DETERMINATION OF SITE PERIOD FOR NZS1170.5:2004

Tam Larkin[1] and Chris Van Houtte[2]

SUMMARY

The fundamental site period, *T*, is a key parameter for site classification in NZS 1170.5:2004. Many sites in New Zealand will fall into site classes C and D, where the boundary between the site classes is $T = 0.6$ seconds. NZS 1170.5 offers several methods of determining site classification. The intent of this paper is to expand on NZS 1170.5 and guide practising engineers towards more accurate and efficient methods for determining site period. We review methods to calculate the shear-wave velocity, then give specific examples for calculating the site period for five types of soil profile (uniform layer, shear-wave velocity increasing as a power of depth, shear modulus increasing linearly with depth, two-layer profile and three-layer profile). We find that NZS 1170.5 clause 3.1.3.7 for calculating site period at layered sites is unconservative and inconsistent with two other well-accepted methods for calculating site period. We consider the most accurate and efficient method of calculating site period for layered sites is to represent the profile as a lumped mass system, then calculate the fundamental frequency from the eigenvalues of the system. The successive application of the two-layer closed form solution is also considered an acceptable method.

INTRODUCTION

The New Zealand earthquake loadings Standard, NZS 1170.5:2004 [1], contains response spectra for structural design. The seismic loading on a specific structure will depend on, amongst other factors, the type of foundation soils where the structure is sited. Sites are categorised into five classes, A to E. The sites classes range from "rock sites" (Class A and B) to very soft soil sites, Class E.

The intent of the code is to classify the site according to its broad vibration properties as represented by the low amplitude fundamental site period, *T*. Empirical data and theoretical studies show conclusively that near surface materials have a significant impact on the surface motion; both the amplitude of the motion and the frequency content. The site period is determined by the geometry and nature of the geologic units present at the site. The site period used in NZS 1170.5 is independent of the strength of earthquake design motion, because the low amplitude period assumes very low strain response to represent the strain-independent soil properties.

Figure 1 shows the spectral shape factors for each of the site classes. There are significant differences in the shape factor between the various classes, illustrating the importance of assessing the site class. Class A, strong rock, and Class B, rock, were not found to be significantly different in the hazard study carried out for NZS 1170.5 and hence they are grouped together. It is clearly important to distinguish between Class C, Class D and Class E. The maximum shape factor (at short periods) is the same in the case of Class D and Class E but the plateau extends to 0.6 seconds in the case of Class D and 1.0 seconds in the case of Class E. Class C is important since many sites in New Zealand will fall in this category. For sites to fall into this class, the site period needs to be less than 0.6 seconds or to have depths of soil not in excess of those listed in Table 3.2 (page 14 of the Standard), which is reproduced here in **Table 1**.

NZS 1170.5 clause 3.1.3.1 specifies a hierarchy of methods to assess the site class. The stated hierarchy in order of preference is:

1. From the site period based on four times the travel time of shear waves from the underlying rock to the ground surface.

2=. From borelogs, including measurement of geotechnical properties.

2=. From a method known as Nakamura ratios.

2=. From recorded earthquake motions.

5. From borehole descriptors but with no measurement of geotechnical properties.

6. From surface geology and estimates of the depth to rock.

Most (if not all) sites are heterogeneous, i.e. they contain a number of soil layers of differing properties. Each layer will influence the site period. For these cases, there is another clause in NZS 1170.5 (clause 3.1.3.7) for determining the site period for layered sites. This clause states that the natural period of the site may be estimated by summing the contributions to the natural period of each layer. The contribution of each layer is defined by multiplying 0.6 seconds (the boundary between class C and class D) by the ratio of the layer's thickness to the maximum soil depths in Table 3.2. It appears the intent of this clause is for determination of site period without calculating the shear wave velocity.

Benefits in true cost and safety are likely to arise from using accurate methods to assess the site period and this is the motivation for this work. The intent of this article is to advise

[1] *Senior Lecturer, Department of Civil Engineering, University of Auckland (member)*

[2] *PhD Candidate, Department of Civil Engineering, University of Auckland (member)*

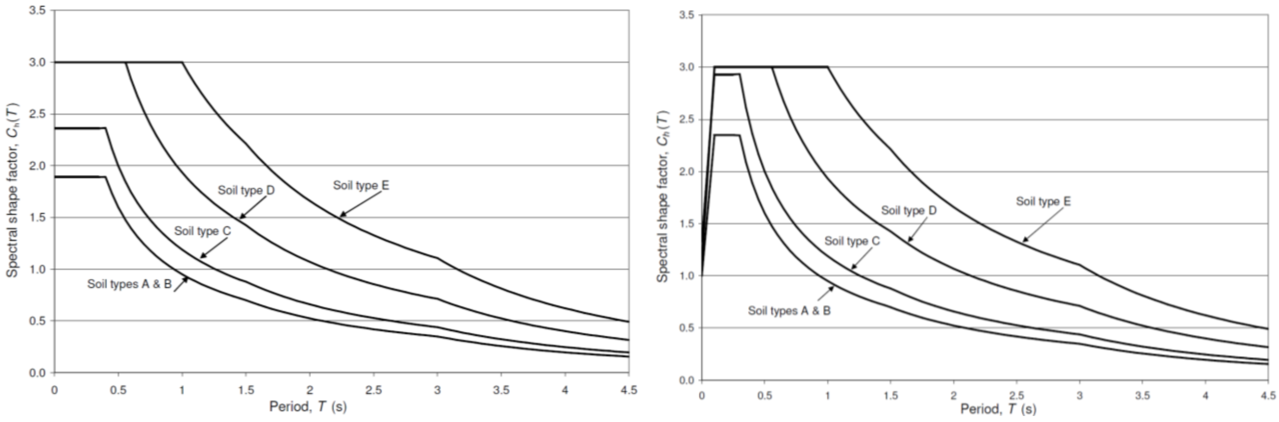

Figure 1: NZS 1170.5 spectral shape factors for general cases (left) and for modal response and numerical integration time-history methods (right).

Table 1. Reproduction of NZS 1170.5 "Table 3.2 – maximum depth limits for site subsoil class C".

Soil type and description		**Maximum depth of soil (m)**
Cohesive soil	**Representative undrained shear strengths (kPa)**	
Very Soft	< 12.5	0
Soft	12.5 – 25	20
Firm	25 – 50	25
Stiff	50 – 100	40
Very stiff or hard	100 – 200	60
Cohesionless soil	**Representative SPT N values**	
Very loose	< 6	0
Loose dry	6 – 10	40
Medium dense	10 – 30	45
Dense	30 – 50	55
Very dense	> 50	60
Gravels	> 30	100

the earthquake engineering community on the methods to assess site class, with particular focus on the specified hierarchy of methods detailed in NZS 1170.5. We begin with a background on several available methods to calculate shear-wave velocity (V_S) at a site, as this is a key parameter for accurate determination of site period. We subsequently present five example soil profiles (uniform layer, V_S increasing as a power of depth, shear modulus linearly increasing with depth, a two-layer profile and a three-layer profile) and discuss methods for calculating site period in each case. For the two-layer and three-layer cases, we compare NZS1170.5 clause 3.1.3.7 with conventional methods for determining site period.

