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1. Editorial Foreword

This paper is reproduced from the proceedings of a seminar on "Seismic Problems in Structural Engineering" arranged by the Departments of Civil Engineering and Extension Studies of the University of Canterbury, and held in Christchurch from May 13 to 16, 1968. Another paper from that seminar, also published in this issue of the Bulletin, discusses requirements for ductility in reinforced concrete structures. The present paper makes a comparison of prestressed concrete with reinforced concrete, and discusses the factors to be considered in the design of prestressed structures for earthquake resistance.

2. Summary

Prestressed concrete has been used in primary seismic resisting frames and towers of moderate height, particularly in long span frames or where precast elements are assembled into frames by post-tensioning. The response of fully prestressed concrete structures is generally considered to be greater than "comparable" reinforced structures because of bilinear elastic rather than elastoplastic behaviour, and because of a lower degree of damping. The main difference between prestressed and reinforced concrete (p.s.c. and r.c.) in the range of interest is in the properties of the steel. The rules for the design and detailing of the concrete, including ductility requirements, are similar to those for the concrete in reinforced concrete, except that the residual stress in prestressing tendons in the compressive zone can lead to over-reinforced characteristics appearing earlier than in comparable reinforced concrete members.

The notes briefly discuss the factors to be considered in the seismic design of prestressed structures and a suggested design procedure for a typical frame is given. Comparisons are made with reinforced concrete. A design example is presented.

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General.

Prestressed concrete is widely used in structural elements not contributing to the primary vertical seismic resisting system, (e.g. flat plates and precast floor units), although these are required to provide horizontal diaphragm action. The seismic resistance required by any prestressed member is therefore one of degree.

Moment resisting frames may have:

- (a) Beams and columns fully prestressed.
- (b) Beams prestressed, columns normally reinforced.
- (c) Beams partially prestressed, columns normally reinforced.

If an increased response is to be taken into account for (a), then intermediate values should be taken for (b) and (c).

Walls or towers can be fully prestressed vertically, thus eliminating the laps that would be required in normally reinforced walls.

Post-tensioning can be used to assemble precast units into rigid frames - this method may be used in short or long span structures. Normal reinforcing has been introduced as well as post tensioning tendons in one large building assembled from precast waffle slab units⁽¹⁰⁾.

The behaviour of the concrete in p.s.c. members is similar to that of the concrete in r.c. members except that:

- (a) Concrete strengths are likely to be higher.
- (b) Less permanent damage is likely since cracks are more likely to close again.
- (c) Prestressing tendons in the compression zone act to provide a variable additional compressive force which can lead to over-reinforced characteristics occurring before those of comparably sized r.c. members.

The steel used for prestressing differs from mild steel, but has some similarities with the higher tensile steels (unstressed) sometimes used in r.c. The stressing steel has:

- (a) A strength approximately six times that of mild steel and steel areas therefore substantially less than r.c.
- (b) No well defined yield point.

- (c) A lower minimum elongation than mild steel (generally 4% minimum which is adequate for the seismic behaviour of the steel in practical members).
- (d) Special anchorage and bond requirements (large forces have to be anchored in joints with complex stress patterns, and cracks in the length of members will generally open wider and be spaced further apart than in r.c.).

P.s.c. remains uncracked longer than r.c. during a loading cycle and therefore comparable structures of p.s.c. will be stiffer than r.c. for small displacements but less stiff for large displacements, ("comparable" structures in these notes means p.s.c. and r.c. structures with the same overall dimensions, member sizes and concrete strengths - reinforcing quantities and ductility requirements will differ).

The obtaining of a sufficient degree of seismic resistance in a structure is necessarily dependent on:

- (a) The intensity and number of governing earthquakes during the life of the structure.
- (b) The required resistance to damage of the remainder of the building and its contents.

If (b) is the more important in a particular structure, then a comparison of the likely actual displacements of comparable p.s.c. and r.c. structures would be important. Further information is needed to verify that a "comparable" p.s.c. structure would have a maximum displacement for a particular earthquake exceeding that of the r.c. structure.

