EXPERIMENTAL RESPONSE OF FRAMED MASONRY STRUCTURES DESIGNED WITH NEW REINFORCING DETAILS

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ABSTRACT

This paper describes an experimental programme in which two one-storey single-bay specimens were tested under cyclic lateral loading. The first specimen was designed according to the normal practice for framed masonry structures, whereas new reinforcing details were provided in the second specimen, with the objective of enhancing the structural response. Special precautions were taken for the application of the lateral forces, considering that experimental and analytical results indicated that the loading system could markedly affect the structural response. Consequently, an adequate procedure should be adopted for laboratory tests in order to obtain realistic data. The most important conclusion of the experimental programme was that the response of reinforced concrete frames with masonry infills can be significantly improved by a rational design aimed at reducing the distortion of the masonry panels while the plastic deformations are concentrated in selected regions of the structure.

INTRODUCTION

Reinforced concrete frames with masonry infill walls have been used since the beginning of the 20th century for low and medium-height buildings. Experimental and analytical research related to infilled frame structures has progressively increased in the last 40 years. Despite this effort, a review of the literature (Comité Euro-International du Béton, 1994, Crisafulli, 1997) shows that the behaviour of this type of structure is not completely understood, that some uncertainties remain for the analysis, and that earthquake-resistant design procedures should be improved.

Observations from past earthquakes showed that severe damage and loss of life could occur in infilled frame buildings, which has lead to the idea that this type of structure exhibits poor seismic performance. However, there is also evidence that masonry infills may produce some beneficial effects on the response of the building. These contradictory conclusions indicate that infilled frames exhibit a poor or good performance depending on how the masonry is used in the earthquake-resistant structure. For this reason, it is very important to understand clearly the complex composite behaviour of infilled frames, and to develop rational design procedures in order to obtain a safe and economical solution. It is worth noting that this type of structure is extensively used to provide lateral resistance against earthquakes in several regions of high seismic risk, especially in developing countries where masonry is still a convenient construction material due to economical or traditional reasons.

The term "framed masonry structures" is used in this paper to describe the case in which the reinforced concrete frame is cast after the construction of the masonry panel, whereas "infilled frame" is used in a general sense to consider other construction techniques.

OBJECTIVES OF THE TEST PROGRAMME

The principal objectives of this experimental programme can be summarized as follows:

a. To observe the performance of framed masonry structures and to improve the understanding of the structural behaviour.
b. To use a realistic loading system for the application of the lateral forces intended to simulate seismic actions.
c. To investigate the behaviour of new reinforcement details proposed to improve the seismic response.
d. To obtain experimental data for the calibration and validation of analytical models.

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TEST UNITS

Design considerations

The research programme involved the testing of two, single-bay, single-storey, framed masonry structures. The units, constructed to a reduced scale of 3/4, represented the lower part of a two-storey structure. The reduced scale was adopted in order to fit the dimensions and loading capacity of the existing testing frame. The dimensions of the masonry panel and reinforced concrete frame were selected according to the common practice followed in South American countries, where framed masonry is used as a structural system.

According to the objectives described above, Unit 1 was designed under the assumptions that cracking of the masonry panel should occur before large plastic deformations develop in the reinforced concrete frame, and that the columns should yield in tension as a result of the axial forces induced by the equivalent truss mechanism, as suggested by Paulay and Priestley, 1992, and San Bartolomé *et al.*, 1992, to obtain a "flexure mode of failure". Furthermore, it was assumed that the tensile strength of the beam should be greater than the applied horizontal force to avoid yielding of the longitudinal beam reinforcement. No special considerations were taken to design the beam-column joints of Unit 1.

Unit 2 was designed according to a new criterion proposed for cantilever infilled frames, in which the ductile behaviour is achieved by controlled yielding of the longitudinal reinforcement at the base of the columns (Crisafulli, 1997; Crisafulli *et al.*, 2000). In order to avoid stiffness and strength degradations, due to the brittle behaviour of masonry, shear distortions in the panel need to be limited. For this reason, the beams and most parts of the columns are designed to remain in the elastic range. The main steps of the proposed procedure can be summarized as follows:

- Evaluate the actions in the infilled frame based on a simple model, such as the equivalent truss mechanism.

- Design the columns of the lower storey considering that yielding should occur at the base of these columns, while other members remain in the elastic range. The longitudinal reinforcement calculated under this assumption should be properly anchored in the foundation. Provide additional longitudinal reinforcement in the lower columns, which is not anchored to the foundation, to induce a "weak region" or "fuse" at the base.

- Evaluate the design shear force taking into account the probable flexural overstrength and the influence of the higher modes of response.

- Design the floor beams and the columns of the upper storeys to resist the axial tensile forces without excessive elongation (considering flexural overstrength). Yielding of the longitudinal reinforcement should be avoided.

- Provide haunched beam-columns joints with diagonal reinforcement, assuring that the inclined surface of the joint is perpendicular to the diagonal of the masonry panel. The vertical and horizontal dimensions of the tapered joint should be greater than 1.5 times the depth of the beam and the column, respectively. This detail reduces of the distortion of the masonry panel by limiting the opening of the joints, improves the transfer of the lateral forces from the frame to the panel and increases the width of the compressive strut.

- Design the transverse reinforcement of the frame members to resist the shear forces, to provide confinement to the concrete and to avoid premature buckling of the longitudinal reinforcement.

Description of the test units

Figure 1 illustrates the reinforcing details and dimensions of Units 1 and 2. The masonry panel was 2.00 m high and 2.52 m long, and was formed by 23 courses of solid concrete bricks with 10 ½ bricks per course; the thickness of the panel was 90 mm. Solid concrete bricks were selected because their dimensions (230 x 90 x 75 mm) represented adequately the bricks in the prototype structure, and also because they were commercially available when constructing the test units. It must be noted that concrete masonry units usually exhibit a larger strength than clay units, however this is not important when a shear-friction failure of the masonry is expected (debonding of the mortar-brick interfaces and sliding shear).

The reinforcement of the frame members of Unit 1 consisted of 4 - 10 mm diameter longitudinal bars with 6 mm diameter stirrups. Different reinforcing details were used in Unit 2, as a result of the design criterion described previously. Two additional 10 mm diameter bars were added to each column. These bars were not anchored to the base of Unit 2 to assure that the axial and flexural strengths of the bottom end of the columns were significantly weaker. Therefore, plastic deformations due to tensile axial loads or bending moments were expected to develop mainly in these regions. The beam was also reinforced with two additional 10 mm bars to reduce the elongation produced by the axial force. Haunched beam-columns joints were used for Unit 2, with an inclination selected to be perpendicular to the diagonal of the masonry panel. A diagonal 10 mm diameter stirrup was placed in each joint with three perpendicular U-shaped 6 mm bars tied to the longitudinal reinforcement of the frame (see Figure 2).