BACKGROUND: DETERMINING SHEAR-WAVE VELOCITY

In an engineering context, the most important mode of seismic response is that associated with shear waves. These waves cause horizontal motion, and hence horizontal shear deformation of the foundation soil. Most of the damage to infrastructure originates from foundation response to shear waves. Of particular relevance to the topic of site period is the velocity of propagation of these shear waves, V_s, through the soil. A linear elastic analysis based on dynamic equilibrium shows that this velocity is related to the shear modulus through equation (1):

$$V_S = \sqrt{\frac{G_{\max}}{\rho}} \quad \mathrm{m/s}, \qquad (1)$$

where G_{max} is the tangential shear modulus at zero shear strain (MN/m^2) and ρ is the mass density (kg/m^3). Frequent use is made of this relationship in geotechnical earthquake engineering.

In situ measurements

The shear wave velocity of a soil may be measured in the laboratory or the field. In a general sense the "best" method is *in situ* measurement since the effective stress is correct and there is no sample disturbance. While in principle the methods are simple there are many complicating factors. What is presented here is a general overview. There are four main methods of *in situ* measurement:

(i) Downhole measurement, where the shear waves (SH) are created at the surface and the travel time to various positions at depth in a borehole are measured. A bi-directional source is useful to enable the shear waves to be readily separated from the compression waves. A relatively recent development is the use of a seismic cone penetrometer to carry out shear wave velocity measurements. A conventional cone penetration (CPT) device is used that has an accelerometer or geophone incorporated in the cone. In this case there is no need for a borehole, the CPT data is interrupted at the depth required and a shear wave velocity test carried out. Some site investigation companies now have such a device and this method represents an effective method of assessing the shear wave velocity profile at a site. As with all CPT work a calibration borehole is recommended.

(ii) Crosshole measurement, where shear waves are propagated between adjacent boreholes and the travel time measured. These tests usually invoke vertical particle displacement, i.e. SV waves. These tests are often used for investigations beneath existing foundations or for the purpose of machine foundations.

(iii) Geophysical refraction and reflection surveys, where a shear source generates shear-waves, and the subsequent arrival times of shear-waves are detected by a line of horizontal geophones. Difficulties arise in generating a large enough shear source to be clearly recorded by the geophones.

(iv) SASW / MASW – Spectral analysis of surface waves, or multi-channel analysis of surface waves, are geophysical methods which utilise the dispersive properties of surface waves (typically Rayleigh waves generated by a sledgehammer hitting a plate) to calculate dispersion curves (phase velocity vs frequency plots). A 1D or 2D shear-wave velocity profile is obtained by inverting (i.e. back-calculating from) the dispersion curve. This technique was used very infrequently in New Zealand prior to the 2010-2011 Canterbury earthquake sequence but is becoming more popular. The data processing does require some specialist knowledge and therefore these methods should only be conducted by experienced personnel.

Of the four methods, downhole measurements are only representative of shear-wave velocity at a single point, while the other three methods represent an average value over a 2D line, which may be more beneficial at many sites. However, if the sites are more complex with 2D or 3D variations, the refraction, reflection and surface wave methods become very difficult. It is also considered best practice to use multiple methods at the same site, to validate the results and quantify uncertainties.

Laboratory measurements

Laboratory methods usually employ what are known as bender elements. These are wafers of a piezo-ceramic material about 6 mm long that generate a small electrical current on flexing. A bender element is installed in the top and bottom of a triaxial specimen, one element being the source the other being the pick-up. A small current is supplied to the source that causes flexure of the bender element with the consequent production of shear waves. These waves travel through the specimen and create flexure of the pick-up with consequent generation of a small current. By using an oscilloscope the travel time between the source and pick-up is measured.

Other laboratory devices, such as a Torsional Resonant Column, are used to measure the shear modulus. Such laboratory studies are usually very detailed and seek data that allows evaluation of the shear modulus as a function of shear strain. Laboratory tests suffer from specimen disturbance but are used when there will be changes in the effective stress at a site since this will produce a change in the shear wave velocity.

Empirical correlations

The most reliable method of assessing V_S is from site specific *in situ* measurement. If this data is not available, then empirical methods may be used to furnish an estimate for V_S or G_{max} based on *in situ* test results e.g. $(N_1)_{60}$ or q_c values, where $(N_1)_{60}$ is the normalised SPT value at 60% energy efficiency and q_c is the cone resistance from a CPT test. The following are suggestions from the literature for an initial estimate of G_{max} or V_S. It needs to be kept in mind that almost none of the data from which the empirical relationships were estimated were derived from New Zealand soils. When using these relationships it is important to consult the reference to understand the geological/geotechnical setting of the data.

Cohesive soil deposits

As an approximate method, the shear modulus is correlated with the undrained shear strength i.e. the ratio of G_{max} / s_u. From the limited data available for a residual New Zealand soil, Meyer (1999) [2] suggested an appropriate value of G_{max} / s_u is approximately 500. V_S may then be calculated from:

$$V_S = \sqrt{\frac{500 \cdot s_u}{\rho}} \quad \text{m/s}, \tag{2}$$

Table 2. Values of G_{max} / s_u from Weiler (1988).

Plasticity Index	Overconsolidation Ratio (OCR) 1	2	5
15-20	1100	900	600
20-25	700	600	500
35-45	450	380	300

Based on values of s_u obtained using triaxial compression, Values of G_{max} / s_u as a function of over consolidation ratio and plasticity index have been suggested by Weiler (1988) [3], shown in **Table 2**.

Based on a wide ranging series of field tests, Mayne and Rix (1993) [4] have suggested the following relationship:

$$G_{\max} = 406 \cdot q_c^{0.695} \cdot e^{-1.13} \tag{3}$$

where e is the void ratio, and both G_{max} and the cone tip resistance, q_c, are in kPa.

Cohesionless soils

There are a number of correlations of shear wave velocity (or G_{max}) with either SPT or CPT values. Most of this data relates to sedimentary soils from overseas. There are some data for New Zealand pumice soil from triaxial testing. Richart *et al* (1970) give further relationships for G_{max} for silica sand as a function of confining stress and void ratio [5]. There is great uncertainty about whether overseas correlations involving q_c and N obtained on quartz sands (all those below except (vi)) can be applied to New Zealand pumice soil. At this point these correlations should not be applied to New Zealand pumice soils.

(i) Based on CPT field tests in Italy on uncemented silica sands, Baldi *et al*. (1989) [6] propose the correlation shown in **Figure 2**.

(ii) Rix and Stokoe (1991) [7] have proposed the following relationship, where all variables are in kPa:

$$G_{\max} = 1634 \cdot q_c^{0.25} \cdot \left(\sigma_v'\right)^{0.375} \quad . \tag{4}$$

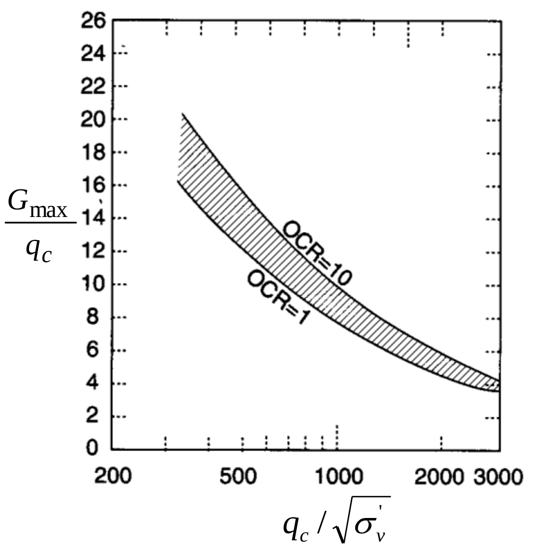

Figure 2: Shear modulus from CPT data (Baldi et al., 1989 [6]).

