with the MR damper that utilized LQR could not reduce displacements as much as the proposed method. Therefore, MR dampers that used the proposed method provided improved response during earthquakes and counteract the undesirable responses.

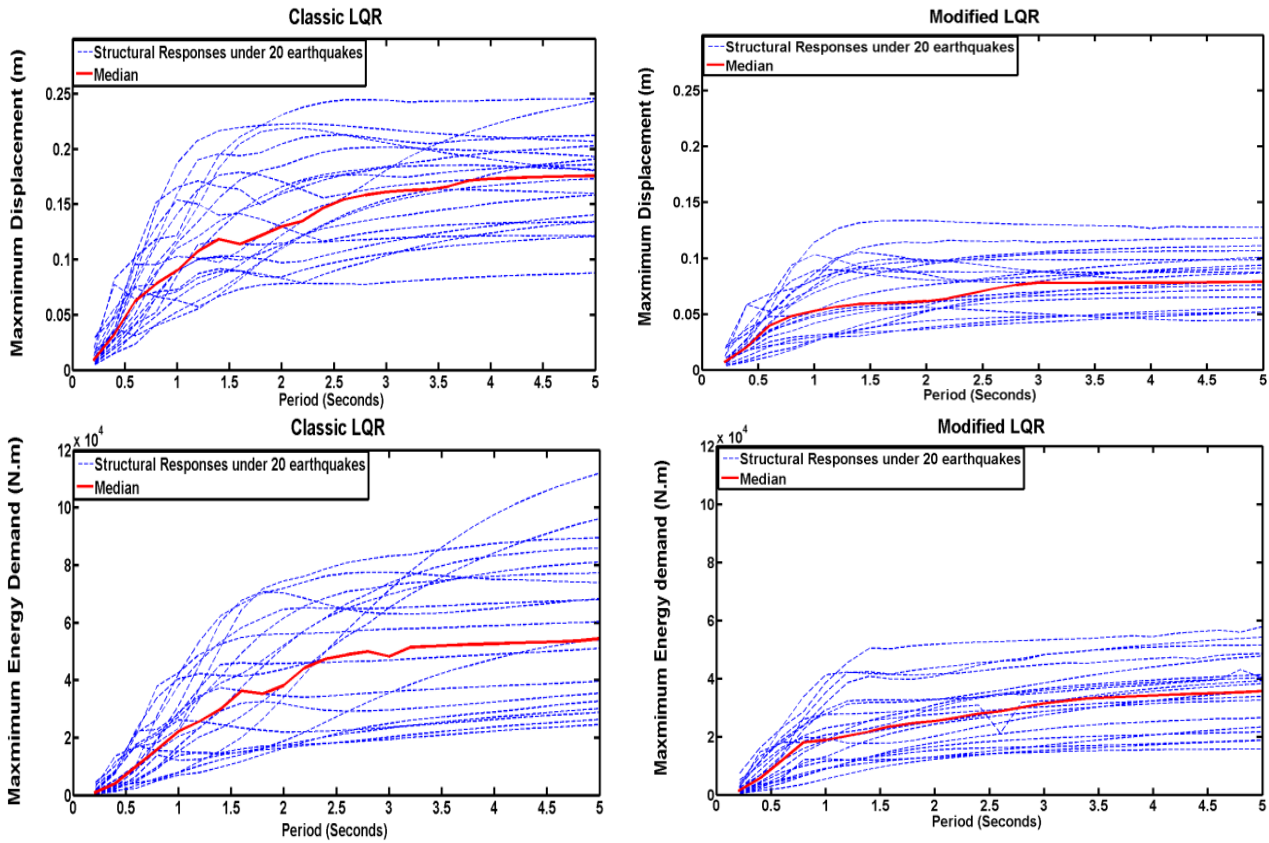


Figure 9: Maximum displacement and energy demand of structures with considering two method of control of structure.

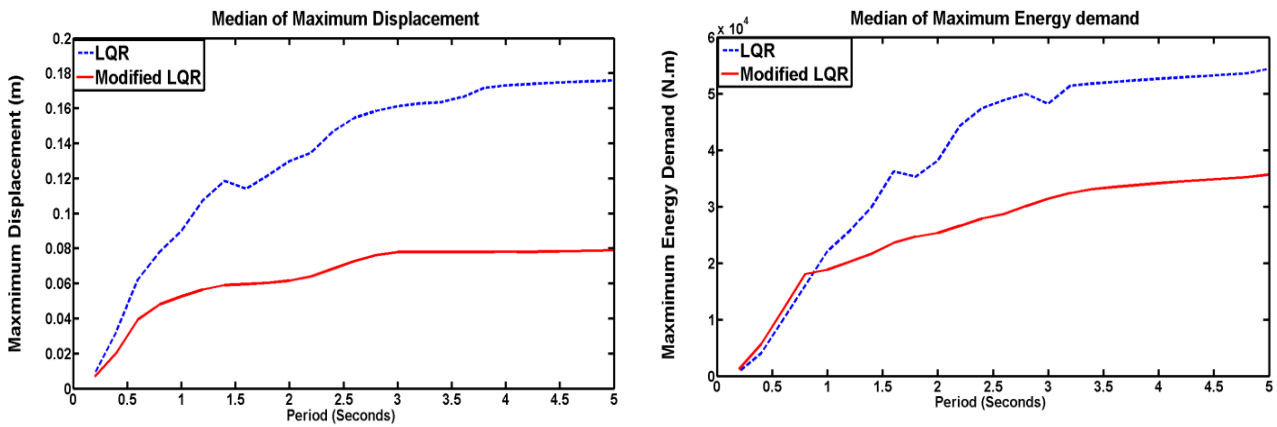


Figure 10: Median of Maximum displacement and energy demand for the classic LQR and the modified LQR.