Grouting of all tendons is recommended to increase damping and improve safety.

Rosenblueth^(1,3), Guyon⁽¹¹⁾, Despeyroux^(4,5), and Zavriev⁽⁸⁾ have commented on the theoretical aspects of p.s.c. seismic behaviour. Sutherland⁽⁶⁾ has reported on the development and performance of p.s.c. and on current research projects. He also described a number of buildings using p.s.c. for seismic resistance. Lin⁽⁹⁾ gives information on current practice in California and describes some buildings. Nakano⁽⁷⁾ has tested a 4-storeyed model structure and considerable research and testing is being done in Japan and Russia.

P.s.c. members have been reported to have performed well in actual earthquakes although there have been connection and other failures not attributable to p.s.c. itself. A continuing survey should be made to obtain up to date information on earthquake performance.

Response of Prestressed Concrete Structures.

Experience of the behaviour of p.s.c. in actual earthquakes has not been sufficient to determine response by observation.

The static force deformation curve for fully prestressed members tends towards that for a bilinear elastic material (Fig.1) whereas the curve for a normally reinforced member is often assumed as that for an elastoplastic material (Fig. 2).

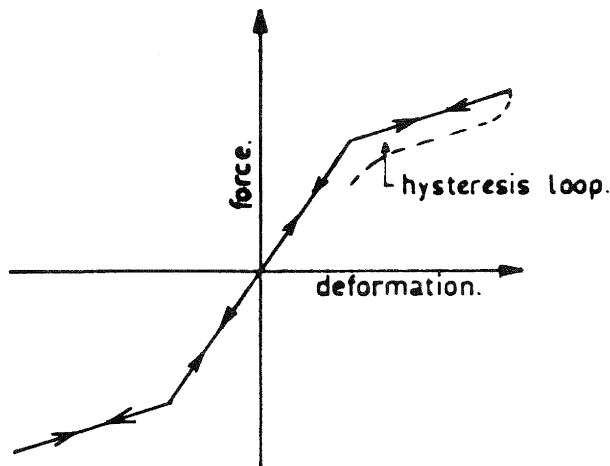


Fig.1 Idealised Bilinear Elastic System.

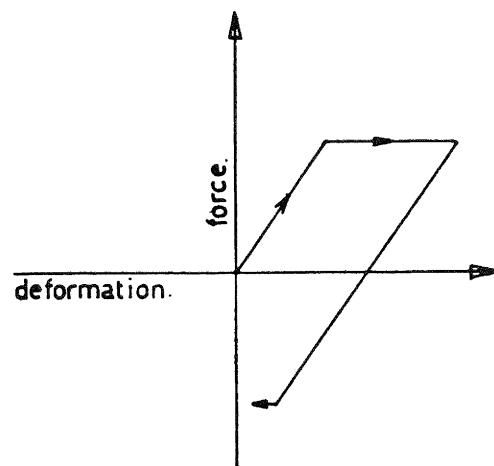


Fig.2 Idealised Elastoplastic system.

The energy absorbed is largely dissipated by plasticity in the elastoplastic case, whereas the energy at first cycle in the bilinear elastic case is largely released as kinetic energy. It is generally assumed therefore that the responses for p.s.c. and r.c. will differ and may be larger for p.s.c.

Hysteresis effects could be significant in p.s.c., particularly at near ultimate conditions. These effects have been the subject of recent research at the Universities of Auckland and Canterbury.

R.c. suffers "stiffness degradation" with repeated loadings, presumably due to increasing damage. This leads to reduced response. The concrete of p.s.c. could suffer similarly if not well detailed, but recovery could be expected to be better due to prestress effects.