(iii) Seed *et al.* (1986) [8] propose:

$$G_{\max} = 138{,}900 \cdot (N_1)_{60}^{0.33} \cdot \left(\sigma_m^{'}\right)^{0.5} \quad , \qquad (5)$$

where the stress units are Pa and $\sigma_m^{'}$ is the mean effective stress, given by:

$$\sigma_m^{'} = 0.33\left(\sigma_v^{'} + 2\sigma_h^{'}\right) \quad . \qquad (6)$$

(iv) Sykora and Stokoe (1983) [9] propose:

$$V_S = 107 \cdot N_{60}^{0.27} \quad \text{m / s} \quad . \qquad (7)$$

Note that the *N* value is not normalised.

(v) Imai and Tonouchi (1982) [10] propose:

$$V_S = 107 \cdot N_{60}^{0.315} \quad \text{m / s} \quad . \qquad (8)$$

Note that the *N* value is not normalised.

(vi) Marks *et al*. (1998) [11] proposed a relationship for New Zealand pumice sand:

$$G_{\max} = \left(14 \cdot D_R^{0.3}\right) \cdot \left(\frac{p_0^{'}}{p_a}\right)^{0.6} \quad \text{MPa} , \qquad (9)$$

where D_R is the relative density, p_0' is the effective confining pressure of a triaxial specimen and p_a is atmospheric pressure. The determination of relative density is problematic for pumice soils, the reference should be consulted before using this relationship.

Finally, a Japanese study (1978) [12] presents the results of a large number of *in situ* wave velocity measurements correlated with *N* values, soil type, depth and geological age. Interested readers will find information on the background to the study within the reference.

CALCULATION OF SITE PERIOD

The term site period refers to the fundamental period of vibration of a horizontal site of linear elastic material when responding to vertically propagating shear waves with horizontal particle motion, known as SH waves. In the context of NZS 1170.5, site period is a key parameter because it defines the boundary between class C and class D (T = 0.6 s). This threshold is critical, given that the design loads change significantly either side of the boundary. To mitigate the large increase in forces at T = 0.6 s, McVerry (2011) [13] proposed intermediate spectra between the existing class C and class D spectra, which are defined entirely in terms of site period. Robust calculations of site period can justify the use of these alternate spectra for calculating earthquake loadings.

For NZS 1170.5 the site is assumed to be one dimensional, i.e. the lateral boundaries are far removed and have no influence on the motion of the site. The properties needed for calculation of the site period are the mass density, ρ, the thickness of the layer, *H*, and the shear modulus, G_{max}, or shear wave velocity V_S.

There are an infinite number of modal frequencies and mode shapes, the fundamental mode being the lowest value of ω, i.e. ω_1 . The fundamental period, *T*, may be calculated from ω by

$$T = \frac{2\pi}{\omega_1} \quad \text{seconds} \quad . \qquad (10)$$

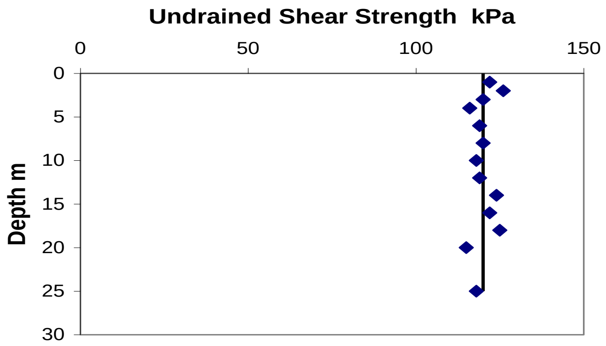

Figure 3: Variation of s_u with depth.

If the mass density is considered constant with depth (within an engineering approximation) then the only property needed to calculate *T* is the distribution of G_{max} or V_S with depth. For some simple distributions of V_S with depth, closed form solutions are available, while for more complex layered site numerical solutions prevail. For more information, Dobry *et al*. (1976) [14] provide a summary of the closed form methods.

To assist site period calculations for a variety of soil profile types, we now present five worked example calculations.

(i) Case 1: uniform layer

For this simple case, the shear wave velocity and mass density are constant with depth, and the lateral boundaries are assumed to be infinite in both planes. From the closed form solution, the first modal frequency is

$$\omega_1 = \frac{\pi V_s}{2H} \quad \text{rad/s} \quad , \qquad (11)$$

from which the fundamental period is

$$T = \frac{2\pi}{\omega} = \frac{4H}{V_s} \quad \text{sec} \qquad (12)$$

The first mode shape is $X(z) = \cos(\frac{\omega z}{V_S})$.

Note that equation (12) is only applicable for sites with constant mass density, ρ, and exactly represents the preferred method in the NZS 1170.5 clause 3.1.3.1 hierarchy for determining site period.

Example

Consider a uniform 25 m layer of saturated clay soil with OCR = 5, PI = 25, ρ = 1950 kg/m^3. A profile of the measured undrained shear strength, s_u, is shown in **Figure 3**. Weiler (1988) [3], gives G_{max} / s_u = 500 and G_{max} = 6 x 10^4 kPa (see Table 2). Applying equation (1):

$$V_S = \sqrt{\frac{1000 \times 6 \times 10^4}{1950}} = 175 \text{ m/s and}$$

$$T = \frac{4H}{V_S} = 0.57 \text{ sec} \quad .$$

Fundamental period = 0.57 s, therefore class C.

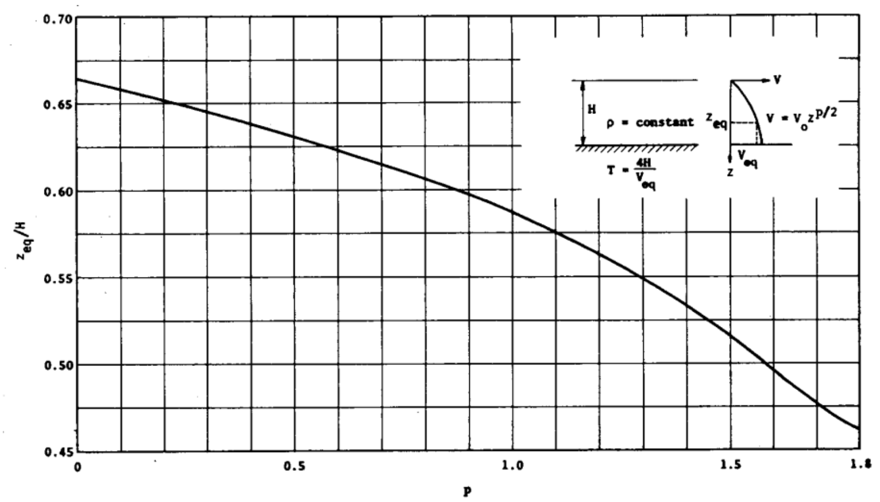

***Figure 4:** z_{eq} / H as a function of p, the closed form solution for equation (14), from Dobry et al. (1976) [14].*