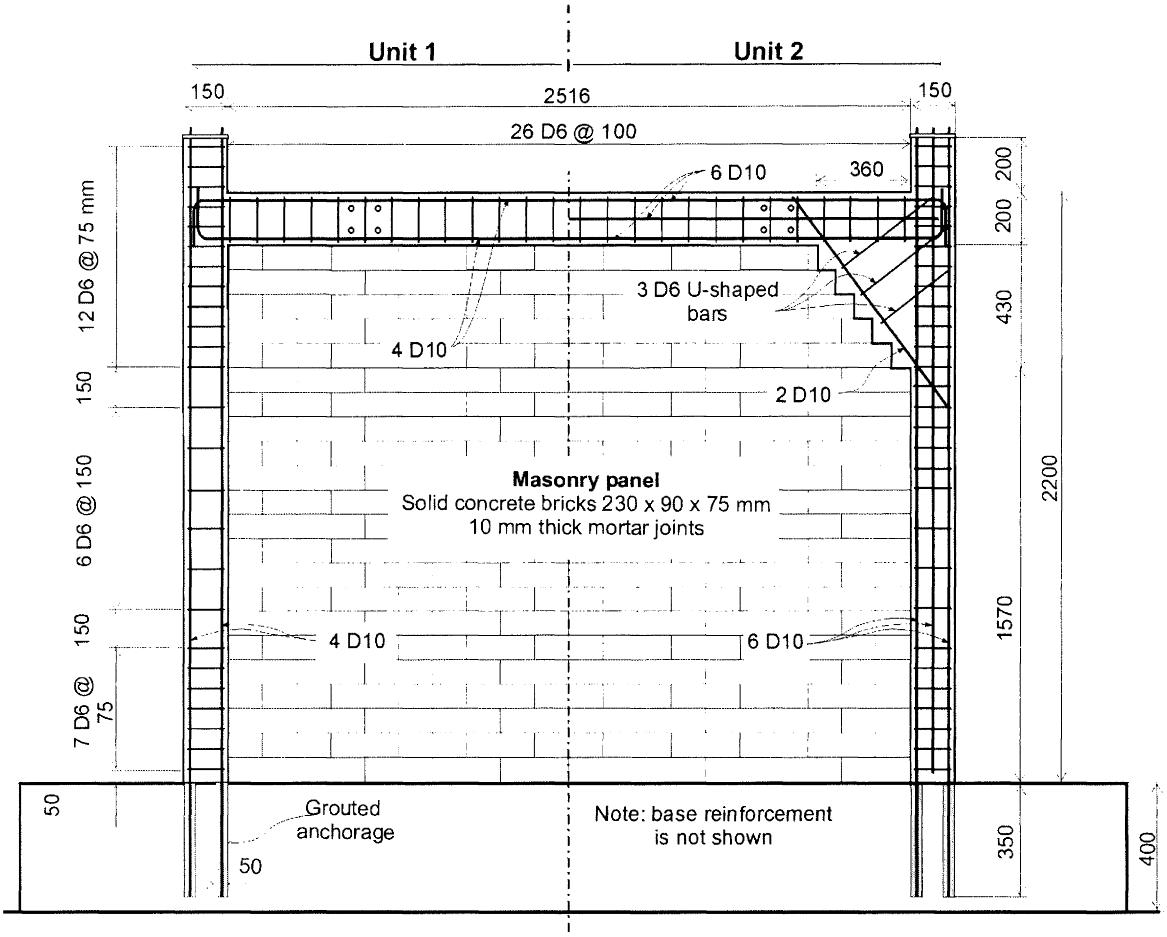


Figure 1. Reinforcing details and dimensions of Units 1 and 2.

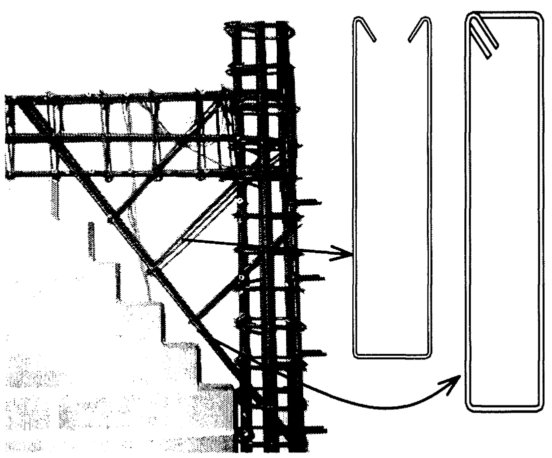


Figure 2. Detail of the joint reinforcement for Unit 2.

The masonry walls were built before casting the frame, as usual in framed masonry structures. A professional bricklayer constructed the walls on the top of the reinforced concrete bases. Solid concrete bricks were layered with mortar. The thickness of the mortar joints was 10 mm. The entire reinforced concrete frame for both units was cast in one operation, using fresh concrete provided by a local company.

Material properties

The compressive strength of the bricks was evaluated by following different standards and testing techniques. Based on these results, it was concluded that the compressive strength of the bricks was $f'_m = 26.2$ MPa (Crisafulli, 1997). The measured value of the elastic modulus was 12,980 MPa, defined as the secant modulus at 1/3 of the compressive strength. This value, equal to 495 f'_m, is smaller than usual values for the elastic modulus of concrete units. Splitting tests were conducted using two 6 mm diameter steel rods to apply the compressive load. The tensile strength, obtained as the average value from five tests, was 2.8 MPa.

The mortar used in the construction of the masonry panels was prepared by mixing one part of cement, two parts of bricklayer lime mortar and two parts of sand. The mechanical properties of the mortar were evaluated from compressive tests conducted on 50 mm diameter by 100 mm high cylinders cast during construction of the masonry panels. The cylinders were cured under the same conditions as the panels. The compressive strength obtained from testing five specimens was 8.0 MPa. The elastic modulus of the mortar was 8,540 MPa. This value was defined as the secant modulus at 1/3 of the compressive strength.

The compressive strength of masonry, obtained as the average value of five tests, was 19.3 MPa (considering the correction factor proposed by Page and Marshall, 1985). The elastic modulus in the direction perpendicular to the mortar joints was 11,550 MPa, defined as the secant modulus at 1/3 of the compressive strength. Shear strength parameters were assessed by testing masonry triplets with three different levels of axial load. The shear force was applied with an Avery universal testing machine, whereas the axial load (perpendicular to the mortar joints) was applied with a hydraulic actuator mounted on a special steel rig. The supports on which the masonry triplets were placed during the test were designed to allow the lateral movement of the bricks and to assure that the resultant of the reactive forces passed through the plane of the mortar joints, in order to obtain a pure shear state in the joints. It was found that the initial bond strength was τ_o = 0.41 MPa and the coefficient of friction was μ = 0.70. According to the shear theory proposed by Crisafulli *et al.*, 1995 (which was formulated as a modification of the shear theory developed by Mann and Müller, 1982), the reduced shear parameters of the masonry were τ_o^* = 0.30 MPa and μ^* = 0.52. These reduced parameters, which take into account the composite nature of masonry and the weakness of the head joints, were used for the evaluation of the lateral strength of the masonry panel according to the diagonal strut model.

The concrete was provided by a commercial ready-mix supplier. The maximum aggregate size was 13 mm because of the congestion of the reinforcing cages and the reduced scale factor of the test units. The mechanical properties of the concrete, obtained from compressive and splitting tests conducted in a 2,500 kN Avery universal testing machine were: compressive strength (at test) 22.5 and 31.2 MPa, tensile strength 2.4 and 2.9 MPa and elastic modulus 22,100 and 25,200 MPa, for Unit 1 and 2, respectively.

The mechanical properties of the reinforcing steel were evaluated from tensile tests at a quasistatic rate. The yield strength, f_y, and the ultimate (maximum) strength, f_{su}, were 353 and 466 MPa for the 6 mm diameter plain bars and 323 and 441 MPa for the 10 mm diameter deformed bars; the strains at maximum load, ε_{su}, were 15 and 25% for each type of bar, respectively.

TEST ARRANGEMENT

General description

The general details of the loading frame are illustrated in Figure 3. The test units were anchored to the laboratory floor, using four steel beams and eight 38 mm diameter steel rods, which fixed the ends of the reinforced concrete base. In order to avoid horizontal movement, the base was compressed between the steel columns with two hydraulic actuators. A steel beam was located at each side of the unit to restrain it against out-of-plane displacements. The restraint was provided by four nylon shims glued to the internal faces of the steel beams, which were in contact with the top of the columns. Small steel plates were glued on the columns at the contact points. In this way, the units were able to move under the action of the lateral forces without significant friction at the contact points.