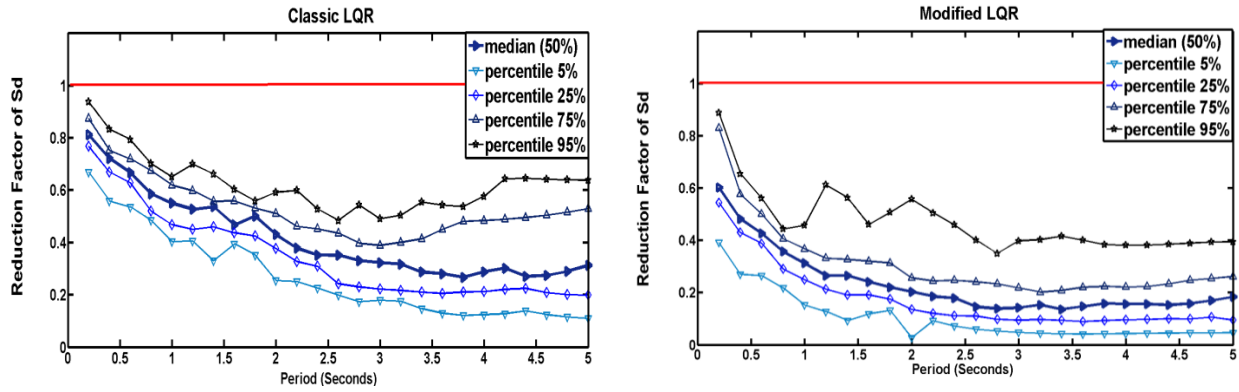


Figure 11: The different percentile of displacement ( $S_d$ ) and total base-shear ( $F_b$ ) response reduction factor ( $R_f$ ) spectra for the MR damper that used the LQR control method and the modified LQR control method.

The different percentile  $R_f$ s for  $S_d$  for two control methods are shown in Figure 11. The classic LQR provides the range of 0.3 to 0.8 for 50<sup>th</sup> (median) percentile reduction factor. However, the median  $R_f$  of the modified LQR is between 0.2-0.6. The results indicate that using the proposed method to control the MR damper reduces the seismic response of the structure significantly. The modified LQR control method could capture the resonance time, so the MR damper could apply more control force to counteract the resonance effect. Figure 12 shows the median  $R_f$  and smoothing reduction factor of displacement for each control methods. An expression proposed by Figure 12 is defined by Eq.20 and Eq.21 where  $T$  is the period of the structures for the LQR control law and modified LQR control law, respectively, which are defined:

$$R_f = 0.033 * T^2 - 0.27 * T + 0.82 \quad (20)$$

$$R_f = 0.037 * T^2 - 0.25 * T + 0.57 \quad (21)$$

For example, a MR damper with the capacity of 3kN can be installed to reduce the displacement of a structure with period of 1 second. The reduction of displacement when using this MR damper is dependent on the control method. If the classical LQR method is used, the structure's displacement reduces by about 41 percent ( $R_f = 0.58$ , Eq.20). However, the reduction of maximum displacement of this structure is 64 percent when the modified LQR is used ( $R_f = 0.357$ , Eq.21). Therefore, the modified LQR is more effective at mitigating the structural responses than the classical LQR when the same device is used.

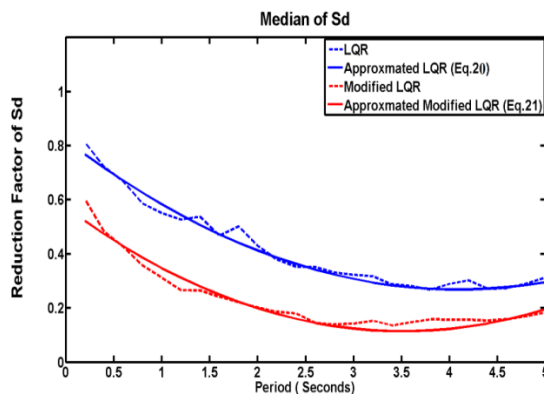


Figure 12: Median reduction factors of displacement for the LQR and modified LQR control methods.

## CONCLUSIONS

A modified LQR algorithm based on the modified Bouc-Wen model has been implemented in the present study to control the seismic vibrations of a structure with MR dampers. To lead the MR damper force close to the optimal control force during the control time, the modified LQR algorithm is used. In this method, the optimal control force is obtained by modifying the conventional LQR controller by updating the weighting matrices applied to the response energy and the control effort, over time intervals. Maximum displacements and reduction factors of maximum energy demand and displacements of structure with periods of 0.2-5.0 seconds are shown to determine the impact and efficiency of the conventional and proposed modified LQR controller on seismic structural performance over a range of ground motions with equal probability of occurrence. The modified LQR controller performs better than the classical LQR controller in reducing the control energy demand and displacement response of the structure. The proposed control method has the ability to vary its properties according to the external load to more effectively control the structure. Overall, the modified LQR controller appears to be an appealing solution for reducing seismic response, with minimal risk of damage to the structure.

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