Bulletin of the New Zealand Society for Earthquake Engineering, Vol. 57, No. 1, March 2024

STRUCTURES INCORPORATING DAMPING DEVICES: ARE WE CORRECT IN OUR DESIGN THINKING?

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(Submitted *December 2022*; Reviewed *March 2023*; Accepted *September 2023*)

ABSTRACT

The use of discrete damping devices in New Zealand buildings is increasing as designers seek to limit damage to both architectural and structural building elements in moderate earthquakes. The overall damping of the structure becomes a combination of inherent material damping, hysteretic damping from yielding of building elements at specific locations and damping from discrete devices. For over 60 years, engineers have used simplified analysis procedures (e.g., modal response spectrum analysis) to predict building response under seismic actions. The design of structures incorporating viscous dampers requires a paradigm shift in approach as the force each damper resists is a function of the relative velocity between its ends. Popular pseudo-static design methodologies promote the use of a *viscous strut* in the analysis model to represent them. However, it is incorrect to use elastic, mode-based analysis for a structure incorporating damping devices and relying on significant inelastic behaviour. This paper elucidates the complex mechanics of damper-structure interaction by reviewing some of the established classical, modal-based, simplified analysis techniques used in seismic design. A series of numerical investigations demonstrate how these techniques are usually invalid for structures that incorporate viscous dampers because they violate the laws of physics. The reason why mode-based techniques are invalid is explored with reference to the imaginary components of the natural modes of vibration and the effect on them of significant inelasticity within the structure. The dynamic characteristics of viscous dampers challenge conventional design approaches such as displacement-based design. This paper establishes that nonlinear time-history analysis is the only meaningful way of predicting the dynamic response of a structure incorporating viscous dampers when significant inelastic behaviour of the structure is expected.

https://doi.org/10.5459/bnzsee.1625

INTRODUCTION

A conventional seismic design strategy relies on a structure enduring large inelastic deformations in a ductile fashion in a major earthquake thereby absorbing/dissipating energy. Traditionally, inelastic deformations equate to material damage in structural and non-structural elements and heavy economic losses. Recent earthquakes such as Christchurch 2011, Tohoku 2011, Kaikoura 2016, Kumamoto 2016 have all resulted in heavy damage. For example, the loss incurred in the Christchurch earthquake was greater than NZD 45 billion which is about 20% of New Zealand's Gross Domestic Product. For the development of a sustainable human society, these levels of losses are unacceptable and should be minimised. This calls for alternative thinking in the way we design our built environment. Seismic resilience that achieves our sustainability goals needs to be embedded in the design process.

One way to achieve this might be to design new and retrofit existing structures with viscous dampers. These provide internal actions which are out of phase with the usual structural actions during an earthquake. Properly specified, they can minimise inter-storey drift, absolute floor accelerations and foundation loading. Consequently, damage and loss of use can be reduced considerably – even in a major seismic event, significantly larger than the design event. Additionally, when used for retrofit they require much less foundation intervention compared to other retrofitting technologies - including base-isolation when that is an option.

However, the design of such a structure is not straightforward as the dampers interact with the structure in a complex dynamic way (particularly when it is responding inelastically) because of their velocity dependence.

Lots of simplified techniques have been developed over the years that treat each damper as a viscous strut. We have investigated the validity of such methods by using mathematically-rigorous, physically-consistent, nonclassical modal analysis. It is shown that these simplified techniques violate some of the fundamental mechanics applicable to a viscously-damped structure and thus their results may be incorrect.

We begin with an explanation of how a viscous damper works and then summarise a critical review of the existing classical pseudo-modal methods. The well-known concept of nonproportionality is introduced with a physics-based explanation. Next, we present a numerical study which illustrates the physical inconsistency in the so-called pseudo methods as applied to both conventional structures and those using additional damping devices. We finish with a discussion of the ramifications of our findings and our conclusions.

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HOW DOES A VISCOUS DAMPER WORK?

A viscous damper is a fluid-mechanical device which exhibits an axial resistance proportional to the relative velocity of its ends. The axial force is out of phase with the force that would be in an equivalent elastic strut/brace in the same position. A longitudinal section of a typical viscous damper is shown in Figure 1. The damper consists of a stainless-steel piston inside a steel cylinder which is divided into two chambers by the piston head. The cylinder is filled with a fluid and there is a pressure accumulator for smooth fluid circulation. The characteristics are controlled by the number, size and types of orifices in the piston head.

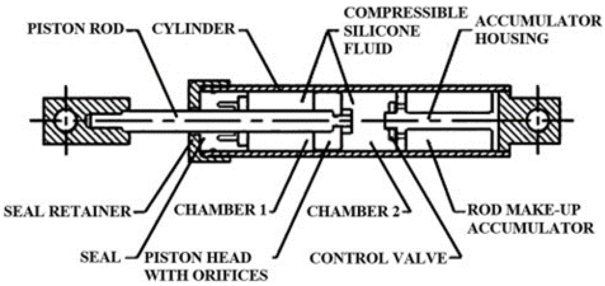

Figure 1: A typical cross section of a viscous damper (Taylor Devices, Inc.).

REVIEW OF METHODOLOGIES FOR THE SEISMIC DESIGN OF STRUCTURES INCORPORATING DAMPERS

The existing analysis methods can be broadly classified thus as shown below in Figure 2.

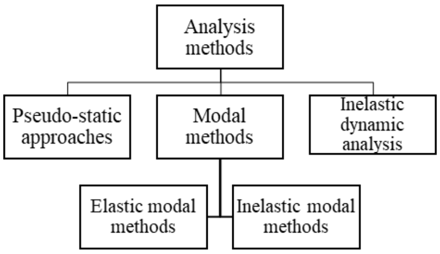

Figure 2: Broad classification of analysis methods.

Each of these methods is described below along with their associated pitfalls and common misunderstandings.

Pseudo-Static Methods

Pseudo-static methods are very popular mainly because of their simplicity. Structural engineers are generally well versed in static, elastic analysis and the thinking that goes with it. Wind, snow, and gravity forces are predominantly treated as being static load cases. This was clearly an incentive for the development of equivalent-static methods to approximate the dynamic loading of earthquakes. A considerable amount of research effort over the last six decades was directed towards this goal.

Mathematically, the response of a structure to an earthquake maybe represented as an equation of forces:

$$M\ddot{u}(t) + C\dot{u}(t) + Ku(t) = -MR\ddot{u}_g(t) \quad (1)$$

Here *M*, *C* and *K* are the mass, inherent structural damping and stiffness matrices, respectively. The vector $-MR\ddot{u}_g(t)$ represents the earthquake loading where $\ddot{u}_g(t)$ is the ground motion acceleration record and *R* is the directionality influence vector. (t) indicates the variation of $\ddot{u}, \dot{u}$ *and* u (acceleration, velocity and displacement relative to the structure's base, respectively) with time. The damping is assumed to be viscous (i.e., velocity-dependent). Note that Rayleigh made this assumption in 1877 [1-2] when he assumed that the decay observed in a free-vibration motion was, for mathematical convenience, viscous. If it is not viscous, but instead hysteretic or internal-friction-based then, in a dynamic sense, Hooke's Law does not strictly apply!

In most of the static methods, the mass and damping terms in (1) are simply set to zero with some adjustments to the effective load vector to mimic the effects of mass and damping. In other words, in a qualitative mathematical sense, most of the pseudo-static methods ignore the direct effect of mass and damping and try to reflect their effects by modifying the demand or imposed load vector in an indirect manner. There are simple and advanced ways of doing these modifications. As the focus of this paper is to highlight some of the key pitfalls and fallacies in using such methods for the design of structures that include viscous dampers, we have not discussed the various intricacies of the different methods.

Some of the most common pseudo-static methods presently used in seismic design are force-based design, displacement-based design [3-4] and energy methods [5-6]. There are also a number of variations within these approaches, but the key assumptions for all these methods mostly remain the same. Only the key assumptions inherent in the existing methodologies are discussed in this section. No specific method is discussed in detail.

Although this paper is to review the appropriateness of these methods being applied to viscously-damped structures, these methods are firstly reviewed for their capability in capturing the dynamics of an inelastic structure without dampers. Later, the same review approach has been adopted to assess the capability of these methods for the design of inelastic structures with dampers.

Key Assumptions Common to the Existing Methods

The key assumptions present in whole or in part in a majority of the pseudo-static methods are:

1. The effective damping ratio is based on the assumed structural ductility.
2. The multi-degree-of-freedom structure (MDOF) can be converted to a single-degree-of-freedom (SDOF) one.
3. An inelastic displacement shape can be approximated by a close-to-elastic mode shape.
4. For a damped structure, the proportions of base shear attributed to the parent frame without the dampers can be de-coupled from the damped system itself.

Conceptual Review

Here are reviews of the concepts of these assumptions, backed up by some numerical results of their effects:

1. Assumption of an effective damping ratio based on assumed structure ductility:

The concept of structural ductility and its equivalent forms are fundamental to most of the design philosophies used in seismic engineering. Regardless of whether it is force-based design or displacement-based design, the concept of structural ductility (or choice of structural ductility) is the starting point. Here, we delve into the concept of structural ductility and its validity. The primary question is: Does the structure really exhibit a constant structural ductility? Or, in other words, what do we really mean when we talk of structural ductility?