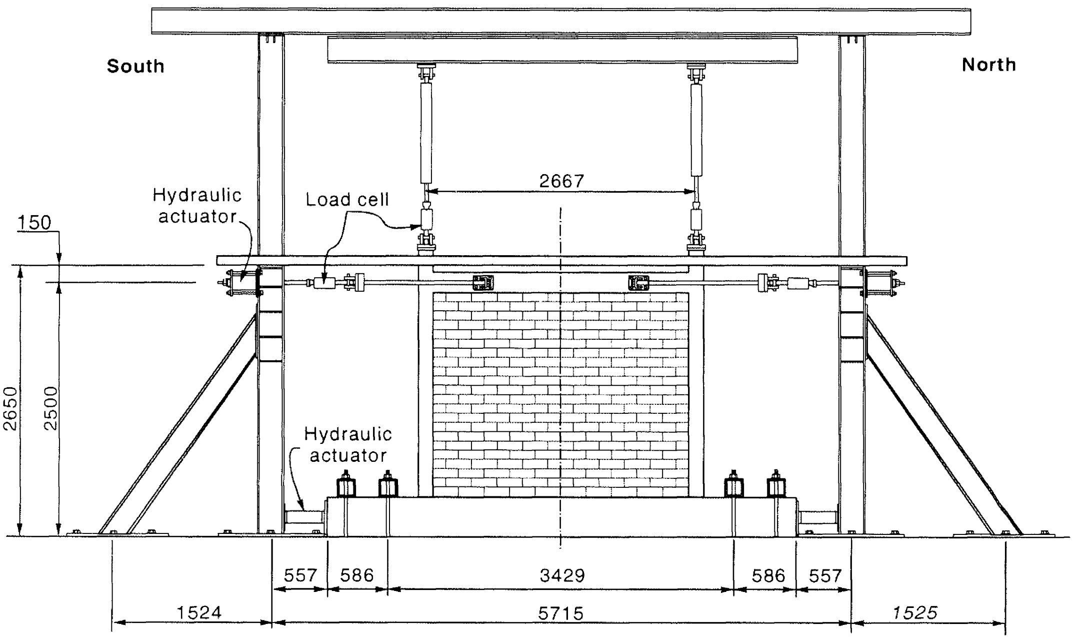


Figure 3. Loading frame used in the tests.

Lateral forces were applied using two hydraulic actuators connected to the units 2.50 m above the testing floor with high strength steel rods. During the test, the hydraulic actuators were used alternatively depending on the direction of the applied force or displacement. This procedure was adopted to represent more adequately the condition in real multi-storey framed masonry structures, in which the beams of the frame are subjected to tensile forces independent of the direction of the lateral force, as explained later in this section. In order to simulate the gravity loads and the overturning moment corresponding to a two-storey structure, vertical forces were also applied at the top of each column. It was assumed that the equivalent lateral forces induced by seismic action followed a linear variation with height. The total overturning moment, measured at the base of the infilled frame, was equivalent to that produced by the same lateral force applied at 3.48 m. The vertical hydraulic actuators were automatically controlled by servo-valves operated by a computer controlled system in order to assure that the vertical forces, P, were related to the applied lateral force, V, according to the following expression:

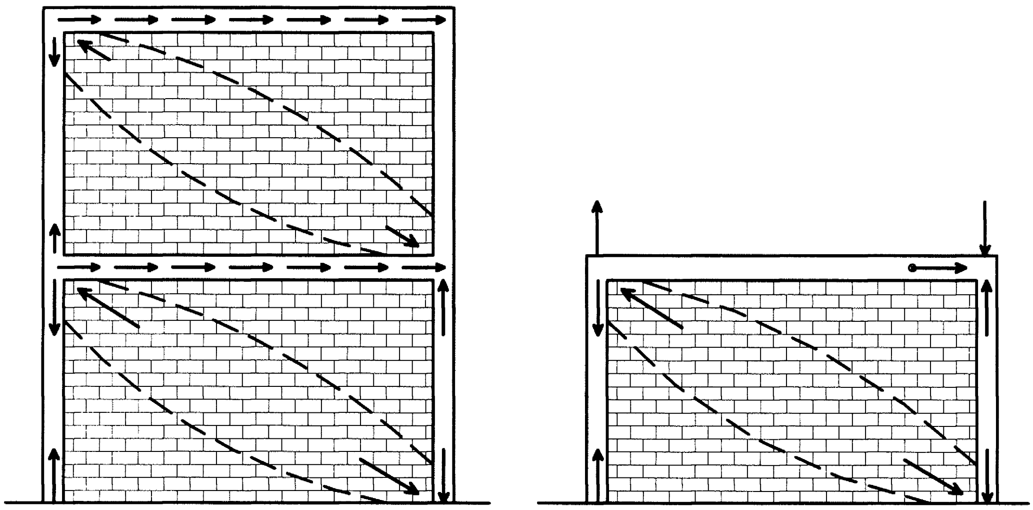
$$P = -20\text{kN} \pm 0.52V \qquad (1)$$

where the term "–20kN" represents a constant compressive force, due to permanent gravity loads. In this way, the overturning moment to shear force ratio remained constant during the test.

A review of previous quasistatic experiments indicated that two loading systems are commonly used to apply lateral forces. In the most common system, two hydraulic actuators alternately push the unit at the external faces of the beam-column joints. The second system employs only one hydraulic actuator capable of pulling and pushing the unit to apply cyclic loading. This actuator is connected either to the external face of the beam-column joint or to a rigid steel beam bolted along the top of the unit.

In real infilled frame structures, however, the situation is different. The inertial forces resulting from seismic actions are distributed between the resisting elements depending on the structural configuration of the building. In the lower storey, a large part of the shear force is transferred from the upper storeys through a truss mechanism, whereas the remaining part is transmitted to the beams from the floor slabs. Figure 4 (a), shows the inertial forces, which mainly concentrate at the floor slab level, the truss forces in the masonry panels and axial forces in the columns (axial forces in the beams are not represented for sake of simplicity). It must be noted that the beams are subjected to variable axial tension independent of the direction of the lateral forces. This effect can be reproduced in dynamic tests using a shaking table, but it is difficult to consider in static tests, particularly with the loading systems described previously.

In the tests reported in this paper, the lateral forces, in both directions, were applied to the units by pulling the beam at 0.50 m from each end of the beam, with the aim of representing the resultant shear force transmitted by the floor slab and from the upper storey, as shown in Figure 4 (b). The exact location of the resultant, which can vary during an earthquake, depends on the structural configuration and on the dynamic properties. However, the assumed position was adopted as a compromise, based on calculations of the resultant force considering the compressive force in the upper masonry strut and distributed inertial forces in the beam.



(a) Real structure (b) Test unit

Figure 4. Seismic forces transfer via a truss mechanisms and loading conditions for the tests.

Due to the lack of experimental evidence related to this topic, finite element analyses were conducted to investigate the effect on structural response of different loading systems, namely, (a) when the force is applied by pushing the face of the beam-column joint, and (b) by pulling the beam close to one of its ends (Crisafulli, 1997). The numerical results indicated that the magnitude of the maximum bending moments in the reinforced concrete members were similar in both cases, except at the joint in contact with the masonry panel. The distribution of the shear forces exhibited some variation, although the maximum values were comparable and occurred at the ends of the members. The axial forces in the beam exhibited significant differences. Large tensile axial forces developed when the force was applied by pulling the infilled frame, whereas reduced compressive forces occurred in the other case. When the lateral force was applied by pulling the beam, the core of the beam-column joint was subjected to a stress state similar to pure shear (see Figure 5). Contrarily, when the force was applied by pushing the face of the joint, the stress state in the core was predominantly compressive, with tensile stresses developing only in the region close to the internal corner of the joint. In the latter case, the behaviour of the joint can be significantly improved.

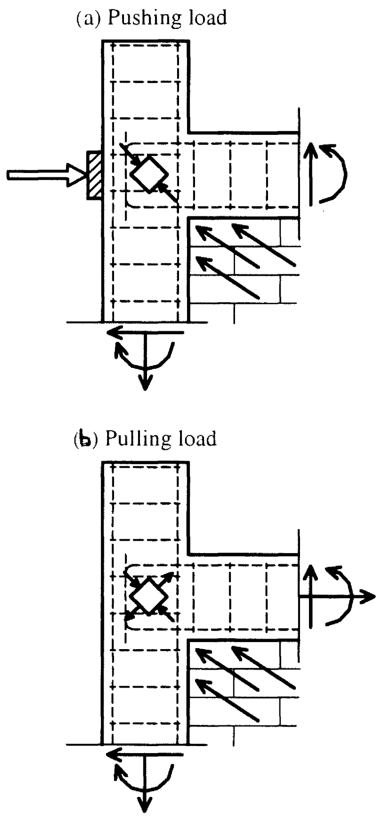


Figure 5. Schematic representation of the transfer mechanism of the lateral force.