The bilinear curve for p.s.c. is made up of a lower part which represents the behaviour of the uncracked concrete and an upper part which represents the cracked concrete in which the steel and concrete are both still in the elastic range (Fig.3). Continued loading leads to a flattening of the upper part due to inelastic effects in steel and concrete. The "apparent ductility factor" for p.s.c. can be defined as the total dis-

placement divided by the displacement at first crack (the tensile strength of the concrete should be neglected). For low "ductility factors", the concrete and steel will remain elastic whereas for higher factors, a true ductility factor will be added. The prestressing steel is itself ductile and the ductility factor achieved will depend mainly on the concrete.

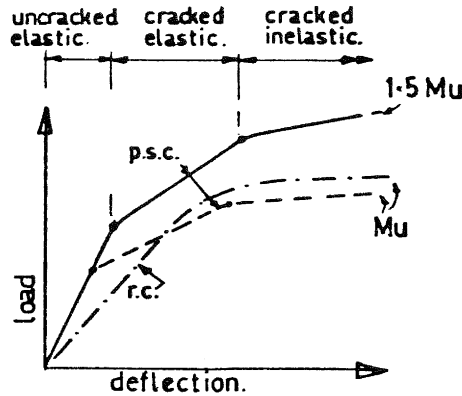


Fig.3 Typical Load-Deflection Curves for Comparable p.s.c. and r.c. Members.

Despeyroux⁽⁴⁾ has compared the cracking moment of p.s.c. with the yield moment of r.c. and therefore implies a considerably stronger p.s.c. member than the comparable r.c. member. Other non-published information indicates that on this basis, the displacement of p.s.c. in a particular case could be greater than r.c.

The actual situation would be affected by hysteresis in p.s.c., and by the differing initial stiffness of comparable p.s.c. and r.c. structures. A less severe approach may therefore be justified.

American and Russian practice does not appear to take any increased response of p.s.c. into account (7,8,14).

A conservative approach is recommended at this stage and where the same seismic coefficients are used for p.s.c. and r.c. as at present required by N.Z.S.S. 1900 Chapter 8, it is suggested that the necessary seismic part of the strength of a p.s.c. member be made up to 50% greater than for r.c.

Member sizes for p.s.c. structures are not necessarily smaller than r.c. and it should not necessarily be assumed that p.s.c. is more flexible and for this reason responds less than r.c. to a particular earthquake.

Vertical Acceleration.

P.s.c. members are more economical when designed to resist a permanent loading in one direction only, (e.g. dead load). An unfavourable loading case can therefore exist when the assumed permanent load is reduced or removed by vertical acceleration. The effects can be analysed if necessary, taking into account the reduction in cable loads due to strain of surrounding concrete, and an ultimate strength approach would be suitable.

Applications of Prestressing.

The most common applications of p.s.c. for primary seismic resistance are:

- (a) Long span rigid frame concrete structures where prestressing is provided to react the permanent dead load and part of the live load. Such structures have to be relatively high before earthquake governs the design of the column/beam connections, since the gravity moments in the span increase as the square of the span whereas the seismic moments increase at a lesser rate (linearly as a first approximation). When earthquake moments exceed the capacity of the tendons extended through columns, then additional mild steel would probably be preferred rather than additional prestress.
- (b) Structures incorporating precast elements, in cases where the preferred method of assembly is by post-tensioning into rigid frames. Such structures may be short span and gravity loadings may not be significant (e.g. wall spandrels). In these structures, fully prestressed members may be designed solely for seismic resistance.
- (c) Walls and Towers, vertically prestressed.

Full and Partial Prestress.

Members can be fully prestressed with no normal reinforcing at critical sections or various combinations of tendons and bars can be used (partially prestressed). Design rules established for full p.s.c. and normal r.c. could be interpolated for partial prestress.

Columns.

Columns are sometimes prestressed, particularly when precast, but in high buildings, r.c. columns will have a greater direct load capacity than p.s.c. for a given size. Suggested design procedures for non-seismic prestressed columns are given by Zia and Guillermo⁽¹⁴⁾.

Design Notes.

A suitable design reference is N.Z.S.R. 32 "Prestressed Concrete" (2) or ACI 318/63 (13) (seismic design rules are not included in N.Z.S.R. 32).