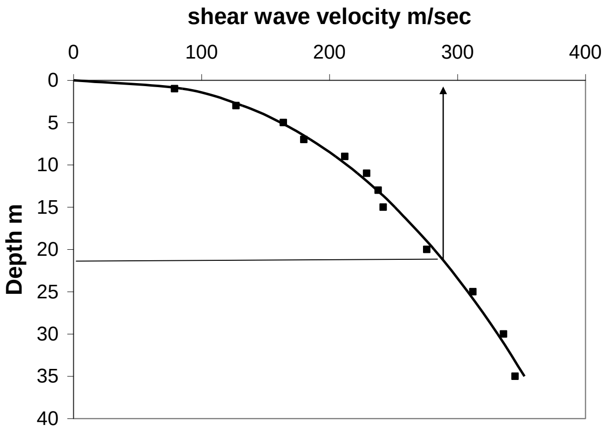

***Figure 5:** Example V_S profile for the case of velocity increasing as a power of depth. Solid black line is Eq (20) fitted by eye.*

(ii) Case 2: velocity increasing as a power of depth

This case has been reported by Idriss and Seed (1968) [15] and Dobry *et al*. (1971) [16], and may be representative of uniform normally consolidated saturated clay deposits or uniform deposits of sand (water table at ground surface). The preferred method in the NZS 1170.5 clause 3.1.3.1 hierarchy can still be applied in this case, using the following closed form solution. The distribution of the shear-wave velocity is taken to be

$$V_S = V_{S0}\, z^{\frac{p}{2}} \quad . \tag{13}$$

Values of p are usually taken to be between 0.5 and 1. The fundamental period is given by

$$T = \frac{4\pi H^{(2-p/2)}}{(2-p)V_{S0}\, q} \quad \text{for} \quad 0 \le p < 2 \; , \tag{14}$$

where q is the first root of $J_n(q) = 0$, J_n is the Bessel function of order $n = (p-1)/(2-p)$. The solution for the period for this case may be found from the equation for a uniform layer

$$T = \frac{4H}{V_{S\,eq}} \tag{15}$$

where $V_{S\,eq}$ is the value of V_s at the "equivalent depth" z_{eq} . **Figure 4** is a graphic representation of the closed form solution, giving the value of z_{eq} / H as a function of p, which may then be used to solve for T.

Example:

Consider a 35 m layer of sand with the water table at the surface. **Figure 5** shows hypothetical results of SPT tests, converted to V_S e.g. by using equation (7). The results are fitted with equation (13) by eye, with p = 0.8 and V_{S0} = 85 m/s. Figure 4 with p = 0.8 gives z_{eq} / H = 0.608, thus z_{eq} = 21.3 m. Comparing with Figure 6, this gives $V_{S\,eq}$ = 290 m/s.

Entering in this value into equation (15) gives a fundamental period T = 0.48 seconds, therefore class C.

(iii) Case 3: Shear modulus increasing linearly with depth

This case for constant ρ has been presented by Ambraseys (1959) [17] for G_{max} increasing with depth, and by Urzua (1974) [18] for G_{max} decreasing with depth. For both of these cases, the site period can still be obtained using the preferred method in the NZS 1170.5 clause 3.1.3.1 hierarchy, using the closed form solution detailed here. If G_0 and G_H are the shear modulus at the surface and base of the layer respectively then the variable K is used where K is

$$K = \sqrt{\frac{G_{\max 0}}{G_{\max H}}} \quad , \tag{16}$$

and

$$\frac{G_{\max}}{G_{\max H}} = K^2 + \left(\frac{z(1-K^2)}{H}\right) \quad . \tag{17}$$

When K < 1 the modulus increases with depth, and when K > 1 the modulus decreases with depth. The fundamental period is given by

$$T = \frac{4\pi HK}{a_1(1-K^2)V_{S0}} \qquad 0 \le K < 1 \quad ,$$

$$T = \frac{4\pi HK}{a_1(K^2-1)V_{S\,0}} \qquad K > 1 \quad , \tag{18}$$

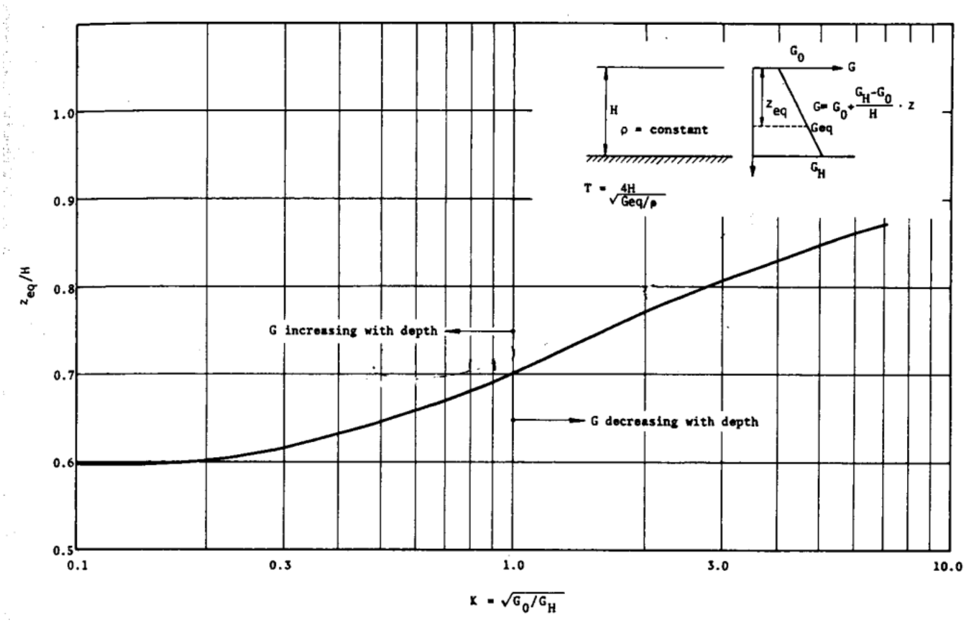

Figure 6: ***A graphical representation of the closed form solution to the shear modulus increasing linearly with depth, z_{eq} / H as a function of K, from Dobry et al. (1976) [14].***

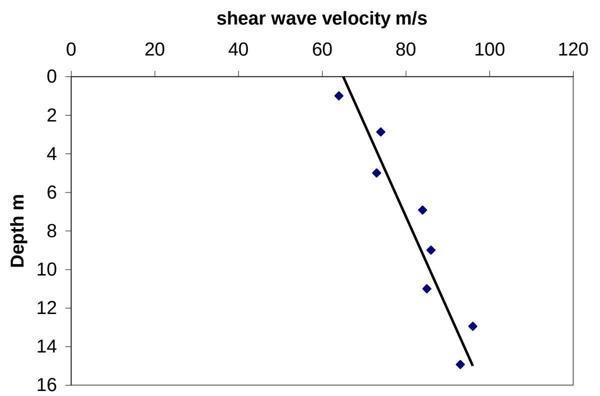

Figure 7: ***Example V_S profile for case of linearly increasing shear modulus.***

where $V_{S\,0}$ is the wave velocity at the free surface and a_1 is the first root of

$$J_0(a_1)Y_1(Ka_1) - J_1(Ka_1)Y_0(a_1) = 0 \,, \qquad (19)$$

where $J_0(\,)$ and $J_1(\,)$ are Bessel functions of the first kind and Y_0 and Y_1 are Weber's Bessel functions. In this case also the period may be expressed using the equation for a uniform layer, $T = 4H / V_{S\,eq}$ where $V_{S\,eq}$ is the wave velocity at depth z_{eq} and the expression for z_{eq} for all K is

$$\frac{z_{eq}}{H} = \left(\frac{a_1}{H}\right)^2 (1-K^2) - \frac{K^2}{1-K^2} \,. \qquad (20)$$

Figure 6 is a graphical representation of the above closed form solution. This may be used to determine z_{eq} / H as a function of K. The value G at z_{eq}, G_{eq}, may then be found from equation (20) and used to evaluate $V_{s\,eq}$. The period is then calculated using $T = 4H / V_{s\,eq}$, equation (15), as in the previous case.