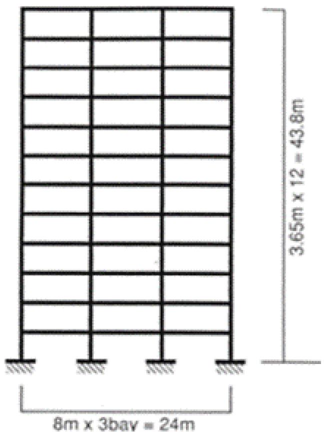

Figure 3: 12-storey reinforced concrete moment-frame.

To answer the above questions, results of the study of a 12-storey reinforced-concrete moment-frame shown in Figure 3 have been used. Details of the frame are given in [7]. This example shows the difficulty in justifying the use of the simplified concept of structural ductility used in current design. The frame members are designed using capacity-design concepts and are assumed to exhibit elastic/perfectly-plastic hinge behaviour, as shown in Figure 4, at the ends of the beams and at the base of the ground-floor columns. The frame has been modelled in the *Ruaumoko* software platform maintained by the first author [8].

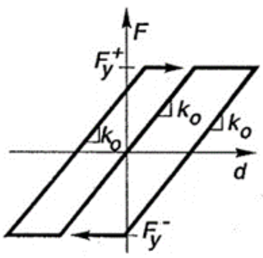

Figure 4: Elastic/perfectly-plastic hinges used.

The frame is subjected to the North-South component of the May 1940 El Centro earthquake record and the base shear vs. 1st storey drift is shown in Figure 5.

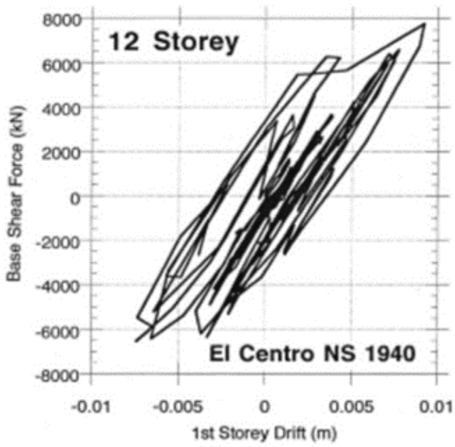

Figure 5: Base Shear vs. 1st storey drift.

The plot in Figure 5 shows a relatively bi-linear hysteresis behaviour and, as a storey yield displacement can be observed in the plot, an estimate may be made of the storey ductility demand. However, if one examines the plot of roof displacement vs. base shear shown in Figure 6, no simple hysteretic behaviour can be observed. The roof displacement may be considered as dominated by the first mode of free vibration, but the base shear is a function of at least all the 12 lateral modes of free vibration. This is a very important observation; even for a simple 2D symmetrical structure like this one, the forces in the structure are a function of all contributing modes of free vibration. There is no obvious yield displacement for the structure. For any real, framed structure there is no sharply-defined yield point although one may estimate a so-called yield roof displacement from a pushover analysis. For a 2D frame this is possible with a carefully chosen lateral load pattern. Such a definition becomes much more difficult for a general 3D structure where, with the progressive development of inelastic behaviour, the structural displacements will be a mixture of torsion and translation.

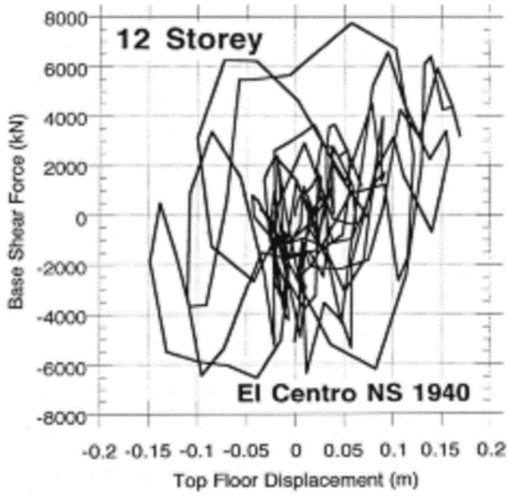

Figure 6: Base shear vs. roof displacement.

An approximate estimate of the structure ductility may be made by comparing the roof yield displacement (computed/estimated as above) to the maximum roof displacement. One may then relate the storey ductility demand and the member curvature ductility demands to the so-called structure ductility demand. These computed ratios are shown in Table.1. The table clearly shows that an approximate ratio of 8 to 1 exists between maximum member curvature ductility and the so-called structure ductility. Many similar analyses carried out over the past two decades on other similar framed structures have also shown similar trends between the two measures.

Table 1: Ratios of structure ductility to member ductility.

12 Storey	Structure Ductility	Max Storey Ductility	Max Member Ductility
El Centro	1.13	2.07 (1.83)	9.49 (8.40)
Bucharest	3.58	7.14 (1.99)	26.79 (7.48)
Park Field	2.85	6.74 (2.36)	24.32 (8.53)
Pacoima	4.74	9.22 (1.95)	33.85 (7.14)

( ): ratio to Structure Ductility

The late Professor Paulay noted that a well-detailed plastic hinge could achieve a curvature ductility of about 30 (once) and, as damage is accumulative, this implies a ductility of 15 twice. One sees immediately that the available structure ductility for a frame structure that is to survive the cyclic earthquake deformations, and particularly if it is to have post-earthquake use, is in fact, extremely limited. Some earlier New Zealand design codes suggest that structure displacement ductilities of the order of six were acceptable in design.

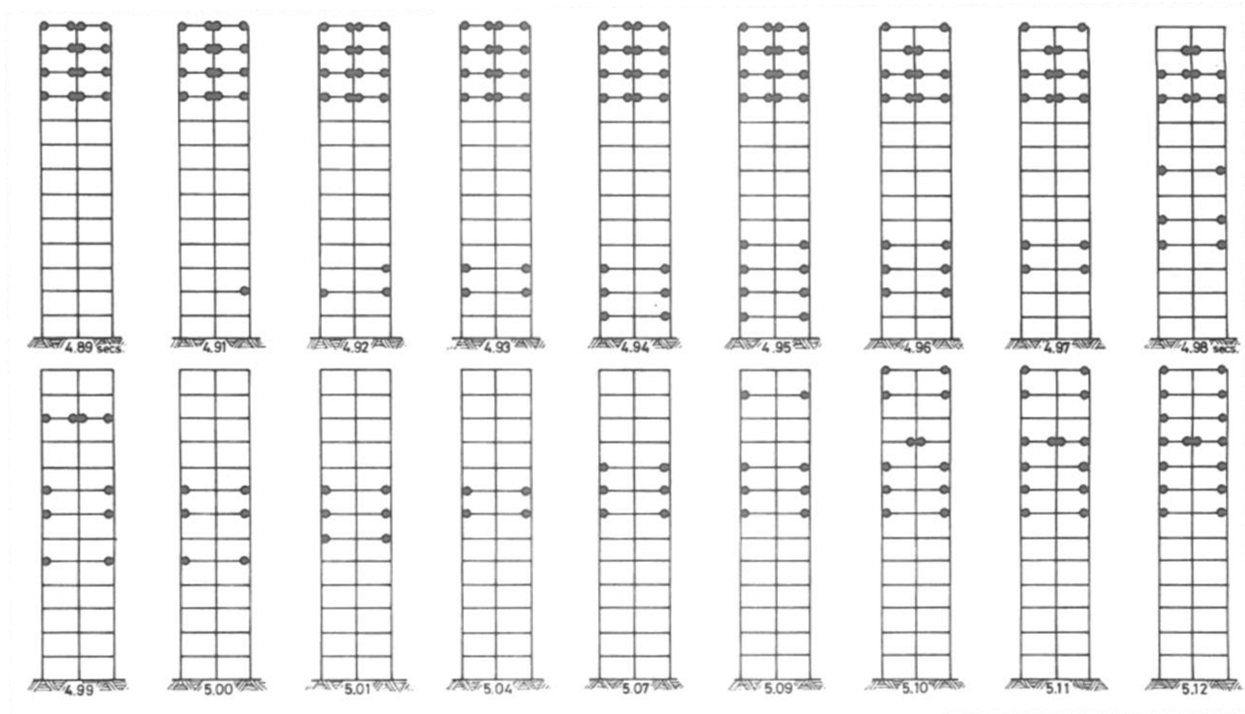

Figure 7: Hinge migration in a multi-storey structure when subjected to El Centro May 1940 [9]).

Figure 7 shows snapshots of hinge migrations of a 13-storey frame when subjected to the El Centro 1940 ground motion [9]. The black dots depict the plastic hinges and the value below is the time from the start of the earthquake in seconds. It can be clearly seen that in every snapshot the pattern of hinging is different. Similar studies have been done by Maniatakis et al. [10]. Also, each of those hinges in different snapshots will be in a different state of yielding - giving non-uniform dissipation capability. The direction of the rotation in the plastic hinges is not shown. It is common to observe in a dynamic analysis that a wave of plasticity propagates up the structure and reflects when it reaches the top. It can be expected that the rotation of hinges in one part of the structure will be in one direction and in the opposite direction in other parts of the structure. This can be likened to higher-mode effects in the structural response. So, a choice of uniform structure ductility for all modes (and hence, therefore, a choice of effective viscous damping ratio representing the hysteretic dissipation) exhibits a clear violation of the physics of the problem. This is again illustrated in a later section for the four-storey numerical example where three snapshots as shown below in Figure 7 are adopted to show the complex mechanics of the system.