It is concluded that both loading systems exhibit important differences, which may affect the observed response of the test units. Consequently, an adequate procedure should be adopted for laboratory tests in order to obtain realistic data. Based on rational considerations and analytical results obtained from finite element models, the application of the lateral forces by alternatively pulling the beam seems to be more convenient, despite its limitations.

Instrumentation and test sequence

The magnitude of the vertical and horizontal forces was measured using four load cells placed between the hydraulic actuators and the test unit. Displacements and deformations of the test units were measured with linear potentiometers. The horizontal displacement of the centre of the beam was registered with one linear potentiometer mounted on steel angles fixed to a concrete wall of the laboratory. A linear potentiometer was used to measure elongation of the beam subjected to tensile axial forces. The diagonal deformation of the masonry panel was registered by measuring the change of length of two Kevlar wires with a length of 3.2 m.

Local strains in the longitudinal and transverse reinforcement were measured with strain gauges. Before the test, all the strain gauges were checked for continuity and resistance to earth. Average strains in the longitudinal reinforcement were also measured with 36 clip gauges. The clip gauges were placed at the ends of the frame members, which represented potential plastic hinge regions.

The test units were subjected to quasistatic cyclic loading. The initial cycles applied to the units were conducted to evaluate the initial stiffness. In Unit 1, one cycle was applied to a force level equal to 20 kN, whereas in Unit 2, three cycles were imposed to 20, 40 and 60 kN. It should be noted that framed masonry structures exhibit an early nonlinear behaviour due to separation of the frame-panel interfaces and cracking of the masonry. Furthermore, this type of structure is usually very stiff, having a small initial yield displacement, and any small change in the definition of this point may result in a large variation of the ductility factor. As a result, the definition of displacement ductility factors is rather arbitrary in this case. In the inelastic range, the test was displacement controlled and several cycles with increasing storey drift, δ, were applied. For each displacement level, three consecutive cycles were repeated to capture adequately the effect of strength and stiffness degradation up to a storey drift of 0.5% and 0.75%, for Units 1 and 2, respectively. The test was finished after applying 34 and 39 cycles up to a maximum storey drift of 1.5% and 2.0%, for Units 1 and 2, respectively.

Limitations of the tests

Special care was taken to design the units and the test setup in order to simulate accurately the conditions in real structures. This proposal, however, could not be completely fulfilled due to some practical limitations. The effect of a floor slab in the prototype structure was not considered in the test units, resulting in reduced flexural and axial strengths of the beam when compared with those of the prototype. Furthermore, the presence of the floor slab and additional reinforcement could decrease the elongation of the beam due to tensile axial forces. Even though the units were loaded to simulate the actions in a two-storey structure, the upper masonry panel was not constructed. When the structure is subjected to lateral forces, the panel usually separates from the surrounding frame, except at diagonally opposite compression corners. In those corners, the masonry panel restricts the deformation of the beam-column joint. This effect was not properly represented in the test because the upper masonry panel was not constructed.

The additional overturning moment applied with the vertical actuators was calculated considering a linear variation of the equivalent seismic forces. It was also assumed that this relationship remains constant. Obviously, when the structure is subjected to real seismic actions, the shear forces induced at each level depend on the dynamic properties of the building. In order to overcome this problem, pseudodynamic or shaking table tests should be conducted.

Hydraulic actuators were used to apply the gravity load at the top of the columns. Part of these loads should have been applied as a distributed load in the beam to represent the load transferred through a floor slab. This fact can modify the stress state in the masonry panel and axial forces in the columns. The conditions modelled in the tests represented a framed masonry structure connected to a floor slab with the main reinforcement parallel to the structure, as can occur when precast slabs are used. In this case, gravity loads are transferred directly to the columns.

TEST RESULTS - UNIT 1

General behaviour

The test of Unit 1 was performed over a period of 3 days, in which 34 cycles were applied with a maximum storey drift $\delta = 1.5\%$. At this stage, the test was finished due to the excessive damage observed in the masonry panel. The unit was able to resist the applied history of cyclic displacements without significant strength decay in the positive direction. However, the residual strength in the negative direction was 79% of the lateral capacity attained in load run 58. The hysteresis loop showed severe pinching only in the final stage of the test.

Unit 1 was designed with the aim of obtaining a reasonably ductile response, involving tensile yielding of the longitudinal reinforcement of the tension column. However, this objective was not achieved. Severe cracking of the upper part of the masonry panel occurred and large elongations developed in the frame members, which led to a sliding shear failure at the top of the columns. The lateral resistance measured during the test was only 50% of that predicted assuming the hypothesis of tensile failure.

The first crack started in the south beam-column joint, during the second cycle, and continued horizontally through the mortar joint closest to the beam. Below the steel plate used for the application of the lateral force, the crack changed to a stepped crack, running alternatively through head and bed joints. Another crack formed in the beam, between the steel plates connected to one of the hydraulic actuators. A similar crack pattern in the masonry panel was observed in two units tested by Fiorato *et al.*, 1970, in which the lateral forces were applied simultaneously at the third points of the beam. At a storey drift of $\delta = 0.1\%$, another stepped crack formed in each corner of the panel and several vertical cracks developed in the central region of the beam. Cracks also formed at the column and beam faces of the joints, leaving the core uncracked. As the test progressed, cracking in the upper part of the masonry panel continued and propagated through the interfaces between the columns and the panel. Tensile cracks developed along the columns and yielding commenced at the ends of the bottom reinforcement of the beam. This was a local effect and yielding did not propagate to other regions of the beam. Flexural cracks formed at the top region of the columns, and yielding started in the external longitudinal bars at a storey drift of $\delta = 0.2\%$, indicating that a plastic hinge was developing. In this stage, the beam was bent upward, as a result of an inclined compressive field developing in the masonry panel. In the final cycles, the beam, one layer of bricks and the corner of the panel slid over the rest of the panel, forming large gaps between the bricks. Sliding also occurred along the shear crack formed at the top of the column. This crack suggested that dowel action in the longitudinal reinforcement of the tension column became the main mechanism for transferring the shear force. Figure 6 shows Unit 1 at the end of the test.

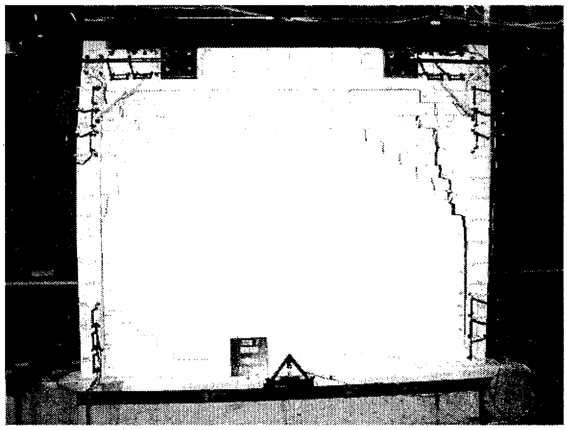


Figure 6. View of Unit 1 at the end of the test.

Figure 7 shows details of the south beam-column joint after the test was finished and large displacements were imposed to emphasize the failure mechanism. The cracks formed in these regions and the local deformation of the reinforcing bars indicated that the force applied to the beam was mainly transferred to the column through the bottom longitudinal reinforcement of the beam. From the column, the shear force was transferred progressively to the masonry panel.

It is worth noting that the type of failure of Unit 1 has been rarely observed in laboratory tests. This fact could be explained considering that the most common loading system employed in previous research consists of two hydraulic actuators, which are alternately used to push the unit. As discussed previously, this loading system can markedly modify the stress state in the beam-column joints. Figure 5 illustrates schematically the transfer mechanism of the lateral force corresponding to the two testing procedures. When the force is applied by pushing the test unit, the joint is mainly subjected to a diagonal compressive field, which can be adequately resisted by the concrete. However, a different mechanism develops when the force is applied by pulling the beam. In this case, the force is primarily transferred to the joint by the longitudinal reinforcement because the beam is already cracked. As a result, large shear stresses are induced at the bottom of the joint in order to transfer the force to the column and then to the masonry panel.