Example:

Consider a 15 m layer of soft normally consolidated clay with a mass density of 1,720 kg/m^3. A series of seismic cone tests have produced the measured shear wave velocity profile shown in **Figure 7**. The shear wave velocity at the ground surface, V_{S0} = 65 m/s. The shear wave velocity at the rock line, V_{SH} = 96 m/s.

Thus $G_0 = V_{S0}^2 \rho = 7.27$ MPa

$$G_H = V_{SH}^2 \rho = 15.85 \text{ MPa}$$

$$K = \left(\frac{G_0}{G_H}\right)^{0.5} = 0.68$$

$\frac{z_{eq}}{H} = 0.67$ from Figure 6; hence $z_{eq} = 10.05$ m

$$G_{zeq} = \left(G_0 + \frac{(G_H - G_0) z_{eq}}{H}\right) = 13.06 \text{ MPa}$$

$$V_{S\,eq} = \left(\frac{G_{zeq}}{\rho}\right)^{0.5} = 87.1 \text{ m/s}$$

$$T = \frac{4H}{V_{S\,eq}} = 0.69 \text{ s.}$$

Therefore class D.

(iv) Case 4: Two layer profile

The solution for calculating site period at a two layer profile is more complex, as continuity of shear stress and displacement at the interface of layers needs to be enforced. NZS 1170.5 specifies a different method for calculating site period for layered sites, by summing the contributions of each layer to the overall site period. Here we compare two alternative methods with the method detailed in NZS 1170.5 clause 3.1.3.7, and show that the alternative methods give more precise results.

Closed form solution

Solutions for the two layer profile were presented by Madera (1971) [19], Chen (1971) [20] and Urzua (1974) [18]. The key variables in these solutions are:

$$\frac{\rho_A H_A}{\rho_B H_B} \ ,$$

and the ratio of the fundamental periods of the lower and upper layers (T_B / T_A). **Figure 8** allows calculation of the period of the compound system as a function of the period of the upper layer.

Consider the two-layer site profile shown in **Figure 9**. For the clay layer, Weiler (1988) [3] gives G_{max} / s_u = 600, and therefore G_{max}= 4.2 x 10^7 Pa. This results in:

$$V_{S\,clay} = \left(\frac{G_{\max clay}}{\rho_{clay}}\right)^{0.5} = 152.7 \text{ m/s}$$

$$T_{clay} = \frac{4H_{clay}}{V_{S\,clay}} = 0.21 \text{ s} \quad \text{from equation (12).}$$

For the sand layer, use Sykora and Stokoe (1983) [9]. As the layer is approximately uniform, average the N values, thus $\overline{N}$ = 8.9.

$$V_{S\,sand} = 107\overline{N}^{0.27} = 193 \text{ m/s , from equation (7)}$$

$$T_{sand} = \frac{4H_{sand}}{V_{S\,sand}} = 0.25 \text{ s}$$

$$\frac{T_{sand}}{T_{clay}} = 1.19$$

$$\frac{\rho_{clay} H_{clay}}{\rho_{sand} H_{sand}} = 0.65$$

$$\frac{T}{T_{clay}} = 2.1 \text{ (from Figure 8)}$$

$$T = 2.1 T_{clay} = 0.44 \text{ s .}$$

Therefore the fundamental period from this approach is **0.44 seconds**, which corresponds to a site class C.

Lumped mass solution

The second method we discuss is known as the lumped mass solution. In this approach, the soil profile may be idealised as a series of masses interconnected by shear springs. The mass is calculated to represent the surrounding soil and the stiffness of the shear spring is computed from the shear modulus. This is sometimes referred to as a 1D shear beam model. **Figure 10** shows the system. Generally the thickness of the sub-layers needs to be approximately 3 m or less for good accuracy. The dynamic equation of motion of the system under free vibration includes the mass matrix, *[M]* and the stiffness matrix *[K]* and may be written as

$$[M]\langle \ddot{x} \rangle + [K]\langle x \rangle = 0 \ , \tag{21}$$

where $\langle \ddot{x} \rangle$ *and* $\langle x \rangle$ are the vectors of acceleration and displacement of each mass relative to the base. They are of dimension n, where n is the number of masses. Equation (21) may be transposed into the classic eigenvalue form.

$$[A] - \omega^2 [I] = 0 \ , \tag{22}$$

where $[A] = [M]^{-1}[K]$, $[I]$ is the identity matrix and ω are the fundamental frequencies of the system. Since the system is closely coupled, the *[K]* matrix is tri-diagonal and symmetric and has the form:

$$K = \begin{matrix} k_1 + k_2 & -k_2 & 0 & 0 \\ -k_2 & k_2 + k_3 & -k_3 & 0 \\ 0 & -k_3 & k_3 + k_4 & -k_4 \\ 0 & 0 & -k_4 & k_5 \end{matrix} \tag{23}$$

The mass matrix is diagonal:

$$M = \begin{matrix} m_1 & 0 & 0 & 0 \\ 0 & m_2 & 0 & 0 \\ 0 & 0 & m_3 & 0 \\ 0 & 0 & 0 & m_4 \end{matrix} \tag{24}$$

The values of the fundamental frequencies can be easily solved using software e.g. SAP, MATHCAD or MATLAB, with the smallest eigenvalue (ω_1) of the [A] matrix corresponding to the fundamental period of the site, i.e. equation (16):

$$T = \frac{2\pi}{\omega_1} \quad .$$

The method is useful especially in the case of relatively thin soft layers of soil contained within a soil profile and where the profile contains layers where the shear wave velocity is a function of depth. Each soil layer is subdivided into sublayers, of thickness h_i, to represent what is actually a continuum. The magnitude of each mass is calculated to represent the soil over one half the sublayer on each side, i.e.

$$m_i = 0.5(\gamma_i h_i + \gamma_{i+1} h_{i+1}) \quad \text{kg} \tag{25}$$

where γ is the unit weight and h_i is the inter-mass distance. The lumped mass system represents a unit plan area since a one dimensional model is employed and thus the shear stiffness of the interconnecting spring is

$$k_i = \frac{(G_{\max})_i}{h_i} \quad \text{N/m} \quad , \tag{26}$$

Taking a finer subdivision of the profile can prove that the solution is converging.

An application of this method to the two-layer profile in Figure 10 is shown here. The mass and stiffness matrices may be formed by compiling the parameters **Table 3**. The soil profile has been discretised into sublayers, with both the sand and clay layers having four sublayers each. Note that the first sublayer is adjacent to the rock interface.