Columns of frames with p.s.c. beams can be either prestressed or normally reinforced. (For information on p.s.c. columns see ref. (14).)

For seismic frames with p.s.c. beams, grout all tendons and effectively anchor the tendons in or through the columns.

The following design procedure can be used for p.s.c. beams in a typical long span seismic frame with tendons extended through the columns:

- (a) Estimate beam size assuming simple span support (an allowance for end fixity can be made).
- (b) Estimate column size.
- (c) Calculate number of tendons required at bottom of beam at mid span, based on ultimate strength requirements for dead load and full live load. (Except for the allowance for end fixity, this is the minimum number of tendons required).
- (d) Calculate number of cables required to "balance" dead load plus seismic live load (a T beam can be assumed to have a flange width equal to the beam width at the supports varying parabolically to normal effective flange width at mid-span). The eccentricity for load balance will then be the T beam eccentricity at mid span, and the tendon c.g.s. can be raised to pass through the mid beam depth at the column centreline where the eccentricity is then zero. Ideally, the number of cables required to "balance" dead load and seismic live load will be the same as required for ultimate strength, but this will depend on the dead load : live load ratio and the seismic live load : live load ratio. The design procedure would be varied to suit the particular case.
- (e) In the "balanced" state with dead load and seismic live load acting downwards and full prestress acting upwards (equivalent to a uniform load equal to the dead load and seismic live load), there will be no

end rotation in the beam and the columns and beams will be subjected to direct loads only.

- (f) Carry out a moment distribution for the worst case of a gravity load equal to the difference between live load and seismic live load (i.e. either $1/3$ or $2/3$ live load depending on the code value of live load). This gives the mid span moment to be added to the uniformly stressed T beam when checking stresses at mid span for dead load and full live load.
- (g) Determine the seismic shears according to N.Z.S.S. 1900 and analyse the frame for these shears.
- (h) Check the adequacy of the beam at the column face for the design earthquake moment using an ultimate strength method and applying an appropriate load factor to the design seismic moment. This ultimate seismic moment only is applied to the uniformly stressed net beam section without any applied dead load or live load gravity moments. (These moments have in effect been redistributed as positive moments to the mid span section.) Ref. (16) gives the basic approach to the ultimate strength calculation and additional account need only be taken of the initial strain in the tendons (say 0.0045). Over reinforced characteristics should be avoided.
- (i) Check the ultimate capacity of the beam at all intermediate sections, taking a moment envelope of ultimate seismic moment diagram combined with an ultimate simply supported moment diagram for all the necessary seismic gravity loading cases.
- (j) Check concrete ductility, shear, deflection, and deferred strain effects as necessary.
- (k) Check stresses at critical sections for design loads (allowing for T beam effects).
- (l) If the tendons alone are insufficient to provide the required seismic resistance, provide additional unstressed reinforcing to make up the difference.

The New Zealand Prestressed Concrete Institute's publication "Seismic Design Recommendations for Prestressed Concrete"⁽¹²⁾ gives general information on design requirements. The load factors suggested should be considered as tentative only and

subject to review as further information becomes available. Meanwhile some increase may be desirable.

In many cases, the capacity of the prestressed members, predetermined by gravity loading requirements, will be more than sufficient for the seismic requirements so that the beam sections will not be critical in seismic calculations.

Detailing.

The principles of detailing of shear reinforcement, secondary reinforcement and of ductility reinforcing will be similar to those for r.c. Special attention should be paid to areas where tendons occur in compressive zones, and hooping or additional longitudinal reinforcing may be required.

The transfer of all loading components in the highly stressed beam column junction should be examined and a rational analysis done to verify that cable tensions can be transferred to column reinforcing. Where the anchors lie within the columns, additional stirrups may be required. In the case of small columns, the only satisfactory solution may be to extend the tendons through the columns and anchor them in external end blocks.