It can be clearly seen that the concept of an assumed structural ductility has no physical or mathematical basis. The concept is applicable for a single-degree-of-freedom (SDOF) structure because in that case the storey ductility and structure ductility coincide. However, for a multiple-degree-of-freedom (MDOF) structure with varying inelastic hinging characteristics, is there any basis for this assumption of constant structural ductility?

So, why do all the present design methodologies adopt the concept of structural ductility?

To answer this question, let's review the idealised plastic mechanism by Park & Paulay [11] which forms the basis of the classic *capacity-based design* which is followed by all the seismic codes across the world, including NZ 1170.5. Figure 8 shows the classical Park & Paulay mechanism. It clearly illustrates the fact that hinges are assumed to appear uniformly and are assumed to have equal dissipation power. It is also expected that, in the event of an earthquake, the whole mechanism happens instantaneously. Compare this with Figure 7, where realistic hinging migration for a simple 2D structure is shown for different instants of time; it can be clearly seen that no similarity exists between the two figures as at no instant does the pattern exhibited in Figure 8 appear.

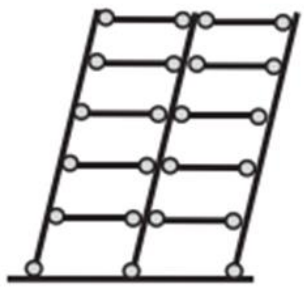

Figure 8: Idealised plastic mechanism by Park & Paulay [11].

Why, therefore, adopt such an assumption which violates the laws of physics in its entirety?

The main benefit is that it allows the whole structure dissipation to be characterised by a single number, structure ductility, which may be related to an equivalent damping ratio that may be applied to the effective SDOF. The complex, MDOF dynamics are thereby simplified to SDOF statics.

The approach is very clever and had its justification in the 1960s when the computational power and the available earthquake records were limited. But what is the justification for this assumption in the present day? In the authors' opinion, there is no justification.

It has been the first author's experience that, over the past 60 years of inelastic dynamic analyses of various highly-complex structures, this mechanism has only been observed in a single time-step in a single structural earthquake analysis and, in general, such a mechanism is extremely unlikely to be exhibited by a real 3D structure.

What does this mean? It simply means that the fundamental assumptions in our building codes exist only for mathematical convenience and simplicity but may have no physical basis. At this juncture, it might be prudent to remember that, if something is simple to apply and claims to represent a very complex phenomenon in its entirety, *then there is a high chance that it will be too good to be true.*

Until now, we have discussed in this section how the assumption of uniform hinging is applied to conventional structural systems. The standard approach to extend this to a structure with added viscous dampers is to add the effective hysteretic damping ratio calculated assuming uniform hinging in the parent frame to the effective viscous-damping ratio. In mathematical terms:

$$\xi_{net} = \xi_{viscous} + \xi_{hysteretic} \quad (2)$$

The most interesting aspect of (2) is that it implies that the distributed viscous dampers and the hysteretic damping are treated as if they provide proportional (viscous) damping without recognition of the sign of relative velocity across a damper. This is a gross violation of the law of physics as (2) is oversimplifying the complex phenomenon of damper-structure interaction with no recognition of the dynamics at each time-step throughout the earthquake. This is, again, illustrated in examples in a later section.

2. Conversion of a MDOF to a SDOF

Approximating a MDOF structure as a SDOF structure has been a primary assumption in seismic engineering and it is only possible if damping proportionality with both mass and stiffness is assumed. This equivalent-SDOF approach for inelastic structures largely dates back to the classic paper of Veletsos and Newmark [12] who stated that the displacements of an inelastic structure were the same as those if the structure was fully elastic. The study was predominantly conducted on inelastic SDOF structures. This is the equal-displacement concept which has been used in design codes around the world for the past six decades. If one reads the Veletsos and Newmark paper, it is very hard to see how they could imply that the displacements were *equal* - even *similar* would be a stretch. Maybe they considered them to be equal because they were not orders of magnitude different? By approximating the structure as a SDOF structure allowed the designer to use a response spectrum to compute the displacements and forces and adjust the spectral values/forces for ductility by using the assumption previously discussed. Later, the method was extended to a multi-mode method with concepts such as SRSS (*square root of the sum of the squares)* to combine modal maximums of internal forces and displacements. Many design codes suggest methods to determine how many of the modes of free vibration should be used; these methods are only really valid for an elastic structure as the contribution of the structure's higher modes is required to represent the localized deformations related to the member inelastic excursions as shown by Crisp [13].

The elegance of these approximations is that they allow a designer to apply simple rules to design a conventional, complex MDOF structure without getting into the intricacies of inelastic multi-modal dynamics. Although two years later, Clough, Benuska & Wilson [14] disproved the findings of Veletsos & Newmark, the *equal-displacemen*t assumption has remained as the backbone of seismic engineering since - mainly because of the simplicity of the mathematics and the pseudo-confidence instilled in a designer because of the easy application of this principle.

In the Displacement-Based Design (DBD) method [3], a single displacement pattern for the structure is used to make a SDOF model. A yield point for the structure is defined and an assumed hysteresis model is chosen. Using the secant stiffness to the maximum displacement and the area under the hysteresis curve to estimate a natural period and equivalent damping is not really realistic as, for most of the ground motion, the structure has deflections much less than the once-only maxima. Also, the energy dissipated over most cycles is much less than that under the hysteretic model chosen. The difficulties with this approach, even for base-isolated structures with their much simpler yield definition and energy dissipation modelling, were highlighted by Andriono and Carr [15].

Approximating a damped elastic MDOF structure with a SDOF structure is only possible if *proportionality* is assumed in the damping of the structure. By adopting a SDOF analytical representation, the Displacement-Based Design procedure is inherently assuming proportionality. In the penultimate section, in the numerical examples for conventional structures, it is shown that when a structure goes inelastic, the concept of proportionality is highly erroneous and hence the MDOF structure cannot be disaggregated into a set of SDOF structures.

Proportional damping assumes that the damping matrix is orthogonalized (de-coupled) in the same manner as the mass and stiffness matrix by the free-vibration mode shapes. The only effect that the damping then has is to change the free-vibration natural frequencies to the damped frequencies. If the damping is small (less than 10% of critical damping, say), then the effect is small (less than 1%) and most engineers just use the undamped frequencies. The natural frequencies are *real* numbers and the mode shapes are also *real*. This behaviour is assumed by almost all current seismic design codes. The codes also tend to assume that the damping is the same for all modes. As will be shown later, if the damping is non-proportional then the frequencies, mode shapes and damping ratios may be complex (*real* and *imaginary* number components).

As described in the next section, the addition of viscous dampers to a structure introduces *non-proportionality* in the damping matrix - even for when the structure is elastic. The main implication of this is that, even in an elastic scenario (where both damper and structure are elastic), *modal de-coupling* is impossible. In seismic engineering, the effective damping ratio attributed by the addition of viscous dampers to a structure is much higher than the commonly-adopted 5% (critical, equivalent viscous). This means that the modal energy transfer ratio (described in detail in the section on damper structure interaction) will be high as there will be energy transfers between the modes. This is again illustrated in the penultimate section in the numerical example of the four-storey structure. This has also been observed by Bathe [16].

An inelastic method relying on the primary assumption of conversion to an effective SDOF may therefore result in a considerable error in the design of a damped system. In a design office, this is normally reflected in requiring more damping (i.e., more dampers) than originally estimated and more than the structure needs. The primary reason for this increase or anomaly is that the design methodology adopted violates the fundamental mechanics and ignores the phenomenon of damper-structure interaction (see the next section). This has been evidenced in Marriott [17].

It must be noted that in real life there are no single degree-of-freedom structures.

3 Assumption of a close-to-elastic mode shape to approximate an inelastic mode shape

Most of the methods use a close-to-elastic mode shape in approximating the inelastic mode shape. This assumption is necessary in most of the methods in order to project the results obtained by the assumption of SDOF back into the MDOF domain. This assumption literally means that, when the structure goes inelastic, the effective mode shape does not change. Thus, in reviewing Figure 7, the pertinent question is whether or not this assumption is valid?

Firstly, let us review this in terms of an undamped structure. Figure 7 clearly illustrates that, at every instant, the plastic-hinge distribution changes; in other words, this means that at every instant the effective mode shape changes. How much it changes is a function of the distribution of the hinges and what degree of elemental ductility they have incurred. Results are presented for two such instants in the penultimate section for the four-storey frame with no dampers. It is shown that when structure goes inelastic, complex mechanisms are exhibited and the present approach of using a single, constant, mode shape is dubious.

In the case of a damped structure, this is more complex as the mode shapes are no longer *normal* (de-coupled) and they have a complex component even if the system is elastic. Again, the complex mechanics for the damped structure are shown in numerical examples in a later section of the paper for two instances of secant stiffness. A graphical depiction of a complex mode is also presented in the same section which clearly illustrates the inherent error in assuming a standard, real or normal mode shape in projecting the SDOF response back to a MDOF one.

Although, theoretically, it is possible to calculate the basis vectors (mode shapes) at each instant of time, computationally this demands the assemblage of the secant stiffness matrix at each time instance. As the secant stiffness matrix is very difficult to calculate (mainly due to the non-unique nature of the matrix), the degree of change in the shape of a mode is almost impossible to check. As the tangent stiffness matrix is updated in a time-history analysis, it could be used to compute the incremental modes, but they are of no use in obtaining the secant response of the structure.