The [M] and [K] matrices are formed according to equations (30) and (31), although in this case both are 8 x 8 matrices. To obtain the eigenvalues, the [A] matrix is calculated by

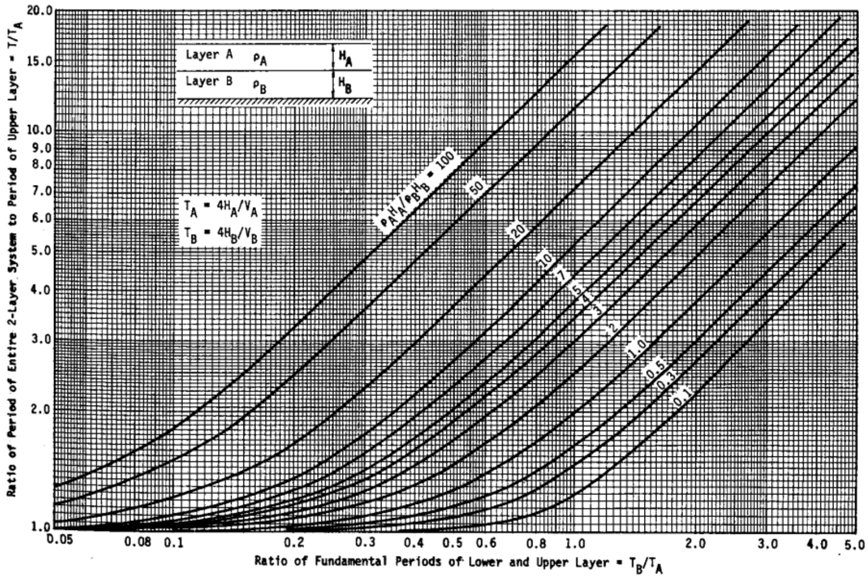

Figure 8: Closed form solution for a two-layer profile, from Dobry et al. (1976) [14].

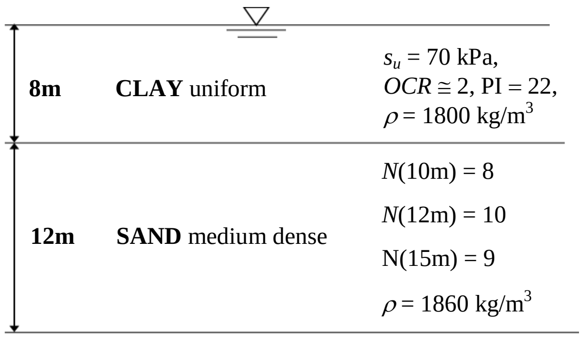

Figure 9: Example two-layer profile.

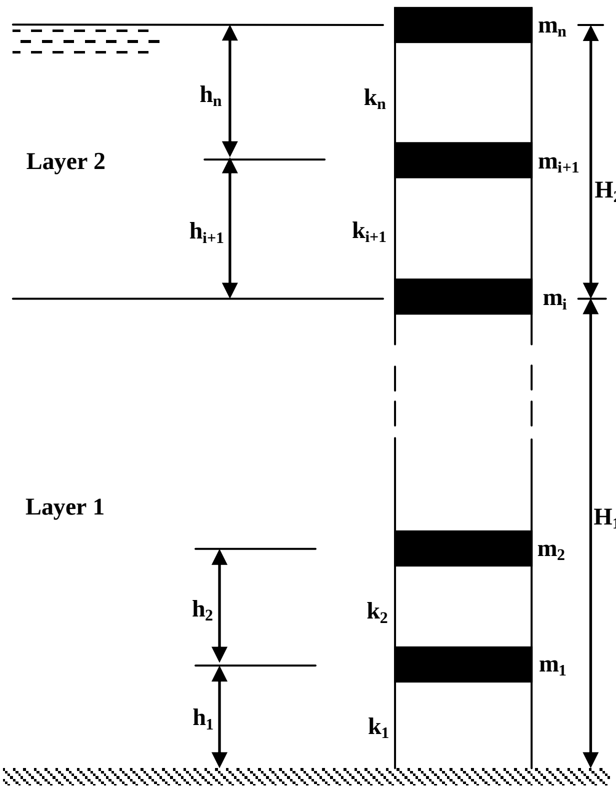

Figure 10: Example of a lumped mass representation of a soil layer system.

Table 3. Properties of the lumped mass system for the two-layer example.

Sublayer	h_i (m)	V_{Si} (m/s)	ρ_i (kg)	$G_{max\,i}$ (MPa)	k_i (MN/m)	m_i (kg)
1	3	193	1860	69.3	23.1	5580
2	3	193	1860	69.3	23.1	5580
3	3	193	1860	69.3	23.1	5580
4	3	193	1860	69.3	23.1	4590
5	2	152	1800	41.6	23.1	3600
6	2	152	1800	41.6	23.1	3600
7	2	152	1800	41.6	23.1	3600
8	2	152	1800	41.6	23.1	1800

$$[A] = [M]^{-1}[K] \tag{27}$$

and the eigenvalues (ω_1^2 to ω_8^2) can be computed using any applicable software. The fundamental frequency, ω_1, corresponds to the lowest eigenvalue, and in this case $\omega_1 = \sqrt{219.6}$ = 14.82 rad/s, calculated using MATHCAD. The soil period, $T = 2\pi/\omega_1$ = **0.42 seconds**. This compares closely with the closed form solution (T = 0.44 sec).

NZS 1170.5 method

Section 3.1.3.7, in conjunction to Table 3.2, gives

$$t_{clay} = \frac{0.6\,H_{clay}}{40} = 0.12\ \text{s}$$

$$t_{sand} = \frac{0.6\,H_{sand}}{40} = 0.18\ \text{s}$$

$$T_{1170} = t_{clay} + t_{sand} = 0.30\ \text{s}$$

This method gives a fundamental soil period of ***T* = 0.30 seconds**. Comparing with the numerical and lumped mass solutions (0.44 and 0.42 seconds respectively), the NZS 1170.5 is unconservative with an error of approximately 30%.

(v) Case 6: Three layer profile

Most sites are comprised of a number of layers of soil with different shear wave velocity, mass density and thickness. The final case we analyse in this article is a three-layer soil profile, an example of which is shown in **Figure 11**. Here we present two alternative methods to calculate the site period and compare it with the NZS 1170.5 method.

Successive application of the two-layer solution

This method was developed by Dobry and Madera (described within [19]) and employs successive use of the two-layer closed form solution. The method assumes constant density for all layers and has been shown by Dobry *et al*. (1976) [14] to yield periods less than 10% in error, given this assumption. Where very large differences in mass density exist, there may be more deviation. The following steps are involved:

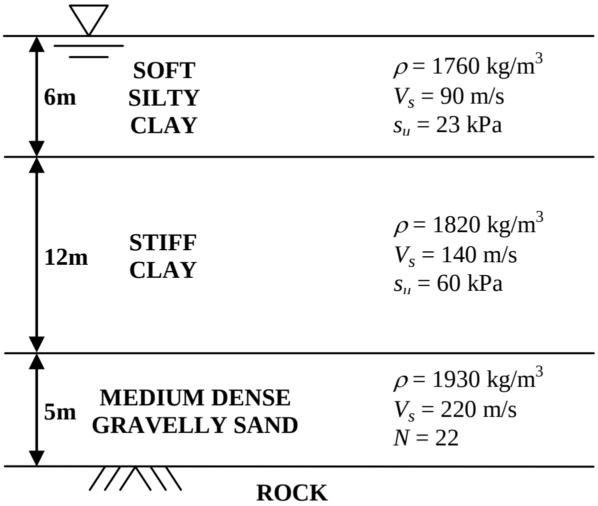

Figure 11: Example three-layer soil profile.