Pretensioned tendons require a considerable length for anchorage⁽¹²⁾ and if this additional length can not be provided, special end anchorage or lapping bars should be provided.

P.s.c. Design Example.

A preliminary design is required for the 1st floor beam in a moment resisting frame as follows: (neglect effect of beam shortening and prestress losses).

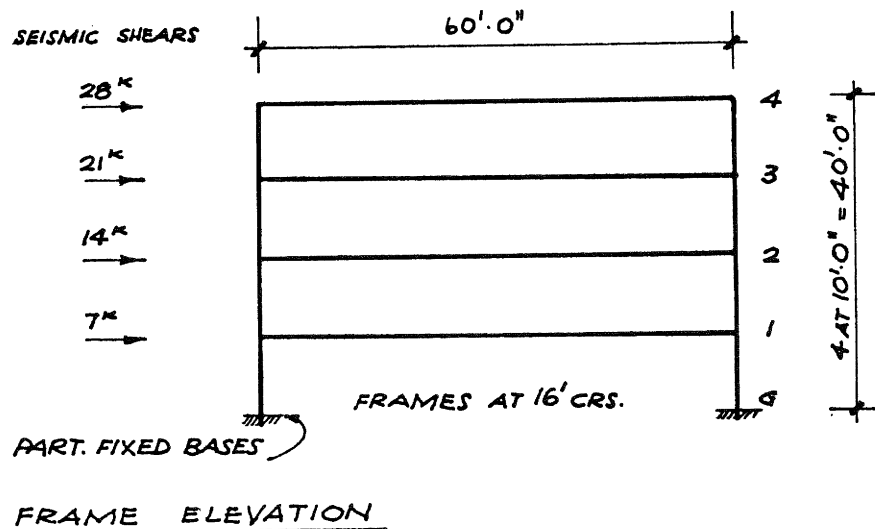


Fig. 4

Design Data

(a) Loads

- Live load = 100 p.s.f. (reduce to say 90 p.s.f. for large area).
- Seis. L.L. = 33 p.s.f.
- Dead load = 120 p.s.f. (gross), 110 p.s.f. (floor system only).
- Seis.coeff. = 0.12g (say) distributed in height according to N.Z.S.S. 1900, Chapter 8.

(b) Structural System

- Assume cast-in-place one way r.c. slab and p.s.c. T beam;
- Columns r.c. cast-in-place.

(c) Beam Cross Section

Assume beam thus:

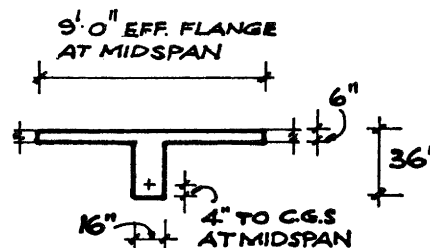


Fig. 5. BEAM SECTION

(d) Column Cross Section

Say 24" x 18"

(e) Materials

Assume $f'_c = 5500$ psi; $f'_s = 224,000$ psi,
 $E_s = 29 \times 10^6$ lbs/sq.inch.

(f) Design References

References (12) and (13).

(g) Seismic Shears

Seismic gravity load/storey

$$d1 + s11 = 60' \times 16' \times (120 + 33) \\ = 147^K$$

$$H = 147 \times 4 \times 0.12 \\ = 70^K$$

Loading Cases

(a) dead load + live load

(b) dead load + seismic live load + earthquake

Section Properties

<u>T beam (9'0" flange)</u>	<u>Net beam 36" x 16" (at column)</u>
$A_c = 1128 \text{ ins}^2$	$A_c = 576 \text{ ins}^2$
$I = 127,000 \text{ ins}^4$	$I = 62,000 \text{ ins}^4$
$Z_b = 5000 \text{ ins}^3$	$Z_b = 3450 \text{ ins}^3$
$y_b = 25.4 \text{ ins}$	$y_b = 18 \text{ ins}$
<u>Gross beam and slab</u>	<u>Column</u>
$A_c = 1630 \text{ ins}^2$	$I = 20,700 \text{ ins}^4$

Idealised Beam

To simplify prestress calculations assume beam has effective flange varying approximately in width parabolically from zero at columns (net beam) to 9'0" o.a at mid span (T beam).