4. De-coupling of the proportion of base shear attributed to the undamped parent frame and the damped system

The equation of motion for a viscously-damped structure with a pure dashpot assumption is given as follows:

$$M\ddot{u}(t) + C\dot{u}(t) + C_d\left(\dot{u}(t)\right)^{\alpha} + Ku(t) = f(t) = -MR\ddot{u}_g(t) \tag{3}$$

Here, all terms are similar to Equation (1), with C_d and α representing the viscous damper parameters.

Now, re-arranging this equation in Newtonian format:

$$M\{\ddot{u}(t) + R\ddot{u}_g(t)\} = -\left\{C\dot{u}(t) + C_d\left(\dot{u}(t)\right)^{\alpha} + Ku(t)\right\} \tag{4}$$

In other words, in total acceleration format, we have:

$$M\{\ddot{u}(t) + R\ddot{u}_g(t)\} = -\{F_{damping} + F_{structure}\} \tag{5}$$

Equation (5) illustrates that the net base shear is resisted by the damping component and the stiffness component. Now, let's compare this with a conventional structure. For a conventional structure, $F_{damping}$ is assumed to be small (in a SDOF structure 5% critical viscous damping produces damping forces of the order of 10% of the other forces in the structure) and, in a classical seismic analysis framework, (5) gets modified to:

$$\left.\begin{array}{c}\{F_{structure}\} = -M\{\ddot{u}(t) + R\ddot{u}_g(t)\} + \{F_{damping,5\%}\} \\ or \\ \{F_{structure}\} = -M\{\ddot{u}_T(t)\} + \{F_{damping,5\%}\}\end{array}\right\} \tag{6}$$

where $\ddot{u}_T(t)$ *is* the total ground acceleration. So, in a conventional system, the net inertial seismic force is modified by the allowance from damping (which is usually the code-based 5% modal damping) and is resisted by the structure.

Although it might feel as if 5% damping is a small ratio, in reality it is not; a 5% damping in a mode almost equates to 47% of energy loss per cycle and makes a marked difference to the dynamic response [18]. A 5% critical damping almost halves an undamped response spectrum. By default, most building codes adopt this value for a normal structure. A question that needs consideration in design is: how applicable is this value to our real structures? In bridge testing on the West Coast of New Zealand in the late 1970s, typical values were of the order of 1.5% of critical viscous damping, and overseas experimental results indicate that steel-framed structures tend to exhibit between 1% and 3% equivalent-viscous damping. In the design of a damper-controlled structure, it would be prudent to check the effects of this.

In a structure with added dampers, the whole of Equation (5) becomes more complex as $F_{damping}$ is a larger quantity exhibiting complex modal mechanics and the whole structure is highly coupled.

To ease the design process for the application of the viscous device and to adapt the complex dynamic design into the well-established static displacement-based framework, some building codes adopt an arbitrary split of base shear between $F_{structure}$ and $F_{damping}$. One such classical approach adopted by the ASCE codes is the 70-30 split of the base shear for symmetric structures where 70% is taken by the structure and 30% is taken by the dampers. This split has no scientific merit or physical evidence other than a mathematical convenience, but it provides a means to adopt a displacement-based approach for the design of a structure with velocity-based devices. Refer to the penultimate section for numerical evidence. An ad hoc reason that may be given for the adoption of such a split maybe the realisation that there is a need for a certain amount of structural stiffness required for the dampers to work against.

Although the focus of this paper does not include the critiquing of building codes, it is important to note that such a split in the base shear force is only an approximation for mathematical convenience. The main reason for the lack of a physical basis is that it is almost impossible to arbitrarily estimate the split of the total base shear between the terms on the right-hand side of Equation (5); in other words, it is impossible to decide at any instant of time during the ground motion, for a given instantaneous base shear value, how much is resisted by the structure and how much is resisted by the dampers. This is because of the complex combined mechanics outlined in the following section on damper-structure interaction.

Modal Methods

The main reason for using modal methods in dynamics is their computational advantage in most of the mathematical processes. Without the use of modal methods, the use of response spectra in design becomes almost impossible. The pseudo-static methods described earlier in this section involve a modified version of the classical modal analysis adapted for inelastic systems. If exact modal methods need to be used for a viscously-damped system, then the method described in the next section needs to be applied in full if the structure remains elastic both in damping and in stiffness. In classical structural engineering, to avoid the use of complex modal methods which follow complex algebra, the method has been simplified as described next.

Classical Modal Method

When full-fledged (nonclassical) modal methods are applied to Equation (3) with discrete dampers, assuming $\alpha = 1.0$, and with some mathematical manipulations, we get,

$$\left.\begin{aligned} \omega_i^2 + \omega_i\left(\frac{(\phi_{n,i}^T \hat{C}\phi_{c,i})}{\phi_{n,i}^T\phi_{c,i}}\right) + \omega_{n,i}^2 = 0 \\ \hat{C} = M^{-1/2}[C + C_d]M^{-1/2} \\ \frac{(\phi_{n,i}^T \hat{C}\phi_{c,i})}{\phi_{n,i}^T\phi_{c,i}} \approx \alpha_i + j\beta_i \end{aligned}\right\} \tag{7}$$

The full details of the mathematical derivation of Eq. (7) are given in the section below.

Here, ω_i is the i^{th} natural frequency of a non-proportionally damped system, $\omega_{n,i}$ is the natural frequency of the proportionally damped system, $\phi_{n,i}^T$ represents the transpose of the i^{th} normal mode vector ignoring the damping term in eq. (3), $\phi_{c,i}$ represents i^{th} the complex mode shape for the M-C-K system, α_i represents the real component terms of the damping matrix and the term β_i represents the complex component. The application of classical modal analysis largely relies on ignoring the term β_i. As already described, if the added damping is small and uniform, this might not incur a very large error as the off-diagonal terms (in the modal damping matrix) may be small. However, for most structures with viscous dampers, this is not generally the case - as illustrated in the penultimate section. No further focus is put on this method - mainly because it completely ignores the physically consistent, non-proportionality phenomenon described in the section below and because most structures also incur inelastic excursions. All classical modal methods lose their significance when structures incur inelasticity as the mode shapes are no longer time-invariant or, if the original modes shapes are treated as the basis vectors, they exhibit coupling effects.

Nonlinear Modal Methods

One of the popular nonlinear time-history modal methods is Fast Nonlinear Analysis (FNA) developed by Prof. Ed. Wilson [18]. Although this approach is better than using pseudo-static methods and elastic modal methods, the authors have strong reservations on the application of this method for a structure incorporating nonlinear dampers with relatively-significant inelastic structural deformations. Also, the use of this method for general nonlinear analysis should be undertaken with a high degree of caution (Puthanpurayil & Sharpe [19]).

This method, as with the other simplified methods, may well be used for initial design, but it is the authors' opinion that the final design of the complete structure should be checked with inelastic dynamic analysis using Direct Integration schemes described below.

Inelastic Dynamic Analysis by Direct Integration

In normal vibration engineering problems, the structure remains linear, and the damper-structure interaction is mostly limited to the attributed non-proportionality described in the next section. In the case of earthquake engineering, structures incur inelastic behaviour of varying degrees when subjected to shaking. Although a properly-designed damped structure minimises the degree of inelasticity, in a major earthquake inelasticity is likely to occur and the combined mechanics of non-proportionality and inelasticity exhibited by the structure will be extremely complex.

When a structure incurs inelasticity (either in the structural members, the base-isolation devices or added energy-dissipation devices), all modal methods struggle to capture the effect because the mode shapes of the structure will incur instantaneous changes during the earthquake. None of the modal methods are capable of reflecting these changes. This is illustrated in the examples in the penultimate section which indicate how much error is incurred when modal methods are applied.

Since none of the modal and frequency-domain methods are applicable, the only way to compute the mechanics is to adopt time-domain methods.

In this section a very brief description of time-domain methods is given. Again, for simplicity, only linear damping is assumed whereas in a real structure both the structure and damping may be nonlinear.

With linear damping, Equation (3) becomes:

$$M\ddot{u}(t) + [C + C_d]\,\dot{u}(t) + K_s(t)u(t) = f(t) \tag{8}$$

Here, $K_s(t)$ represents the instantaneous secant stiffness of the structure. Equation (8) may be solved using direct-integration or time-marching methods [20]. In NZS 1170.5, this method is called nonlinear time-history analysis (NLTHA).

$$\left.\underbrace{\left[\frac{4}{\Delta T^2}M + \frac{2}{\Delta T}C_T + K_T\right]}_{K_{dynamic}}\Delta u = P_{effective}\right\} \tag{9}$$

M, C_T and K_T are the tangent mass, damping and stiffness matrices. Equation (9) is solved to compute the response of the system. Although, in theory, Equation (9) may be used to solve the system, in practice, a total equilibrium format with implicit Newton Raphson iteration at each time step would be required to solve the system. Equation (9) is the usual text-book incremental equilibrium solution and can work for systems with constant structural damping or with damping changing with time but, as indicated above, a total equilibrium formulation is better - but these can only be accomplished if the damping actions are evaluated at every time-step. Further details on the incremental solution may be found in [20].

Equation (9) clearly illustrates that, when a structure incurs inelasticity, the tangent stiffness is reduced and the effect is reflected in the dynamic stiffness of the system. Similarly, when discrete dampers are added, they change the tangent damping matrix which also affects the instantaneous dynamic stiffness. The whole damper design (in a simplistic manner) may thus be described as identifying the trade-off between the contribution of the added dampers and the effect of the inelasticity to achieve a specific performance target throughout the response of the structure to the input ground motion.