1. The top two layers are assumed to lie on rock and their period, $T_{1\text{-}2}$, is computed with the aid of Figure 8. For this, assume $\rho_A = \rho_B$.
2. The top two layers are replaced by a new top layer with $T_A = T_{1\text{-}2}$ obtained in 1 and $H_A = H_1 + H_2$.
3. The rock interface is assumed below layer 3 of the profile and the period, $T_{1\text{-}3}$, of the new system is estimated by Figure 8 using $T_A = T_{1\text{-}2}$, $T_B = 4H_3 / V_{s3}$ and $\rho_A H_A / \rho_B H_B = (H_1 + H_2) / H_3$.
4. The top three layers are replaced by a new top layer with $T_A = T_{1\text{-}3}$ obtained from 3 above and $H_A = H_1 + H_2 + H_3$.
5. The process is repeated until the last layer is considered. The estimated period of the profile is

$$T \approx T_{1-n}\ , \tag{28}$$

where n is the number of soil layers.

Table 4 shows the solution to the three-layer profile in Figure 11 using this method. The fundamental period the site, ***T* = 0.59 seconds**, therefore on the boundary between class C and class D.

Table 4. Solution to the three-layer example profile using the successive application of the two-layer solution.

Layers considered	H_A (m)	H_B (m)	H_A/H_B	V_A (m/s)	V_B (m/s)	T_A (s)	T_B (s)	T_A/T_B	T/T_A	T (s)
1 & 2	6	12	0.5	90	140	0.27	0.34	1.27	2.0	0.54
1 to 3	18	5	3.6	-	220	0.54	0.091	0.17	1.1	0.59

Table 5. Properties of the lumped mass system for the three-layer solution.

Sublayer	h_i (m)	V_{Si} (m/s)	ρ_i (kg)	$G_{max\,i}$ (MPa)	k_i (MN/m)	m_i (kg)
1	2.5	220	1930	93.4	37.4	
						4825
2	2.5	220	1930	93.4	37.4	
						5143
3	3	140	1820	35.7	11.9	
						5460
4	3	140	1820	35.7	11.9	
						5460
5	3	140	1820	35.7	11.9	
						5460
6	3	140	1820	35.7	11.9	
						5370
7	3	90	1760	14.3	4.8	
						5280
8	3	90	1760	14.3	4.8	
						2640

Lumped mass solution

This is the same method as applied in the previous case. Here, we divide the top and bottom layers into two sublayers, and the middle layer into four sublayers. The accuracy of the eigenvalues will depend on the number of sublayers. **Table 5** shows the properties of the lumped mass system. Forming [*M*] and [*K*] matrices as previously gives a smallest eigenvalue of $\omega_1 = \sqrt{116.1} = 10.77$ rad/sec. The soil period, $T = 2\pi/\omega_1 =$ **0.58 seconds**. This compares closely with the successive two-layer solution.

NZS 1170.5 approach

Section 3.1.3.7, in conjunction to Table 3.2, gives

$$t_1 = \frac{0.6H_1}{40} = 0.18 \text{ s}$$

$$t_2 = \frac{0.6H_2}{40} = 0.18 \text{ s}$$

$$t_3 = \frac{0.6H_3}{45} = 0.07 \text{ s}$$

$$T_{1170} = t_1 + t_2 + t_3 = 0.43 \text{ s.}$$

This method gives a fundamental soil period of ***T* = 0.43 seconds**, and the site would be classified as class C. Comparing with the successive two-layer and lumped mass solutions (0.59 and 0.58 seconds respectively), the NZS 1170.5 method is unconservative, with an error of roughly 27%.

(vi) Other cases

For closed form solutions for other example soil profiles (e.g. over consolidated crust overlying normally consolidated clay, shear modulus decreasing with depth), or alternative methods to the previous five cases, we refer the reader to Dobry *et al.* (1976) [14] and references therein.

ISSUES WITH TABLE 3.2 IN NZS 1170.5

As demonstrated in the previous section, NZS1170.5 clause 3.1.3.7 for evaluating of period at layered sites gives results that are inconsistent with other well-accepted calculation methods. For the two-layer and three-layer example soil profiles, the code method is shown to be unconservative by roughly 30%, as shown in **Table 6.**

Accepting the categories of soil and the representative strengths and *N* values from Table 3.2, another assessment of the depth of soil to limit the period to less than 0.6 seconds has been made. This has been done using previously identified empirical relationships from overseas data between strength and shear modulus [3], and N value and shear wave velocity, [9, 10]. **Figure 12a** compares a graphical representation of Table 3.2 with maximum depth limits for a site period of *T* = 0.6 s for a cohesive soil site with a PI of approximately 20 to 25 and an OCR of 1 and 5. **Figure 12b** is the same for cohensionless soils.

The maximum depths specified in Table 3.2 are inconsistent with the site period boundary of 0.6 seconds. For cohesive soils, the maximum depth specified by the curves derived from available correlations is approximately one half of that from NZS 1170.5. The effect is still evident for cohensionless soils, but not to the same extent. The overestimations in the Table 3.2 maximum soil depths mean that some sites (the sites in the shaded areas of Figures 12a and 12b) that should be classified as class D according to site period are instead classified as class C in NZS 1170.5 i.e. the code is unconservative. We believe that either the maximum depths in Table 3.2 should be amended, or removed altogether. Instead, we recommend that the lumped mass solution be adopted as the preferred method for determining site period for layered sites, as this method is simple, efficient, and easily adaptable to complex sites. Successive application of the two-layer solution is also considered an acceptable method to determine site period, despite being less efficient than the lumped mass solution.

Note that Figure 12b only applies to sites containing quartz sand. The curve should not be used for pumice sands since the correlations in the literature between SPT *N* value and shear wave velocity do not apply to volcanic sands. Pumice sands are a crushable material even under relatively moderate levels of stress with the result that this material produces very different behaviour during CPT testing compared with quartz sands.

Table 6. Comparison of methods to calculate site period for two-layer and three-layer examples.