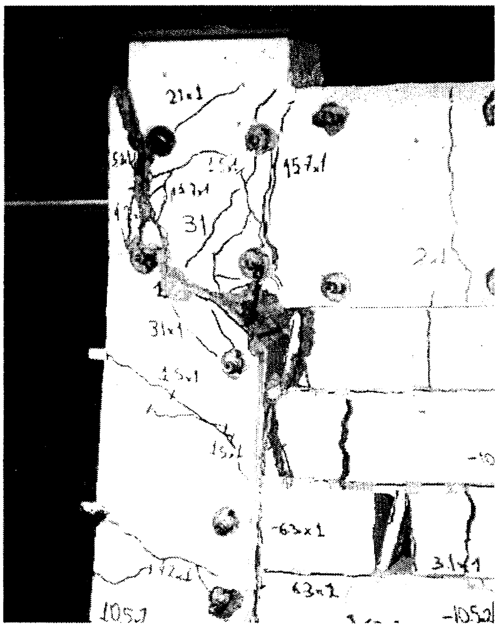


Figure 7. Detail of the south beam-column joint of Unit 1 after the test.

Lateral force-displacement response

The lateral force-displacement response of Unit 1 is shown in Figure 8. The first cycle exhibited nonlinear behaviour, even though no cracks were observed. During the second cycle the stiffness significantly changed at a force level of 30 kN, due to cracking of the masonry panel. The lateral force resisted by the unit slightly increased in subsequent cycles up to 43.0 and -42.9 kN, respectively, in the direction of the positive and negative displacements. Even though the lateral capacity was very similar in both directions, the envelope of the hysteresis loop shows a different behaviour.

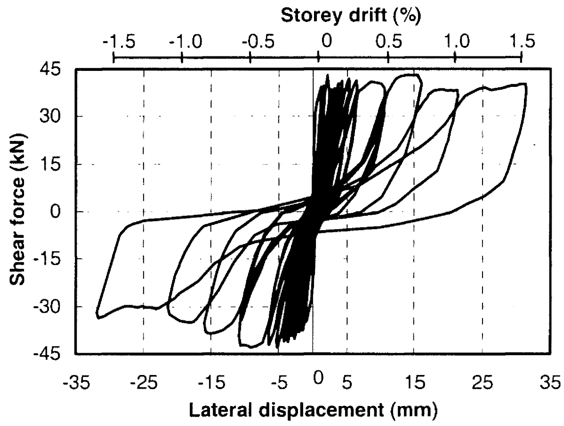


Figure 8. Lateral force-displacement response of Unit 1.

It is to be noted that the lateral resistance of the bare frame, calculated according to a push over analysis, was approximately 13.5 kN. This fact demonstrates the significant influence of the masonry panel in increasing the lateral resistance of the infilled frame, even though the panel was severely cracked.

The elastic stiffness of Unit 1 was predicted according to two simplified models, namely, the beam analogy and the truss model. In the first case, it was assumed that the frame and the panel can be represented as a monolithic member. The stiffness, therefore, was calculated according to standard elastic theory considering the contribution of the flexural and shear deformations. In the second model, the panel was represented with an equivalent compressive strut, in which the width of the strut, w, was adopted as 0.25 of the diagonal length of the panel, d_m =3.21 m (in this case w = 0.80 m). The beam analogy agreed very well with the experimental results up to a force level of 10 kN, which represents 23% of the maximum lateral force and the equivalent strut model gave an adequate value of the secant stiffness at a force level equal to 80% of the lateral capacity of the unit.

The use of the equations proposed by Mainstone, 1971, Liauw and Kwan, 1984, and Decanini and Fantin, 1986, for the analytical evaluation of the width of the equivalent strut leads to values of w/d_m equal to 0.09, 0.17 and 0.19, respectively. It is observed that the best agreement with the experimental response corresponds to the latter equation, which was derived for framed masonry. However, the comparison of results is difficult because the force level at which the equations are valid has not been specified.

Strength and stiffness degradation occurred when the cycles were repeated to the same displacement level. These effects were more significant between the first and second cycle of the series, with a typical strength reduction ranging from 16% to 5% in the initial and final cycles, respectively. The shape of the loops was satisfactory up to storey drifts of 0.3%, even though some pinching occurred. In the final cycles, however, the response was typical of sliding systems. In this case, reloading started with a very low stiffness, which suddenly increased when the dowel action mechanism was mobilized. The slope of the unloading branches was almost vertical and large deformations remained even when the lateral force was removed.

The lateral displacement, measured at the centre of the beam, was mainly due to elongation of the beam, flexural deformation in the columns and sliding shear at the top of the columns and masonry panel. Other sources of displacement, such as shear deformations in the columns and rigid body movements, had no practical importance in this case. In the initial cycles the lateral displacement was principally due to elongation of the beam. Then, cracking occurred in the beam and total beam elongation was about twice the lateral displacement. As the test progressed, the beam elongation slightly decreased and the lateral displacement was larger because the flexural deformations in the columns and sliding shear in the masonry panel became more important. In subsequent cycles, the maximum value of the beam elongation remained almost constant, although in the last cycles increased up to 22 mm.

Readings of the displacement along the diagonals of the masonry panel indicated that the contribution of the panel deformation to the total lateral displacement was significant, especially in the final stage of the test when large gaps formed due to sliding shear. The shape of the hysteresis loops, obtained by plotting the lateral forces against the horizontal component of the diagonal displacement, was very similar to that of the response presented in Figure 8, and showed the same degradation pattern throughout the test.

Internal deformations

The shear distortion in the beam-column joints was obtained from readings of the clip gauges located along the diagonals of the joint. The peaks of the shear distortion in each cycle remained almost constant up to a storey drift of $\delta = 0.25\%$, increasing up to 0.0093 at the end of the test. In relative terms to the storey drift of the unit, the shear distortion was more significant in the initial cycles. Each joint of the frame was mainly stressed in one direction of loading. As a result of the interaction between the frame and the masonry panel, the beam-column joints opened, producing an unfavourable effect on the structural behaviour. This was due to reduction of the restraint provided by the frame to the cracked masonry panel and to the modification of the transfer of the shear forces in the loaded corner. The variation of the relative opening of the joints indicated that the opening started in the early cycles and the joint closed when the forces were applied in the reverse direction. As the test progressed, the opening increased, showing permanent deformations when the forces were reversed.

Data obtained from clip gauges and strain gauges were used to evaluate the strain state in the longitudinal reinforcement of the frame. In each instrumented section, the strain in both layers of reinforcement was measured, which allowed the calculation of the curvature and axial strain of the frame members at different stages of the test. For sake of brevity, the curvature and axial strain diagrams are not presented here. Strain gauges were attached only to the transverse reinforcement located at the bottom of the columns. Recorded strains indicated that the stress level in these stirrups remained very low. The maximum stresses, registered in the last cycle, were 70 and 132 MPa in the south and north column, respectively. The stress was higher in the north column, probably due to the formation of an inclined crack crossing the transverse reinforcement, which did not occur in the other column.

Evaluation of the cracking force and the lateral strength

The cracking force at which sliding shear starts can be calculated according to different procedures proposed in the literature. Considering the material properties measured in the masonry tests, the expressions developed by Stafford Smith and Riddington, 1978, and Paulay and Priestley, 1992, lead to a cracking force of 82.6 and 209.2 kN, respectively, which are significantly higher than the experimental value of 31.6 kN. The cracking force can be obtained using the equivalent strut model, provided that the compressive strength of the strut is properly evaluated. Crisafulli, 1997, proposed that the strength of the masonry to be used with this analytical model can be calculated as a function of the mechanical properties of masonry (using the reduced shear parameters, τ_o^* and μ^*, previously defined) and the inclination of the diagonal strut. For the particular case considered here, the strength level at which cracking occurred in the diagonally loaded masonry was 1.05 MPa, which leads to a cracking force of 35.8 or 47.7 kN, for values of the equivalent strut width of 0.15 or 0.20 of the diagonal length, respectively. These values show a better agreement with the measured cracking force.