DAMPER-STRUCTURE INTERACTION - AN INSIGHT INTO THE MECHANICS OF DAMPED SYSTEMS

This section mainly addresses the question: *What happens when discrete viscous dampers are added to the structure*? The simple answer is that viscous dampers make the net damping of the structure *non-proportional*.

A simplified theoretical explanation of the concept of *non-proportionality* using an elastic structure follows. It is based on published research and describes the novel mechanics exhibited by these device-incorporated structures. It is an elucidated summary of the following: Chopra [20], Clough & Penzien [21], Veletsos & Ventura [22], Hurty & Rubinstein [23], Liang & Lee [24], Adhikari [25], and De Chang et al. [26].

The generic equations of motion for a linear, dynamic, multi-degree-of-freedom structure are:

$$M\ddot{u}(t) + C\dot{u}(t) + Ku(t) = f(t) \tag{10}$$

Here, all the terms are same as Equation (1). $f(t)$ is the force vector varying with time and, in the case of earthquake loading, this becomes $-MR\ddot{u}_g(t)$.

Classical modal analysis is one of the most efficient methods for solving Equation (10). The method works by identifying undamped modes or *classical normal modes* of free vibration which can simultaneously orthogonalize mass and stiffness matrices which results in the de-coupling of the equations of motion. Most building codes and standards assume that the structure possesses classical normal modes. The phenomenon of de-coupling results in the disaggregation of a complex multi-degree-of-freedom (MDOF) structure into a set of single-degree-of-freedom (SDOF) systems and the responses of the complex MDOF structure can be represented as the summation of the responses of the SDOFs.

Strictly speaking, classical modal methods are not applicable in seismic engineering and even less so for structures with added damping devices which make the structure non-proportionally damped. In other words, it means the above-mentioned de-coupling process is not straight-forward and results in complex modes. In this section, a physics-based illustration of the effect of non-proportionality is presented. *The reader may wish to skip the mathematics in this section and focus only on the text without losing continuity.*

Now, if discrete viscous dampers are added, Equation (10) is modified in a generic form as given in Equation (3) - repeated here for convenience:

$$M\ddot{u}(t) + C\dot{u}(t) + C_d\left(\dot{u}(t)\right)^{\alpha} + Ku(t) = f(t) \tag{11}$$

Here, C is the inherent damping of the structure, C_d is the added damping coefficient and α is the velocity exponent which is normally between 0.1 and 1.0. It must be noted that Equation (11) uses a pure dashpot model for the viscous dampers for illustration purposes. It should also be noted that if α is appreciably less than 1.0 (which makes the damper much more effective at lower velocities), then the tangent damping coefficient tends to infinity as the velocity tends to zero. This may lead to computational-precision difficulties in time-history analyses**.**

For simplicity, let's also consider $\alpha = 1.0$**.** Then, Equation (11) becomes:

$$M\ddot{u}(t) + [C + C_d]\dot{u}(t) + Ku(t) = f(t) \tag{12}$$

For notational simplicity, let's assume $[C + C_d]$ to be simply C from here on.

For the structure presented in Equation (12) to have *normal modes* or *classical modes,* the *Caughey criterion* needs to be satisfied. The Caughey criterion states that, for normal modes to exist [25]:

$$KM^{-1}C = CM^{-1}K \tag{13}$$

In the case of Equation (12), when discrete dampers are added ($C_d \neq 0$), this does not generally hold true and thus the damping becomes *non-proportional*. The remaining part of this section illustrates what this means and how it effects the dynamics of the damper-incorporated structure.

To understand physically the effects of *non-proportionality*, let's consider the homogenous form of Equation (12):

$$I\ddot{u}(t) + M^{-1}C\dot{u}(t) + M^{-1}Ku(t) = 0 \tag{14}$$

where I is the unit matrix.

Equation (14) can be transformed to:

$$M^{-1/2}MM^{-1/2}M^{1/2}\ddot{u}(t) + M^{-1/2}CM^{-1/2}M^{1/2}\dot{u}(t) + M^{-1/2}KM^{-1/2}M^{1/2}u(t) = 0 \tag{15}$$

Assigning the following:

$$\left.\begin{aligned} y(t) &= M^{1/2}u(t) \\ \hat{C} &= M^{-1/2}CM^{-1/2} \\ \widehat{K} &= M^{-1/2}KM^{-1/2} \end{aligned}\right\} \tag{16}$$

Equation (15) becomes:

$$I\ddot{y}(t) + \hat{C}\dot{y}(t) + \widehat{K}y(t) = 0 \tag{17}$$

Now, in the modal domain:

$$y(t) = \Phi_n q(t) \tag{18}$$

Φ_n represents the normal mode obtained by ignoring the damping term. Substituting Equation (18) in Equation (17) and pre-multiplying by ${\Phi_n}^T$, we get:

$$\begin{bmatrix} 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \end{bmatrix}\{\ddot{q}(t)\} + [{\Phi_n}^T\hat{C}\Phi_n]\{\dot{q}(t)\} + \begin{bmatrix} \widehat{k_1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \widehat{k_n} \end{bmatrix}\{q(t)\} = 0 \tag{19}$$

If the damping is proportional, the second matrix term on the left-hand side should also have a diagonal form that de-couples the entire MDOF into a series of SDOFs. This is not the case when discrete dissipation devices are added into the structure as the Caughey criterion is generally violated and the damping non-proportionality phenomenon is exhibited. *In other words, systems with non-proportional damping will have off-diagonal terms which are of considerable magnitude - preventing the above-mentioned de-coupling of the entire MDOF structure into a series of SDOFs.*

Therefore, an alternative treatment with damped eigen-decomposition needs to be undertaken. The eigen-decomposition of the $M - C - K$ structure is performed through a state-space formulation. Readers can refer to Hurty & Rubinstein [23] for more details.

$$\begin{Bmatrix} \ddot{u}(t) \\ \dot{u}(t) \end{Bmatrix} = \begin{bmatrix} -M^{-1}C & -M^{-1}K \\ I & 0 \end{bmatrix}\begin{Bmatrix} \dot{u}(t) \\ u(t) \end{Bmatrix} \tag{20}$$

As per [23]:

$$\left.\begin{aligned} \begin{bmatrix} -M^{-1}C & -M^{-1}K \\ I & 0 \end{bmatrix} &= \Phi\Lambda\Phi^{-1} \\ \Phi &= \begin{bmatrix} \Phi_c\Lambda_c & \Phi_c^*\Lambda_c^* \\ \Phi_c & \Phi_c^* \end{bmatrix} \\ \Lambda &= \begin{bmatrix} \Lambda_c & 0 \\ 0 & \Lambda_c^* \end{bmatrix} \\ \Lambda_c &= \begin{bmatrix} \lambda_{c,1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \lambda_{c,n} \end{bmatrix} \end{aligned}\right\} \tag{21}$$

Here, $\lambda_{c,i}$ are the eigen values and Φ_c is the complex mode-shape matrix for $the\ M - C - K$ system.

The interesting question is now: What do we mean by a complex mode shape?

A simple explanation is that, when a structure exhibits a complex mode shape, different masses reach their maximum amplitudes at different times; in other words, there are phase shifts among different masses in each mode. This is illustrated in the next section in the mode shape plots where a comparison between a normal mode and a complex mode shape is given. This is very much in contrast to a normal mode where all masses reach their extreme displacements at the same instant - which allows the concept of effective mass and hence the de-coupling. In other words, the complex mode shape does not allow de-coupling of the MDOF structure into a series of SDOFs as there will be energy transfers happening between the modes. This phenomenon is termed as *non-proportionality*.

To further investigate the concept of non-proportionality, let's rewrite Equation (18) in the complex modal domain.

Assume:

$$y(t) = \Phi_c e^{\lambda_c t} \tag{22}$$

Substituting Equation (22) into Equation (17) and pre-multiplying by ${\Phi_n}^T$, we get:

$${\Phi_n}^T \Phi_c {\Lambda_c}^2 + {\Phi_n}^T \hat{C} \Phi_c \Lambda_c + \Omega^2 \Phi_c = 0 \tag{23}$$

where:

$$\Omega^2 = diag(\omega_{ni}^2) \tag{24}$$

and it is the eigen value matrix of $\hat{K}$. For the i^{th} mode, Equation (23) may be re-written as:

$$\omega_i^2 + \omega_i \frac{(\phi_{n,i}^T \hat{C} \phi_{c,i})}{\phi_{n,i}^T \phi_{c,i}} + \omega_{n,i}^2 = 0 \tag{25}$$

Since $\phi_{c,i}$ is a complex value, we can say that:

$$\frac{1}{2\omega_{n,i}} \left(\frac{(\phi_{n,i}^T \hat{C} \phi_{c,i})}{\phi_{n,i}^T \phi_{c,i}} \right) = \xi + i\gamma \tag{26}$$

In the spirit of the damping ratio in classical modal analysis, Equation (26) describes the complex damping ratio. Alternatively, Equation (26) may be viewed as a combination of the classical damping ratio (which is *real*) and an *imaginary* damping ratio component which reflects the *other mode effects* on the damping ratio. In other words, these so-called *other mode effects* create the phenomenon of *non-proportionality*. According to Liang and Lee [24], this is called the *modal energy transfer ratio* which actually reflects the energy transfer to other modes.