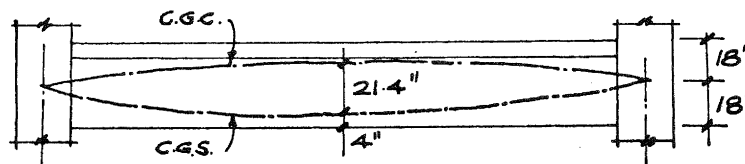
CABLE PROFILE

Fig. 6.

The cable profile is then approximately a linear transformation of a concordant profile.

Loading Case Dead Load Plus Live Load (dl + ll)

The beam will have some end fixity but can conservatively be assumed as simply supported. (The final design would be checked taking into account the beam and column stiffnesses).

(a) Ultimate

$$\begin{aligned} \text{Applied } M_u &= 1.5 \text{ dl} + 1.8 \text{ ll} \\ &= 29,500,000 \text{ ip} \end{aligned}$$

$$\text{Approx. } \phi M_u = \phi 0.9 A_s f'_s d \text{ (T beam) } (\phi=0.9, d=32")$$

$$\text{Hence } A_s f'_s = 1,140,000 \text{ p}$$

$$\text{Say 5-12 w 315 cables (} A_s = 5 \times 0.93 = 4.65 \text{ ins}^2)$$

$$A_s f'_s = 1,040,000 \text{ p}$$

(b) Working Stress

$$\begin{aligned} M_{dl + ll} &= 18,200,000 \text{ ip (simple support),} \\ P &= 5 \times 120^K = 600 \text{ kips} \end{aligned}$$

$$f_{bt} = -3640 \text{ psi (tension from dl + ll only)}$$

$$f_{pe} = +3120 \text{ psi (Bott. comp. due to pre-stress only with } P/A \text{ on gross beam and slab section)}$$

Hence 500 psi tension if simple span - end fixity can counteract this. (Alternatively apply mid-span frame moment for live load only beyond "balanced" loading case).

Loading Case Dead Load + Seismic Live Load + Earthquake
(dl + sll + e.g.)

The example has been chosen so that idealised free beam has no end rotations when loaded with dl + seis. ll + prestress. In this example, the tendon requirements for dl + full ll are consistent with those for no end rotations under dl + sll + prestress - in practical examples this will not necessarily be the case.

Tendon ecc. for
balancing $d1 + s11 = \frac{M_{d1} + s11}{P} = 22''$ (21.4" provided)
(No end rotations
of free beam)

($M_{d1} + s11 = 13,200,000$ ip,
 $P = 600,000$ p)

Hence tendons produce load balance for $d1 + s11$ and ends of beams and columns can be designed for uniform direct loads plus seismic bending only. (The gravity moment at the beam ends from $d1 + se11$ has then been cancelled out by prestress.)

In effect the design seismic moments are applied to a simply supported beam which is subjected to P/A only throughout its length.

(a) End Moments From Seismic Shears

Approximately assuming mid-height points of contraflexure in the upper storey and point of contraflexure say in the lower third point of the lower storey (partial base fixity)