The lateral strength of Unit 1 was calculated according to the mechanism illustrated in Figure 9, considering the imposition of a positive displacement. In the ultimate stage, the beam and the upper part of the masonry panel moved sideways. The north column separated from the lower part of the masonry panel and bent in double curvature. The south column remained in contact with the masonry panel and large curvatures developed at the top, due to the bracing effect induced by the panel. The lateral force applied to the beam, V, was mainly transmitted through the columns, since a large horizontal gap developed at the top of the masonry panel and continued open during the last cycles of the test. Therefore, no significant force could be transferred from the beam directly to the masonry panel.

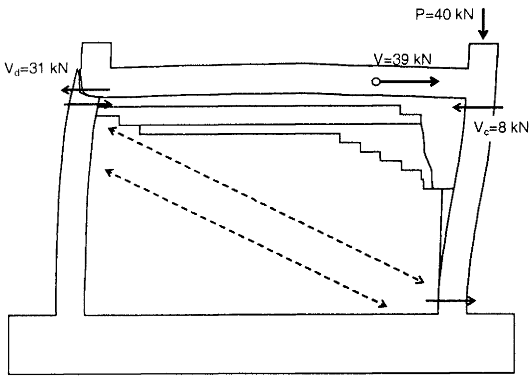


Figure 9. Failure mechanism observed for Unit 1.

In the north column, the shear force V_c can be calculated under the assumption that the flexural strength is developed at both ends of the member:

$$V_c = \frac{2\,M_{uc}}{h_e} \qquad (2)$$

where M_{uc} is the flexural strength of the compression column, equal to 7.8 kNm, and h_e is the effective height between plastic hinges, which can be taken as 1.83 m. This value was obtained considering the usual expressions for the evaluation of the plastic hinge length (Paulay and Priestley, 1992). According to Equation 2, it is found that V_c = 8 kN.

The shear force resisted by the cracked section of the south column resulted from a combined mechanism due to friction between the crack surfaces and dowel action in the reinforcing bars crossing the crack. The participation of each mechanism to the total strength depended on the applied displacement and on the previous loading history. Initially, friction controlled the response and gradually decreased as the roughness of the crack degraded owing to the cyclic loading. In contrast, the development of dowel action required large relative displacements at the horizontal joint. Different mechanisms, namely, flexure, shear or kinking of the longitudinal reinforcement (Park and Paulay, 1975), could occur depending on the magnitude of the relative sliding and the elongation of the reinforcing bars. Therefore, the shear in the cracked section was not equal to the sum of the maximum shear forces developed by friction and dowel action because they occurred at different displacement levels. At the ultimate state, it can be considered that the dowel action was mainly due to a kinking mechanism in the reinforcement. Therefore, the shear force V_d can be calculated as (Park and Paulay, 1975):

$$V_d = A_{st}\, f_y \cos\,\alpha \qquad (3)$$

where A_{st} is the total area of longitudinal reinforcement and α is the angle of kinking (measured between the direction of the shear force and the longitudinal bar). It is to be noted that the angle α could not be measured accurately in the test unit. The equilibrium of horizontal forces required that the force V_d be equal to 31 kN, which implies a kinking angle of 72° according to Equation 3. Assuming that kinking occurs over a length equivalent to 5 times the diameter of the bar, in this case 50 mm, the horizontal relative displacement between the beam and the column should have been 16 mm to obtain a kinking angle of 72°. This value agrees with the relative displacement measured in the last cycle of approximately 14 mm (see Figure 7). Therefore, it is believed that the failure mechanism represented in Figure 9 gives an adequate explanation of the response of Unit 1.

TEST RESULTS - UNIT 2

General behaviour

Testing of Unit 2 was completed after the application of 39 cycles with a maximum storey drift of 2.0%. The test was finished at this stage because the travel lengths of the main linear potentiometers were close to their limits. The overall response of Unit 2 was excellent when compared with that of Unit 1. The lateral resistance was 89.7 and -91.5 kN in each direction, representing an increase of about 110% in relation to Unit 1. Even though large lateral displacements were imposed, strength degradation was only observed in the final cycles. The maximum storey drift imposed to the unit was markedly higher than typical values measured in similar tests (Alcocer *et al.*, 1993) or recommended for design purposes (Vintzeleou and Tassios, 1989), which normally range from 0.4% to 0.6%. The crack pattern in the masonry panel was also improved, with a wide spread of thinner cracks. No significant damage occurred in the haunched beam-column joints.

The first cracks, induced by tensile axial forces, formed at the lower region of the columns and at the beam at a storey drift of 0.05%. Furthermore, a horizontal crack developed in the mortar joint closest to the beam and then propagated, as a stepped crack, parallel to the haunched joints. When the storey drift was increased to 0.08%, additional cracks formed at the frame members and in the masonry panel, running diagonally with an X-shaped pattern. As the test progressed, cracking of the masonry panel continued, especially in its central region, clearly showing the development of the equivalent truss mechanism. Small vertical cracks were observed at the bottom of the columns as a result of the splitting stresses induced by the reinforcing bars. At this stage, cracks formed at the ends of the haunched joints, being perpendicular to the diagonal reinforcement. Yielding of the longitudinal reinforcement started at the bottom of the columns when a storey drift of 0.2% was applied to the unit. In subsequent cycles, yielding also occurred at the top of the columns, produced by the combined effect of bending moment and tensile axial force.

In the final cycles of the test, additional cracks formed in both the frame and the panel. The cracks located at the base of the columns grew considerably due to the large plastic deformations induced in the longitudinal reinforcement. These deformations concentrated mainly at the base of the column as a result of the special reinforcement detail used in the construction of Unit 2. Yielding developed at the ends of the bottom reinforcement of the beam. It was clearly observed that the tension column bent in double curvature. Independent of the direction of the applied displacement, the beam remained bent upwards and a 1-2 mm gap formed between the two layers of bricks located at the top of the masonry panel. Figure 10 shows the cracking of Unit 2 at the end of the test. It is worth comparing this figure with Figure 6 in order to observe the different cracking pattern in both units. No significant damage occurred in the haunched beam-column joints, except for a few cracks in the direction perpendicular to the diagonal reinforcement. This clearly indicated that the objective of restraining the opening of the joints was achieved with the proposed detail.

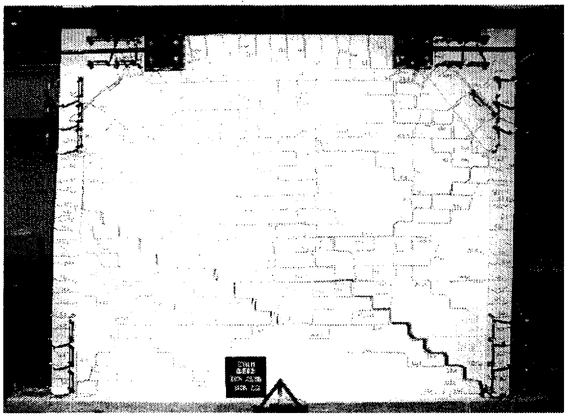


Figure 10. View of Unit 2 at the end of the test.

After the test was finished, the cracked concrete at the base of the columns was removed to expose the reinforcement. The observation of these parts of the unit suggested that the shear transferred to the columns, directly through the beam and through the masonry panel by the compressive strut mechanism, was resisted across the cracked interfaces by a combination of friction and dowel action. The deformation of the reinforcing bars at the base of the column indicated that kinking of the longitudinal reinforcement was probably the main source of the dowel strength in the final stage of the test.

Lateral force-displacement response

Figure 11 shows the lateral force-lateral displacement response of Unit 2 measured during the test. Cracking of the masonry panel and the reinforced concrete frame commenced at a force level of about 65 kN, producing a significant decrease of the stiffness. As the imposed displacement increased, the lateral strength of the unit augmented up to a maximum value of 89.7 and -91.6 kN in each direction, respectively. Afterwards, the resistance continuously decayed to 70.3 and -75.6 in the final cycle. The lateral resistance of the bare frame, obtained from a push over analysis, was approximately 17 kN. The large difference between this value and the measured strength of Unit 2 was due to the interaction between the panel and the frame.