To physically illustrate this, Liang and Lee [24] describe that, when non-proportional damping is present, it almost acts like the presence of an imaginary device which creates virtual modes. The net effect of this phenomenon is that a non-proportionally damped structure can never be de-coupled in the modal or physical domain. Trying to describe the dissipation effect using the classical damping ratio only (ignoring the *modal energy transfer*) may be a violation of physics and hence highly flawed. This is illustrated in examples in the next section for a simple four-storey structure.

NUMERICAL STUDY

This section investigates the aspects discussed in the previous two sections. A simple, four-storey shear frame with one degree of freedom per storey is used as a case study example.

Figure 9 shows a four-storey shear frame structure. The masses have values of $m_1 = 1 \times 10^4$, $m_2 = 8 \times 10^3$, $m_3 = 8 \times 10^3$ and $m_4 = 5 \times 10^3$ kg. Similarly, the structure has uniform inter-storey stiffnesses. For the structure without added dampers, the damping takes the form of Rayleigh or Modal damping [26] [27]. For the damper-controlled structure the coefficients are given by the property values of the discrete dashpots.

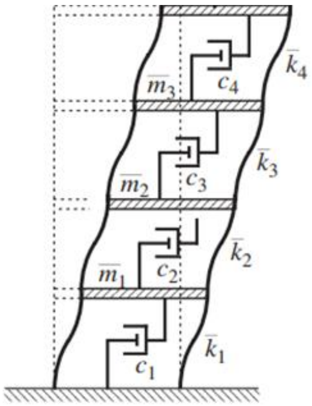

Figure 9: Four-storey shear frame.

Uncontrolled Structure (With No Added Dampers)

Example 1:

Are modal methods valid for inelastic analysis of conventional structures even without added dampers?

Most structural engineers are very familiar with conventional structures with the normally-assumed viscous damping. The four-storey elastic shear building model with no distributed dampers is used as a starting point. To represent the damping exhibited by structures (the free-vibration motion decays with time), both Modal damping (Caughey [29] or Wilson and Penzien [28]) and Rayleigh damping formats are investigated to show that similar trends are exhibited by these so-called *proportional* damping models when the structure becomes inelastic.

In most design codes the damping is assumed to be viscous and have the same fraction of critical damping in all modes. This may be regarded as the Modal damping model. In early computational methods the requirement to solve for all frequencies and mode shapes of free vibration required to form the fully-populated damping matrix was computationally impractical. Therefore, a Rayleigh damping model where one used a combination of the mass and stiffness matrices to form the damping matrix became the norm. The mass and stiffness matrices are already available in the computer analysis so this approach was an easy solution. The Rayleigh damping model was shown to give very unrealistic responses by Crisp [13] where the computed displacements were much lower than they would be if the damping followed the damping ratios specified in the design codes. It was shown that the higher modes in the structure were super-critically damped (up to 3000% of critical damping) such that the damping moments at the beam-column joints were of the order of the beam yield moments. With the Rayleigh damping model the damping ratios can only be specified at two frequencies and the damping ratio at all other frequencies is pre-determined by the model. Crisp showed conclusively that the reduced response was due to the excessive damping associated with the higher modes by the Rayleigh damping model. By the late 1970s the computational memory problems in computing the Modal damping matrix were no longer significant [28], so a more realistic damping approach became available. The option of Modal damping has been available in *Ruaumoko* since 1979 [8]. Unfortunately, the Rayleigh damping model is still the most commonly-used model for the damping matrix in most current time-history analyses. The computational speed and memory constraints of the 1960s no longer exist and the Rayleigh damping model has no mathematical or engineering justification for its continued use [30].

As traditional structural engineers are very familiar with structures without added dampers, a four-storey elastic shear building model with no distributed dampers is used as a starting point for the present study. To represent the damping exhibited by the structure (the free-vibration motion decays with time), both Modal damping [28-29] and Rayleigh damping formats are investigated to show that similar trends are exhibited by these so-called proportional damping models when the structure becomes inelastic.

For the elastic structure with 5% damping for all modes, the assembled Modal damping matrix using the Wilson-Penzien approach is given as:

$$\boldsymbol{C_{Modal}} = \begin{bmatrix} 12631.79 & -7890.36 & -1484.83 & -545.28 \\ -7890.36 & 21882.18 & -7732.65 & -1484.83 \\ -1484.83 & -7732.65 & 23070.05 & -7454.13 \\ -545.28 & -1484.83 & -7454.13 & 26966.54 \end{bmatrix} \quad \text{(R1)}$$

It can be clearly seen from (R1) that the modal damping matrix is now a fully populated one. In the 1970s this was considered to increase computational cost but, with the speed of modern computers, this is now of minor consequence.

In the case of the Rayleigh damping model (C=αM+βK, with α and β defined at the first two frequencies), a 5% viscous damping gives:

$$C_{Rayleigh} = \begin{bmatrix} 15844.57 & -12773.67 & 0 & 0 \\ -12773.67 & 30460.78 & -12773.67 & 0 \\ 0 & -12773.67 & 30460.78 & -12773.67 \\ 0 & 0 & -12773.67 & 31689.13 \end{bmatrix} \quad \text{(R2)}$$

The Caughey and Kelly criterion [25] is satisfied for both the Modal and Rayleigh damping models.

In both cases normal modes exist, and the mode-shape matrix is given as:

$$\boldsymbol{\Phi_E} = \begin{bmatrix} 1.000 & -1.000 & +1.000 & -0.750 \\ 0.875 & -0.429 & -0.571 & +1.000 \\ 0.625 & +0.714 & -0.857 & -0.750 \\ 0.375 & +1.000 & +0.875 & +0.375 \end{bmatrix} \quad \text{(R3)}$$

where each vertical column indicates the modal displacement of one mode.

In order to see what happens when the structure goes inelastic, two scenarios are looked at. In the first scenario, the stiffness at the first storey is reduced by 75% (Inelastic 1 in the tables) (equivalent to the secant stiffness when the storey ductility is 4); and the second scenario the stiffness in the top storey is reduced by 75% (Inelastic 4 in the tables). It must be noted that, when a structure incurs inelasticity, the stiffness changes happen as a function of time - as illustrated in Figure 7. In this example, we are taking two such snapshots in time by estimating the secant stiffness at that instant.

The main purpose of this is to see what mechanics the structure really exhibits during such snapshots. The nonclassical modal analysis procedure described in the previous section is used for computation. The Modal and Rayleigh damping matrices used for this computation are shown in Equations (R1) and (R2), respectively.

Effect on Damping Ratio: First Symptom of Non-Proportionality.

Tables 2 and 3 record the damping ratio changes for both the Modal and Rayleigh damping models. It can be clearly seen that, with inelasticity, an imaginary component of the damping ratio starts to appear. The structure no longer meets the Caughey and Kelly criterion and exhibits non-proportionality. In these two different scenarios, the sizes of the imaginary components are different. It must be remembered that these two inelastic scenarios are simply two snapshots in time of the response to an earthquake of the structure in Figure 7. When the structure returns to being elastic, the properties revert to those of the elastic structure.

Table 2: Damping ratios for individual modes for the Modal damping model.

Mode	Elastic	Inelastic 1	Inelastic 4
1	0.05	0.06 + 0.0001i	0.06 - 0.53i
2	0.05	0.09 - 0.0002i	0.11 - 0.78i
3	0.05	0.05 + 0.000i	0.10 + 1.56i
4	0.05	0.05 + 0.000i	0.05 + 0.0001i

Table 3: Damping ratios of individual modes for the Rayleigh damping model.

Mode	Elastic	Inelastic 1	Inelastic 4
1	0.05	0.06+ 0.0001i	0.08 - 0.53i
2	0.05	0.09 - 0.0002i	0.13 - 0.78i
3	0.06	0.06 + 0.0001i	0.10 + 1.56i
4	0.08	0.07 + 0.0001i	0.07 + 0.0004i

Based on these observations, a pertinent question that arises is: *What is the validity of the concept of an effective damping ratio based on ductility?*

The contribution of the imaginary component in the so-called damping ratio in the inelastic state can be beneficial or detrimental. So, assuming an effective dissipating ratio and reducing the demand uniformly for all modes may not reflect the physics in their entirety.

The importance of the imaginary component of the damping ratio is more pronounced in structures with added discrete dampers. More on this is discussed in detail in Example 2 below.

Frequencies

Tables 4 and 5 clearly indicate that, when the structure goes inelastic, it exhibits non-proportionality and this results in complex frequencies (real and imaginary components). The pertinent question is: *For a structure that exhibits complex modal mechanics when it incurs inelasticity, what is the validity of the response spectra-based approaches?*

Table 4: Frequencies for individual modes for the Modal damping model.

Mode	Elastic	Inelastic 1	Inelastic 4
1	8.38	1.54 + 1.68i	1.95 + 2.06i
2	22.9	3.00 + 3.18i	2.74 + 2.98i
3	34.6	3.97 + 4.18i	3.58 + 3.79i
4	42.5	4.48 + 4.72i	4.32 + 4.54i

Table 5: Frequencies of individual modes in the Rayleigh damping model.

Mode	Elastic	Inelastic 1	Inelastic 4
1	8.38	1.54 + 1.68i	1.95 + 2.06i
2	22.9	3.00 + 3.18i	2.72 + 3.01i
3	34.6	3.95 + 4.22i	3.56 + 3.82i
4	42.5	4.43 + 4.77i	4.26 + 4.60i

The question maybe restated as: *How can one incorporate spectra-based techniques for a structure that possesses complex natural frequencies?*

The significant change in the magnitudes of the modal frequencies should be noted.