Example profile	*Closed form solution (s)*	*Lumped mass solution (s)*	*NZS1170.5 clause 3.1.3.7 (s)*
Two-layer	0.44	0.42	0.30
Three-layer	0.59	0.58	0.43

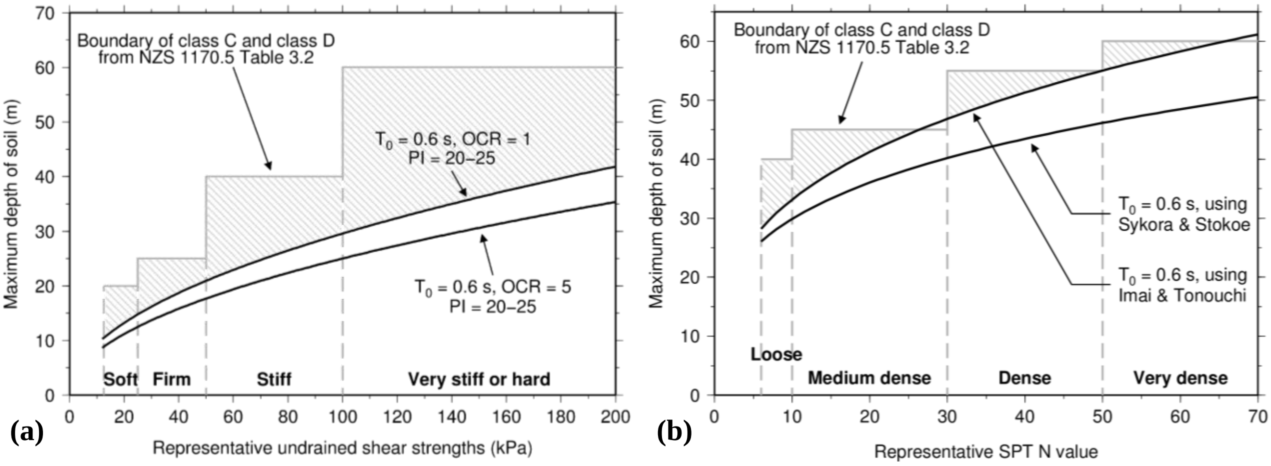

Figure 12: ***(a) Comparison of the maximum depth limits for site class C for cohesive soils in NZS1170.5 Table 3.2 (grey lines) with the depth to give T= 0.6 s, calculated using Weiler (1988) [3] (black curves). (b) Comparison for the maximum depth limits for site class C for cohesionless soils in NZS1170.5 Table 3.2 with the depth to give T= 0.6 s using the correlations of Sykora and Stokoe (1983) and Imai and Tonouchi (1982), equations (7) and (8) respectively. Shaded areas correspond to sites that are classified as C according to Table 3.2, but should be classified as class D according to site period. Note that correlations in (b) are only applicable for quartz sand and should not be applied to pumice sands.***

OTHER METHODS TO DETERMINE SITE CLASS

To this point, this article has discussed methods to determine the fundamental site period with and without shear-wave velocity measurements. These methods correspond to the first two methods from the hierarchy detailed in clause 3.1.3.1 of NZS 1170.5, and clause 3.1.3.7, which is specific to layered sites. However, the code identifies several other methods for determining site class, which we discuss briefly here.

Site period from Nakamura ratios

This technique was introduced by Nogoshi and Igarashi (1971) [21], and popularised by Nakamura (1989) [22], and consists of estimating the ratio between the Fourier amplitude spectra of the horizontal (H) to vertical (V) components of ambient vibrations. While used highly infrequently in New Zealand, this is an inexpensive, simple method of directly measuring the fundamental period at a given site. This method generally performs very well for horizontally layered soil profiles with large impedance contrasts i.e.

$$\frac{\rho_{rock} V_{S,rock}}{\rho_{soil} V_{S,soil}} \geq\sim 4 \quad , \qquad (29)$$

[23] and is generally able to detect surface topographic effects on the site period. However, the results become less certain when the impedance contrasts are more gradual, or when the slope of subsurface interfaces increases. When there is additional noise contamination e.g. from wind or harmonic industrial activity, the interpretation of the H/V ratios can be compromised. This method was the subject of an in-depth European study, SESAME (Site EffectS Assessment using AMbient Excitations). We refer the reader to a special issue of the Bulletin of Earthquake Engineering (2008), Volume 6, Issue 1, which covers the limitations of the method in detail, and gives guidelines for the application of the method.

Site period from earthquake recordings

Another method described in NZS 1170.5 clause 3.1.3.1 for calculating site period is via “recorded earthquake motions”. The code is referring to a method known as horizontal-to-vertical spectral ratios (HVSR) from S-wave shaking (Lermo & Chavez-Garcia, 1993) [24]. This method is similar to Nakamura ratios, in that it involves taking the ratio between Fourier amplitude spectra of horizontal and vertical components, however the data is S-wave windows from recorded earthquake motions, rather than from microtremors. H/V curves are generally averaged for several events to give the final HVSR, and greater than 10 events is considered a reliable average.

As the method is from recorded data, it is generally only applicable in New Zealand for GeoNet recording sites (see http://magma.geonet.org.nz/resources/network/netmap.html). Note that NZS 1170.5 is interested in the low-strain fundamental period, therefore the chosen events for the HVSR should be free of nonlinear soil effects (e.g. with a PGA approximately less than 0.1g). This method is only valid where the recording instrument is free-field.

From boreholes with descriptors but without geotechnical measurements

This method is common in engineering practice. With no measured geotechnical properties, representative strength, modulus or shear-wave velocity values are assumed without consideration of the *in situ* conditions that can significantly affect the actual values. Presumably the assumed properties are used then in conjunction with Table 3.2 to determine the site class. Given the inconsistencies in Table 3.2 that we have outlined in this article, and the fact that the assumed geotechnical properties are unlikely to be representative of the true properties, this site classification method should be used with caution as it is unlikely to yield accurate results.

From surface geology and estimates of the depth to underlying rock

This method is described as the “least preferred method” to determine site class under NZS 1170.5. It is unclear how this method yields an estimate for site classification, however again we presume that Table 3.2 guides the selection of site class. Use of Table 3.2 requires geotechnical information and the depth to bedrock, neither of which are measured in this method, thus we also consider this the least-preferred method. This method is unlikely to give a reliable site classification estimate.

CONCLUSIONS

Under NZS1170.5:2004, the fundamental site period, *T*, is the key parameter to account for the influence of near-surface material on earthquake ground motion. Given that site period is closely related to the shear-wave velocity (V_S), this paper gives a background on methods of obtaining shear-wave velocity using *in situ* measurements, laboratory tests and existing empirical correlations. From there, we examine methods to calculate site period for various types of soil profiles according to the NZS 1170.5 clause 3.1.3.1 hierarchy for site classification. Examples for five types of sites are shown:

- Uniform layer;
- Shear-wave velocity increasing as a power of depth;
- Shear modulus increasing linearly with depth;
- Two-layer soil profile; and
- Three-layer soil profile.

For the two-layer and three-layer profiles, we offer two alternative methods for calculating site period, a closed form solution and a lumped mass solution. The NZS 1170.5 clause 3.1.3.7 is unconservative with respect to these two methods for both the two-layer and the three-layer profiles. We consider the lumped mass solution to be the most accurate and efficient method for calculating site period for layered sites. The Dobry and Madera method of successive application of the two-layer solution, is considered an acceptable, if less efficient method.

An issue is also identified with the bedrock depths in Table 3.2 of the code. The maximum bedrock depths for site class C are inconsistent with the site period boundary of 0.6 seconds. For cohesive soils, the maximum depths are unconservative by roughly a factor of two. For cohesionless soils, the maximum depths are overestimated by roughly 10 to 20%. These inconsistencies result in some sites being classified as class C, when according to site period, the site should be classified as class D. We recommend that Table 3.2 is either amended or removed in the next iteration of the Standard.

Further details are given on alternative methods in NZS 1170.5 to assess site classification, to supplement the information given in the code. Notes are made on their applicability to certain situations.

As a final thought, we suggest that calculation of site period is a task best suited for geotechnical engineers, since they have the greatest depth of technical knowledge with regard to the site characteristics.

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