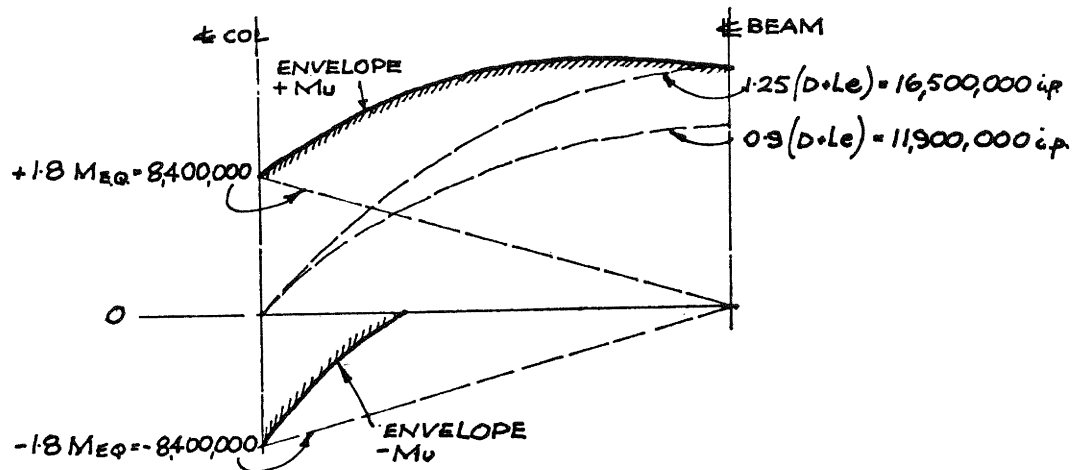
M seismic at end of beam

$$= \left(\frac{63K}{2} \times 5' + \frac{70K}{2} \times 6.7' \right) \times 12 \times 1000 \text{ ip}$$

$$= \underline{4,660,000 \text{ ip}}$$

(b) Ultimate Moments for $d1 + s11 + eq$

Assume required load factors $U = 1.25 (D + L_e) + 1.8E$
or $U = 0.9 (D + L_e) + 1.8E$
(Other load factors might be considered)

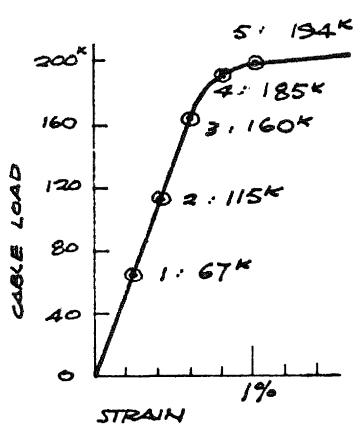
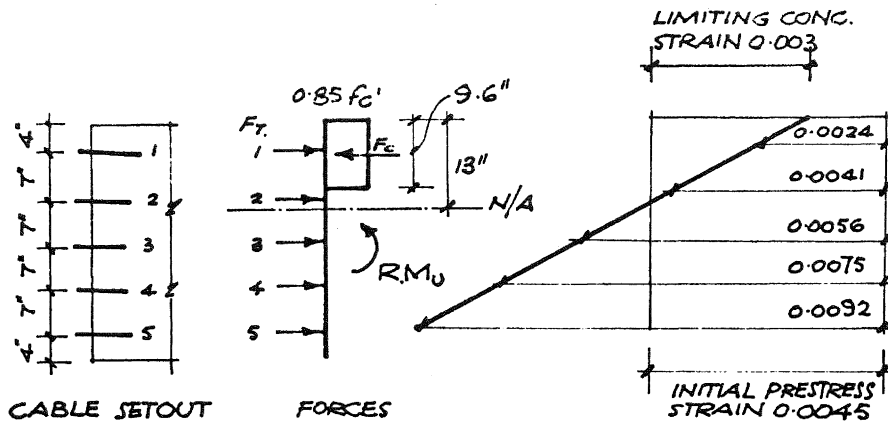


ULT. MOMENTS

Fig. 7.

(c) Check Ultimate Flexural Capacity at Ends of Beam

Assume 5 tendons arranged vertically at 7" c/c symmetrically about beam centre line. Net beam section applies at column face. (An iterative method can be used to find the neutral axis.)



STRAIN DIAGRAM (SHOWING TOTAL STRAINS FOR EA. TENDON)

FROM SEPARATE CALC. $F_C \approx \sum F_T$
 HENCE, $R.M_U = \phi \times 11,790,000 \text{ i.p.}$ ($\phi = 0.9$)
 $= 10,600,000 \text{ i.p.}$

HENCE, $R.M_U = \underline{2.27} \times \text{DESIGN SEISMIC MOMENT}$

CABLE LOADS (FROM STRESS/STRAIN DIAG.)