5. *Mode Shapes*

The normal mode shape matrix of the elastic structure is given in (R3). The non-normalized mode shape matrix of the inelastic structure with the inelasticity in the first storey is given as:

$$\Phi_I = \begin{bmatrix} +0.0050+0.092i & -0.0040-0.049i & +0.0012-0.0090i & +0.00030-0.0017i \\ +0.0020+0.063i & +0.0030+0.017i & +0.0008+0.0240i & +0.00037+0.0110i \\ +0.0014+0.047i & +0.0027+0.025i & -0.0009-0.0036i & -0.00100-0.0200i \\ +0.0007+0.026i & +0.0014+0.018i & -0.0015-0.0260i & +0.00060+0.0110i \end{bmatrix} \quad \text{(R4)}$$

Complex modes appear in pairs as complex conjugates and what is presented in (R4) is only one part of it for simplicity. Note that each column represents one mode similar to (R3).

The first apparent aspect is that the moment the structure goes inelastic, the mode shapes become complex; in other words, the lumped masses reach their maximum amplitude at different instants of time because of phase shifts among the lumped masses in each mode.

A direct comparison of matrices (R3) and (R4) simply indicates that the mode shapes become complex when the structure goes inelastic. To get a better understanding of the qualitative nature of the change exhibited by the mode shapes, normalized plots of the mode shapes are shown in Figures 10 to 12.

Figures 10, 11 and 12 depict the normalized first, second and third mode shapes for the structure with proportional damping and nonproportional damping. The plots are generated by plotting both the real and imaginary components of the mode shape in the horizontal axis. The red line represents the elastic structure with proportional damping. The blue line represents the inelastic structure where the original proportional damping becomes non-proportional due to incurred inelasticity. As can be seen, all the plots for proportional damping (a state when the structure is elastic) only have the real component whereas, when the structure goes inelastic, the imaginary component appears and the mode shape changes in a 3D space.

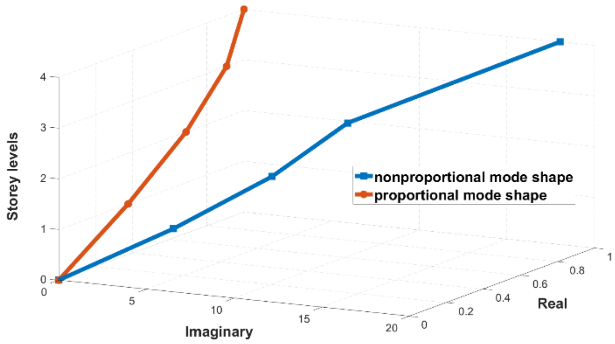

Figure 10: Comparison of proportional and non-proportional mode shape pattern for the first mode.

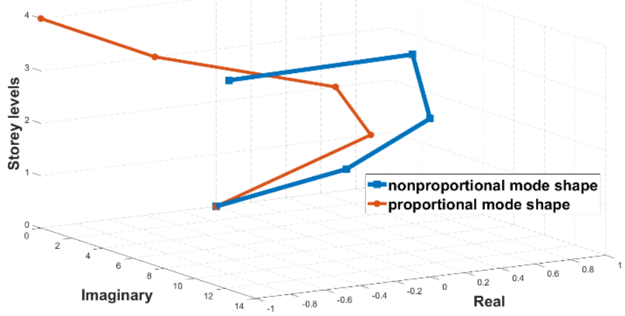

Figure 11: Comparison of proportional and non-proportional mode shape pattern for the second mode.

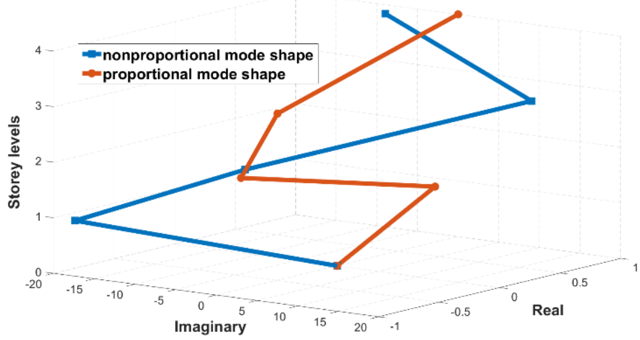

Figure 12: Comparison of proportional and non-proportional mode shape patterns for the third mode.

The plots shown here to illustrate the effects of the non-proportionality can only be done for the one-dimensional structure in this example (vertical height only and only lateral displacement). *For a three-dimensional structure the modes would have to be drawn in a six-dimensional space!*

From the plots, it is very clear that the assumption of a constant shape for a specific mode does not hold when the structure incurs inelastic excursions. Depending on the degree of inelasticity, the shape changes progressively as values of the imaginary component increase. A pertinent question that arises from this is: *What is the validity of the assumption of a close-to-elastic mode shape to approximate an inelastic mode shape adopted in displacement-based approaches?*

Note that, as the shape and amplitude of a mode changes, all modal characteristics such as modal participation factors, etc., also change - which will result in coupled modes and the concept of equivalent SDOF behavior then ceasing to exist.

A corollary question to this might be: *How do we justify the effective SDOF behavior which forms the basis of both force-based and displacement-based design?*

A secondary point is that the appearance of the complex mode shape on the occurrence of inelasticity is probably one of the primary reasons why nonlinear modal superposition methods like Fast Nonlinear Analysis (FNA) struggle to produce realistic results when the system exhibits higher degree of inelasticity. The more degrees of freedom that incur inelasticity, the less accurate these modal methods will be. Although in the paper published by Puthanpurayil & Sharpe [19] it has not been shown explicitly why the FNA results vary, the primary reason is because of what is shown in Figures 10 to 12. The degree of the change in the shape depends on the degree of inelasticity - hence the main reason why Prof. Wilson, developer of FNA, clearly states that the structure should have limited inelasticity, so that the change in shape is not drastic. Puthanpurayil & Sharpe [19] report an error of 400% for FNA when applied to a particular structure with distributed inelastic devices.

Controlled Structures (With Dampers)

The previous section presented an investigative study of the validity of the application of pseudo-modal and modal methods to conventional inelastic structures without viscous dampers. This section furthers the investigation into the validity of those methods for inelastic structures with dampers. It also illustrates the mechanics of *damper-structure interaction.*

In this process we look to answer the following question:

Example 2: Are modal methods/modal-based, pseudo-static methods applicable for either an elastic structure or an inelastic structure if it is fitted with viscous dampers?

The structure in Figure 9 is used. The mass and the stiffness properties remain the same, and damping is attributed to discrete dashpots. Two scenarios exist: one with damping distributed to match the stiffness or the mass and one with non-uniform damping. The first scenario matches the Rayleigh damping model and will then have a very similar behaviour to that described earlier in this section. Here, only the mechanics associated with nonuniform damping are discussed. For practical, real structures, it is more usual to arrive at a non-uniform damping in a Performance-Based Design framework. Three snapshots in time of the stiffness matrix, very similar to those adopted earlier, are investigated.

For the case of non-uniform damping, only the damper coefficients at Levels 1 and 2 are assumed to have a non-zero coefficient value and the rest are assumed to be zero; $c_1 = 2 \times 10^4\ N - s/m$ and $c_2 = 1 \times 10^5\ N - s/m$.

The Caughey and Kelly criteria are violated and the structure is inherently non-proportional.

Damping Ratio

Table 6 records the damping ratio for the three different states of the system. The most unique feature for nonuniform damping is that, even when the structure remains elastic, the damping ratio per mode is a complex number; in other words, structure is non-proportional - even in the elastic state.

Table 6: Damping ratio comparison for nonuniform damping for three different states of the system.

Mode	Elastic	Inelastic 1	Inelastic 4
1	0.13 - 1.156*i*	0.20 + 1.682*i*	0.09 - 0.76*i*
2	0.07 - 1.212*i*	0.20 - 1.866*i*	0.035 - 0.78*i*
3	0.63 + 0.019*i*	0.66 + 0.026*i*	0.06 + 0.09*i*
4	0.02 + 0.058*i*	0.02 + 0.055*i*	0.62 + 0.09*i*

It can also be seen that, depending on the degree of inelasticity, the non-proportionality increases. A feature of Table 6 is that it explicitly shows how complex the overall *damper-structure interaction* phenomenon is when dampers are added to the structure. Therefore, the use of simplified techniques to design such a complex structure needs to be reviewed rigorously.

Frequencies

Table 7 compiles the frequencies of the damped structure in all the three states. The most important point to be noted is that, as the damping is non-proportional even in the elastic state, the frequencies are also complex numbers. So, *how can one apply spectral approaches to such a structure even in the elastic state?*

Table 7: Modal frequencies for three different states of the structure with nonuniform viscous dampers.

Mode	Elastic	Inelastic 1	Inelastic 4
1	1.97 + 2.151*i*	1.53 + 1.69*i*	1.94 + 2.09*i*
2	2.58 + 5.36*i*	2.34 + 5.168*i*	2.60 + 5.34*i*
3	3.34 + 3.50*i*	3.07 + 3.39*i*	2.83 + 2.90*i*
4	4.44 + 4.52*i*	4.43 + 4.52*i*	3.80 + 3.98*i*

Mode Shapes

Figures 13 to 15 depict the mode shapes of both the elastic structure and the inelastic structure with nonuniform damping. As was shown above in Tables 6 and 7, since even the elastic structure exhibits non-proportional characteristics, and to benchmark the nature of change in mode shape, a plot of the elastic structure with proportional damping (red) is also given.