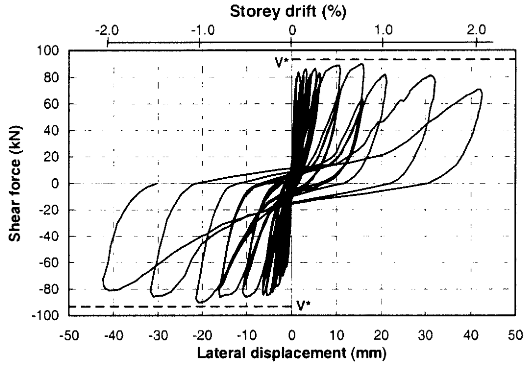


Figure 11. Lateral force-displacement response of Unit 2.

The initial stiffness measured during testing of Unit 2 agrees well with that obtained from the cantilever wall model up to a force level of 12 kN. Afterwards, the stiffness decreased markedly as a result of partial separation between the panel and the frame. In the load range from 40 to 70 kN, the secant stiffness is well represented by the equivalent truss model taking a width of the compressive strut of between 0.25 and 0.35 of the diagonal length of the panel, d_m. In this model, rigid end blocks were provided to represent the stiff region at the haunched joints where the beam and column intersect. The length of this rigid region was set to be half of the dimension of the joint. The effect of rigid end blocks was not significant due to the large axial stiffness of the strut, which controlled the stiffness of the model.

The hysteresis loops measured during the test showed strength and stiffness degradation. These phenomena were observed when the cycles were repeated to the same displacement level, as well as when the lateral displacement was increased. The degradation of the hysteresis loops measured during testing of Unit 2 was similar to that observed in Unit 1.

Measurements taken during the test indicate that the beam elongation represented about 50% of the lateral displacement in the initial cycles. As the test progressed, the beam elongation became less significant. A comparison of results obtained from both Units shows that the additional beam reinforcement placed in Unit 2 was very effective in reducing the axial deformation of the beam.

Internal deformations

The deformation of the beam-columns joints of Unit 2 was efficiently controlled by providing haunched joints with adequate reinforcement. Readings from linear potentiometers, located in similar positions in both units, in order to compare results, indicated that the opening of the joint was less than 0.1 mm up to a storey drift of $\delta = 0.3\%$. Afterwards, the opening progressively increased up to a maximum value of 1.2 mm in the final cycle. These values are markedly smaller than those measured in Unit 1.

Strains measured in the longitudinal reinforcement were used to evaluate curvatures and axial strains in the frame members. The maximum curvatures developed at both ends of the tension column, whereas no significant curvatures occurred in the compression column. The beam bent upwards as a result of the rotation induced in the joint in which the lateral force was applied. The variation of the axial strains conceptually agrees with axial forces predicted according to the equivalent strut mechanism. Tensile strains concentrated at the base of the south column due to the cut off of the longitudinal reinforcement used in Unit 2.

Strains recorded in the stirrups closest to the base of each column indicated that the stress did not exceed 80 MPa up to a storey drift of $\delta = 0.5\%$, with maximum values arising when the column was subjected to tension. In subsequent cycles, the stress increased and the maximum values occurred in both loading directions. During the two final cycles of the test, the yield strain was slightly exceeded. In the other instrumented stirrups, the stresses were smaller than 20 MPa.

Evaluation of the cracking force and lateral strength

Different equations were used to predict the force level that initiated sliding shear in the masonry panel by debonding of the mortar joints and these results were presented for Unit 1. The cracking force, evaluated according to the procedure proposed by Crisafulli (1997), is 59.6 or 83.5 kN for values of the equivalent strut width of 0.25 or 0.35 of the diagonal length, respectively. These results agree well with the measured force equal to 65.0 kN. The comparison between the cracking forces corresponding to Units 1 and 2 (31.6 and 65.0 kN, respectively) clearly indicates that the haunched joints contribute to the increase in the width of the equivalent compressive strut, reducing the stresses in the masonry panel.

The lateral resistance of Unit 2 was primarily controlled by the tensile axial strength of the columns. This is one of the chief objectives considered in the design, which allows the use of a simple procedure for the evaluation of the lateral strength. The lateral force applied to the unit induced a tensile axial force in one of the columns, which can be estimated using the equivalent truss mechanism. In addition, the axial force applied by the vertical actuators should be also considered, see Equation 1. Therefore, the tensile axial force in the column, T_c, is given by:

$$T_c = V\frac{h}{L} + 0.52\,V - 20\,\text{kN} \tag{4}$$

where V is the applied lateral force and L and h are the length and height of the frame, respectively. The lateral strength, V^*, will develop when the longitudinal reinforcement of the column (with area A_{st}) yields. Thus:

$$V^* = \frac{A_{st}\,f_y + 20\,\text{kN}}{\frac{h}{L} + 0.52} \tag{5}$$

From Equation 5, considering that $A_{st} = 316$ mm^2 and $f_y = 323$ MPa, it is found that the shear strength of the unit is $V^* = 93.4$ kN. The comparison of this value with the measured response of Unit 2 indicates a good agreement, although the predicted strength is slightly higher. This fact could be explained considering that the bending moment acting on the base of the column slightly decreases the tensile axial strength of the member.

The experimental data obtained from the tests was used to calibrate a general procedure for the evaluation of the lateral strength of infilled frames. The description of this procedure can be found elsewhere (Crisafulli, 1997; Crisafulli *et al.*, 2002).

COMPARISON OF THE BEHAVIOUR OF BOTH UNITS

In the previous sections, results obtained from the test units have been presented separately. Since Unit 2 was designed with the aim of improving the structural behaviour under lateral loading, it is useful to discuss some aspects of the response of both units. In the initial stage, the lateral stiffness of Unit 2 was higher than that corresponding to Unit 1 at the same force level. This increase can be explained by the use of the haunched beam-column joints and because the deformable length of the frame members was smaller. The maximum lateral force resisted by Units 1 and 2 was 43.0 and 91.6 kN, respectively, which represents an increment of 113%. This increment was mainly achieved by limiting the deformation of the surrounding frame, whereas the mechanical properties of the masonry panel and the tensile axial capacity of the columns were the same. Even though both units exhibited some strength degradation, Unit 2 was able to sustain a storey drift of 2%, which is a high value for framed masonry structures. Figure 12 compares the envelopes of the cyclic lateral force-lateral displacement response for both units.

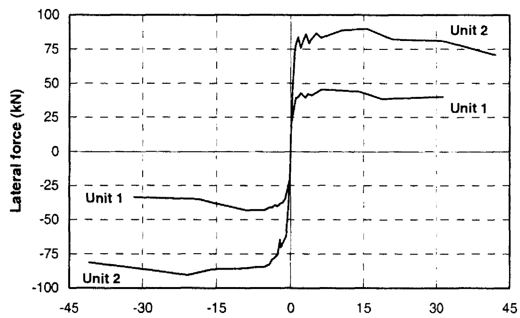


Figure 12. Comparison of the envelopes of the cyclic response for Units 1 and 2.

The comparison of the cyclic response of both units indicates that the shape of the hysteresis loops was similar, changing from approximately linear loops at the beginning of the test to highly pinched loops at the end of the test. The main difference is observed in the initial cycles, in which the area of the hysteresis loops of Unit 1 was larger, in relative terms, due to more severe cracking and nonlinear behaviour.

CONCLUSIONS

The main objectives of the test programme were to investigate the behaviour of framed masonry structures using a realistic procedure to apply the lateral forces, to compare the behaviour of new reinforcement details for framed masonry to that of customarily designed structures, and to obtain experimental data for the validation of analytical models.

The influence of different loading systems was discussed on the basis of analytical results and conceptual considerations, indicating that the loading system can significantly affect the test results. The loading system used in this research programme did not reproduce exactly the conditions in real structures, although it seems to be more realistic than the other procedures commonly used in laboratory tests.