ULT. FLEXURAL CAPACITY Fig. 8.

The assumed section is therefore stronger than would be required by normal load factor. Normally designed columns would yield before the beam reached its ultimate capacity. However, the beam section should be checked for ductility in the final design.

(d) Working Stress Method

The suggested procedure in Ref. (12) is to take three quarters of all moments, loads, prestress forces, and then use normal working stresses. (This method is the p.s.c.)

equivalent of the usual 33 1/3% stress increase for seismic loadings.)

$$\begin{aligned} \frac{3}{4} \times M \text{ seismic} &= 3,500,000 \text{ ip} \\ f_b &= \pm 1010 \text{ psi} \\ \cdot \frac{3}{4} \times P/A &= + 780 \text{ psi} \end{aligned}$$

"Effective" max. comp. stress is not too high. "Apparent" tensile stress of 230 psi is reasonable.

Final Design

The final design would require an ultimate strength and working stress check at any other critical sections. The need for special ductility reinforcing at critical sections should be examined, particularly at the bottom of the beam near the ends. The location of tendon anchorages should take into account the transfer of tension from tendons to column reinforcing. The effects of shrinkage and axial shortening should be examined.

References

- (1) Rosenblueth, E.: "On Earthquake Resistant Design", N.Z. Eng. (Sept. 1966).
- (2) "Prestressed Concrete": N.Z. Standard Recommendation (N.Z.S.R.32).
- (3) Rosenblueth, E.: Discussion on paper by T.Y. Lin, "Design of Prestressed Concrete Buildings for Earthquake Resistance", Proc. A.S.C.E.Struct.Division, (Oct.1965).
- (4) Despeyroux, J.: "On the Use of Prestressed Concrete in Earthquake Resistant Design", Proc. 3rd World Conference on Earthquake Eng. Vol. III (1965).
- (5) Despeyroux, J.: "On the Use of Prestressed Concrete in Earthquake Resistant Design", Travaux, (April-May 1966).
- (6) Sutherland, W.M.: "Prestressed Concrete Earthquake Resistant Structures - Development, Performance and Current Research", Proc. 3rd World Conf. on Earth-

quake Eng. Vol. III, (1965).

- (7) Nakano, K.: "Experiment on Behaviour of Prestressed Concrete Four Storeyed Model Structure under Lateral Force", Proc. 3rd World Conf. on Earthquake Eng. Vol. III, (1965).
- (8) Zavriev, K.S.: "Dynamics and Earthquake Resistance of Prestressed Reinforced Concrete Structures", Proc. 3rd World Conf. on Earthquake Eng. Vol. III (1965).
- (9) Lin, T.Y.: "Design of Prestressed Concrete Buildings for Earthquake Resistance", Proc. A.S.C.E. Struct. Division (Oct.1965).
- (10) Allardice, N.W.: "Engineering Aspects of the Arts and Library Building, Victoria University of Wellington", N.Z.Eng. (Dec. 1966).
- (11) Guyon, Y.: Discussion on Ref. (4) above.
- (12) "Seismic Design Recommendations for Prestressed Concrete", N.Z.P.C.I. (1966).
- (13) "A.C.I. Standard Building Code Requirements for Reinforced Concrete", Chapter 26, Prestressed Concrete (ACI318/63).
- (14) "Combined Bending and Axial Load in Prestressed Concrete Columns". Zia and Guillermo (PCI) Journal, June, (1967).
- (15) "Principles of the Design and Construction of Earthquake Resistant Prestressed Concrete Structures". PCI Seismic Committee - PCI Journal (June 1966).
- (16) Park, R.: "Ultimate Strength Design of Reinforced Concrete Structures", University of Canterbury, Seminar, Vol. I, pp. 61-63, 1967.