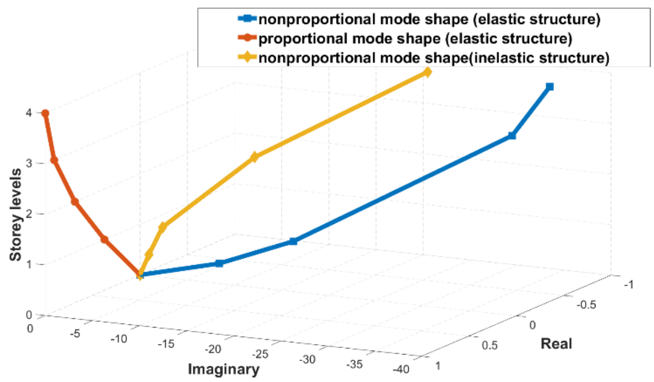

Figure 13: Comparison of proportional and non-proportional mode shape patterns for the first mode.

From the plots it can be clearly seen that, even in the elastic case, the nature or shape of the mode is different - exhibiting phasing effects as different lumped masses reach their maximums at different instants. From this it is very clear that using a constant close to a linear-mode-shape assumption in some of the simplified methods in codes is quite misguiding and violates the physics of the system. The proportional mode shape only has ordinates in the real axis and its imaginary component is completely zero whereas the non-proportional mode shape in both cases (elastic and inelastic state of the system) has ordinates in the imaginary axis. A design methodology assuming a mode shape that ignores the imaginary component will incur errors depending on the non-proportionality and inelasticity. Also, the modal characteristics such as the participation, etc., which drive the base shear will also be highly in error.

Deriving an effective SDOF from the MDOF using such a physically inconsistent mode shape will clearly violate the physics of the reduced-order model. Again, designs obtained through analysis or design methodologies based on such pseudo approaches need to be thoroughly investigated. More on this is given in the next section.

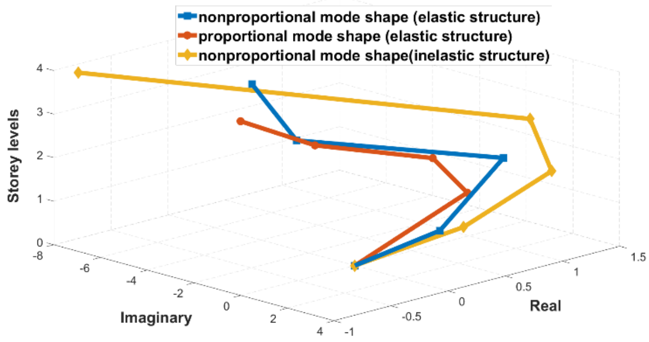

Figure 14: Comparison of proportional and non-proportional mode shape pattern for the second mode.

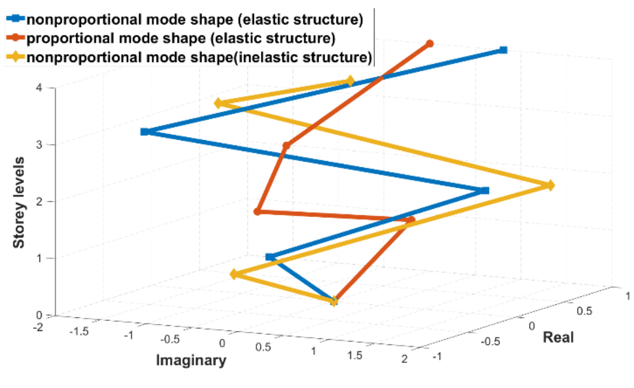

Figure 15: Comparison of proportional and non-proportional mode shape pattern for the third mode.

SUMMARY AND A SUGGESTED WAY FORWARD

As evidenced in the above sections, this paper raises some important questions about some of the fundamental assumptions inherent in seismic design for the past 60 years. To summarize, based on the studies presented in the previous sections of this paper, the following questions are raised concerning the pseudo-static and modal approaches currently in common use by structural designers:

a) *Can modal/pseudo static spectra-based methods be really applied to an inelastic or non-proportionally damped structure when they have complex natural frequencies?*

b) *Is there any mathematical or physical basis for the SDOF reduction of an inelastic MDOF or non-proportionally damped MDOF structure based on the use of elastic mode shapes?*

c) *Can modal methods as advocated in structural building codes capture the phenomenon of damper-structure interaction?*

d) *Is there any validity in the concept of an effective added-damping ratio both in inelastic and non-proportionally damped structures as implied by the use of Displacement-Based design methods?*

e) *Is there any reliable definition of structural ductility in the case of 3D MDOF structures?*

f) *Is there any justification in the continual use of the equal-displacement theory?*

g) *Are we justified in still blindly accepting the use of a 5% critical damping ratio that is assumed by most seismic codes in the production of their design spectra, particularly when we are modelling the inelastic energy dissipation effects directly?*

These questions demand a paradigm shift in design thinking. The authors do understand that raising all these issues is not a solution for going forward. The interesting aspect the authors want to point out is that none of these questions arise if the design is based on nonlinear time-history analysis (NLTHA) by direct integration. The entire complex mechanics described earlier are inherently incorporated when NLTHA with direct integration is used as the analysis tool. However, there are practical difficulties and, even today, the authors do acknowledge that to design by time-history analysis in a conventional practice is often impractical in terms of time, cost and capability.

Proportionality, which is a mathematical construction to allow the mathematical representation of a MDOF structural model as a SDOF structural model and allow for the inclusion of damping across the modes, is no longer valid when the structure goes inelastic or dampers are introduced. Proportional damping is also extremely unlikely to be valid when the damping is represented at the element level rather than at the structure level. Direct-integration analysis (NLTHA) is the only way to track the dependencies of the inelastic actions in structural members and the behaviour of the dampers. The use of time-history analysis becomes even more critical if viscous dampers are nonlinear with respect to their input velocity.

When we know that so many of the methods and assumptions used in current design are, at best, of debatable validity, are we still prepared, as a profession, to carry on as we have for the past several decades? Just because a method is popular, or commonly used, does not mean that is correct. In a future structural failure inquiry, or the equivalent of the Royal Commission into the Canterbury Earthquakes, can we say that our current approaches are *best practice*? A good investigator with an understanding of the requirements of reliable and competent structural analysis would have a field day with what our profession currently does in design.

One of the major concerns with much of the simplified current design methods is how far are we from meeting the *Swiss Cheese* criterion.

An alternative approach would be to do the conceptual design by a pseudo-static approach (not because it is correct in a physics' sense but for convenience) and then definitely verify the design using NLTHA methods. *If there are means to skip the pseudo-static approaches for conceptual design (or in other words do design directly using NLTHA), it is even better.* This can be done for both conventional structures and for those with added dampers. The authors do recognize that these are not ground-breaking ideas and these are known to most design practitioners and researchers across the globe. However, the evidence presented in the previous sections literally begs the designer to adopt this in *a mandatory fashion* to ensure resilience, and also to make sure that the design intent is met, especially when the design is done for highly-seismic regions such as Wellington in New Zealand. In this way it can be ensured that all the aims made in the conceptual design are achieved and that the design behaves as expected. This thinking has been recognized before by practitioners and academics nationally and internationally [3] [31].

The authors are fully aware that the NLTHA approach is not without its difficulties. No method of analysis can be any more accurate than the lowest accuracy of any data that is used as input. There are the problems of uncertainties in material properties, member section data, plasticity modelling, hysteresis modelling, applied loads and the selection of the earthquake excitations and their scaling. The major drawback to using such analyses will be in modelling the structure, in data preparation and in the evaluation of the analysis results. However, such NLTHAs are the only way to evaluate the inelastic behaviour of the members of the structure and/or the effects of dampers added to the structure. This non-linear behaviour cannot be replicated by any of the approximate methods of analysis. Even the supposed accuracy of the traditional modal analysis as proposed by many of the seismic codes cannot represent these effects because the very superposition implied in combining the modes precludes any nonlinearity in member behaviour.

This alternative approach is very important for structures incorporating dissipation devices. As these devices inherently affect the dynamics of the structure, the forces of the devices, the connection forces and attributed stresses in all elements should be estimated from inelastic dynamic analysis using time-marching schemes rather than from pseudo-static modal approaches or other *ad-hoc* approaches presently adopted in the profession.

CONCLUSIONS

The paper presents a critical review of some of the well-established classical pseudo-static and modal-based techniques and reinforces the critical review with numerical evidence. It has been shown that no mathematical or physical basis exists for some of the simplifying assumptions inherent in design methods used in current seismic engineering. The phenomenon of *damper-structure interaction* has been de-mystified, and its effect has been shown. The necessity for a paradigm shift in design thinking has been reinforced and a pathway forward for design engineers has been suggested. The inevitability of using explicit nonlinear dynamic analysis approaches using classical direct-integration techniques for practical structural design of both added-damping structures and inelastic structures has been emphasized and recommended. We feel that the NLTHA methods should be made mandatory for seismic design in regions with a likelihood of large ground shaking. The results from the simplified techniques that may be satisfactory for preliminary design need to be verified by more rigorous analyses techniques.

Notwithstanding this recommendation, it is critical that those using the simple equivalent code-based approximations are aware of the limitations of such methods and factor this into decisions around the robustness of the structural systems that are adopted.

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