The test of Unit 1 showed severe cracking of the masonry panel, which occurred mainly in the upper part by debonding of the mortar joints, and significant elongation of the frame members. The failure mechanism involved sliding shear at a column top and along the horizontal mortar joints. The lateral resistance was primarily determined by dowel action of the longitudinal reinforcement. The mode of failure of Unit 1 was different from that normally observed in similar tests, which suggests that the loading system used for the application of the lateral force could markedly influence the results. Significant opening of the joints, produced by the relative rotation of the beam with respect to the column, was observed in Unit 1. This had an adverse influence on the structural behaviour due to the reduction of the restraint provided by the frame to the cracked masonry panel and to the modification of the transfer of the shear forces in the loaded corner of the panel.

Unit 2 showed a very good response, and the lateral resistance was about 110% higher than that of Unit 1, even though the tensile and shear strengths of the columns and the characteristics of the masonry panels were almost the same. This improvement of the response was achieved by a rational design, in which ductile behaviour was assured by yielding of the longitudinal reinforcement of the columns, and the deformation of the surrounding frame was limited in order to preserve the geometry of the masonry panel. The cracks in the masonry panel of Unit 2 mostly formed along the mortar joints and were spread widely over the panel, with a smaller width than those observed in Unit 1. The use of haunched beam-column joints with diagonal reinforcement was found to be an efficient method of restraining the opening of the joints. Furthermore, this type of joint improved the transfer of the lateral forces from the beam to the masonry panel and increased the width of the equivalent compressive strut.

Both units exhibited significant pinching of their force-displacement hysteresis loops and moderate strength degradation in the final stages of the test. The maximum storey drifts imposed during the test were 1.5% and 2.0% for Units 1 and 2, respectively.

According to comparisons between measured and analytical values of the stiffness and cracking force, the width of the masonry strut to be considered in the equivalent truss mechanism in Unit 1 varied between 0.15 and 0.25 of the diagonal length. In Unit 2, with haunched joints, the width of the strut varied between 0.25 and 0.35 of the diagonal length. In both cases, smaller width values were required for an adequate evaluation of the cracking force, than for the stiffness. It must be noted, however, that the prediction of the stiffness and strength of framed masonry structures according to the model proposed by Crisafulli, 1997, requires a precise evaluation of the width of the equivalent strut. More experimental research is needed to investigate this aspect in detail and consideration of the influence of different parameters, such as dimensions, mechanical properties and relative stiffness of the masonry infill and the frame.

Unit 1 was designed to obtain a mode of failure involving tensile yielding of the longitudinal reinforcement of the columns. This objective was not achieved. The lateral resistance measured during the test was about 50% of that predicted under the hypothesis of a tensile failure of the columns. The lateral resistance of Unit 2, which was designed according to a new criterion, was adequately predicted on the basis of a simple procedure based on an equivalent truss mechanism and the tensile strength of the columns.

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REFERENCES

Alcocer, S. M., Sánchez, T. A. and Meli, R. (1993). "Comportamiento ante cargas laterales de una estructura tridimensional de dos niveles a escala natural construida con mampostería confinada" (in spanish), *Memoria del X Congreso de Ingeniería Sísmica*, Puerto Vallarta, Mexico, pp. 416-423.

Comité Euro-International du Béton, Task Group III/6 (1994). *Behaviour and Analysis of Reinforced Concrete Structures under Alternate Actions Inducing Inelastic Response*, Bulletin d'information No. 220, Vol. **2**, Chapter 5, pp. 310-380.

Crisafulli, F., Carr, A. and Park, R. (1995). "Shear Strength of Unreinforced Masonry Panels", *Pacific Conference on Earthquake Engineering*, Melbourne, Australia.

Crisafulli, F. J. (1997). *Seismic Behaviour of Reinforced Concrete Structures with Masonry Infills*, PhD Thesis, Department of Civil Engineering, University of Canterbury, 404 p.

Crisafulli, F. J., Carr, A. J. and Park, R. (2000). "Capacity Design of Infilled Frame Structures", *Twelfth World Conference on Earthquake Engineering*, Auckland, New Zealand. Paper No. 0221.

Crisafulli, F., Carr, A. and Park, R. (2002). "Rational Evaluation of the Lateral Strength of Infilled Frames", *7th National Conference on Earthquake Engineering*, Boston, USA.

Decanini, L. D. and Fantin, G. E. (1986). "Modelos simplificados de la mampostería incluida en pórticos. Características de rigidez y resistencia lateral en estado límite" (in spanish), *Jornadas Argentinas de Ingeniería Estructural*, Buenos Aires, Argentina, Vol. **2**, pp. 817-836.

Fiorato, A. E., Sozen, M. A. and Gamble, W. L. (1970). *An Investigation of the Interaction of Reinforced Frames with Masonry Filler Walls*, University of Illinois, Urbana, Illinois, Civil Engineering Studies, Structural Research Series No. 370.

Liauw, T. C. and Kwan, K. H. (1984). "Nonlinear Behaviour of Non-Integral Infilled Frames", *Computers & Structures*, Vol. **18**, No. 3, pp. 551-560.

Mainstone, R. J. (1971). "On the Stiffnesses and Strengths of Infilled Frames", *Proceedings of the Institution of Civil Engineers*, Supplement IV, pp. 57-90.

Mann, W. and Müller, H. (1982). "Failure of Shear-Stressed Masonry - An Enlarged Theory, Tests and Application to Shear Walls", *Proceedings of the British Ceramic Society*, Vol. **30**, pp. 223-235.

Page, A. W. and Marshall, R. (1985). "The Influence of Brick and Brickwork Prism Aspect Ratio on the Evaluation of Compressive Strength", *Proceedings of the Seventh International Brick and Masonry Conference,* Melbourne, Australia, Vol. **1**, pp. 653-664.

Park, R. and Paulay, T. (1975). *Reinforced Concrete Structures*, John Wiley & Sons Inc., New York, 768 p.

Paulay, T. and Priestley, M. J. N. (1992). *Seismic Design of Reinforced Concrete and Masonry Buildings*, John Wiley & Sons Inc., 744 p.

San Bartolomé, A., Quiun, D. and Torrealva, D. (1992). "Seismic Behaviour of a Three-Storey Half Scale Confined Masonry Structure", *Proceedings of the Tenth World Conference on Earthquake Engineering*, Madrid, Spain, Vol. **6**, pp. 3527-3531.

Stafford Smith, B. and Riddington, J. R. (1978). "The Design of Masonry Infilled Steel Frames for Bracing Structures", *The Structural Engineer*, Vol. **56B**, No. 1, pp. 1-7.

Vintzeleou, E. and Tassios, T. P. (1989). "Seismic Behaviour and Design of Infilled R.C. Frames", *European Earthquake Engineering*, Vol. **III**, No. 2, pp. 22-28.

APPENDIX 1: NOTATION

A_{st} = total area of longitudinal reinforcement
d_m = diagonal length of the masonry panel
E_A = absorbed energy
E_D = dissipated energy
E_c = elastic modulus of the concrete
E_s = elastic modulus of the steel
f'_c = compressive strength of the concrete
f_{su} = ultimate (maximum) strength
f'_t = tensile strength of the concrete
f_y = yield strength of the reinforcing steel
h_e = effective height between plastic hinges
h = height of the frame
h_m = height of masonry panel
L = length of the frame
L_m = length of masonry panel
M_{uc} = flexural strength of the compression column
P = vertical force
T_c = tensile axial force in the column
V = applied lateral force
V^* = lateral strength
V_c = the shear force in the reinforced concrete columns
V_d = shear force resisted by dowel action
w = width of the equivalent strut
α = angle of kinking of a reinforcing bar
δ = storey drift
ε_{sh} = strain at the beginning of strain hardening of the reinforcing steel
ε_{su} = strain at maximum load of the reinforcing steel
ε_y = yield strain of the reinforcing steel
τ_o = shear bond strength
τ_o^* = reduced shear bond strength
μ = coefficient of friction
μ^* = reduced coefficient of friction