

# SEISMIC BEHAVIOUR AND DESIGN OF REINFORCED CONCRETE INTERIOR BEAM-COLUMN JOINTS

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## ABSTRACT

This paper is aimed at improving the current understanding of the mechanisms of shear transfer in interior beam-column joints of reinforced concrete frames. Simple variable-angle truss models are used to illustrate the joint shear transfer mechanisms. The model is used in the paper to evaluate the relative importance of those variables that are deemed to affect the shear strength of joints in the current New Zealand Concrete Structures Standard, (SNZ, 1995). The analyses suggest that some of the variables currently being considered might not be as important as thought and that the current design recommendations can be simplified and, in general, be eased. The authors propose a simple three-component equation for use in design. The design equation is based on the results of a parametric analysis and was calibrated against a database of tests obtained from the literature.

## INTRODUCTION

Beam-column joints have long been recognized as critical elements in the seismic design of reinforced concrete frames (ACI, 1999; AIJ, 1990; Eurocode, 1994, SNZ, 1995). Beam column joints must provide sufficient strength to transfer the shear forces resulting from the actions of the framing members. The worst possible demand in joints arises when the framing members, usually the beams, develop negative and positive plastic hinges at the joint faces. Joint shear failure, or even incipient joint deterioration, should be precluded as far as practicable in design in order to preserve the structural integrity of the frames in large earthquakes and to avoid costly repairs of these inaccessible members after moderate earthquakes. In fashion with the Capacity Design Philosophy the joints should be made strong enough so that the mechanism of plastic deformation chosen in design can develop and be maintained.

One of the major problems faced in design is that the joint shear strength of a joint cannot be determined with the accuracy of the flexural strength or even with the shear strength of the framing members. Uncertainty in the determination of the joint shear strength arises largely because of lack of fully understanding the mechanisms of shear transfer and the strength degradation that occurs due to the loading history involving inelastic cyclic reversals in the plastic hinges that form adjacent to the joints.

This paper reviews the background to the current seismic design recommendations for interior beam-column joints of moment resisting frames in New Zealand. The paper also describes an analytical solution for evaluating the shear strength of interior beam column joints. The solution is based on the lower bound theorem of the theory of plasticity and

uses struts and ties for obtaining the stresses acting on the diagonal compression field in the joint panel. The trends obtained from the analysis are calibrated using a database of test results. A comparison of the current design recommendations is made in light of the results obtained from the model. A simple three-component design equation is proposed in the paper.

## INPUT ACTIONS IN INTERIOR BEAM-COLUMN JOINTS

Figure 1 (a) shows the input forces in an interior beam-column joint of a moment resisting frame with a cast-in-place slab once plastic hinges have developed in the beams at the column faces. By comparing the slope of the bending moment diagram in the columns and in the joint in Figure 1 (b), it can be concluded that the shear force in the joint is several times greater than that of the framing columns. Figure 1 (c) shows the shear force diagram for the columns and the joint. The horizontal joint shear force  $V_{jh}$  can be determined from equilibrium of horizontal forces in the joint panel. With reference to Figure 1(a) the horizontal joint shear force  $V_{jh}$  is:

$$V_{jh} = T_T + T_S + C_{ST} + C_{CT} + D_B \cos\theta_B - (V_{col} + F_{eq}) \quad (1)$$

where  $T_T$  is the tension force induced by the top beam bars anchored in the joint core,  $T_S$  is the tension force induced by the slab bars plus beam bars anchored outside the joint core,  $C_{ST}$  is the compressive force carried by the top beam reinforcing bars anchored in the joint core,  $C_{CT}$  is the compressive force in the beam plastic hinge carried by the concrete,  $D_B$  is the magnitude of the diagonal compression force carrying the shear in the plastic hinge region of the beam,  $\theta_B$  is the inclination of the diagonal force  $D_B$  with respect to

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the horizontal plane,  $V_{col}$  is the shear force carried by the column above the joint and  $F_{EQ}$  is the fraction of the inertia force induced by the earthquake at the level of the diaphragm

and that is carried by the column below the joint.

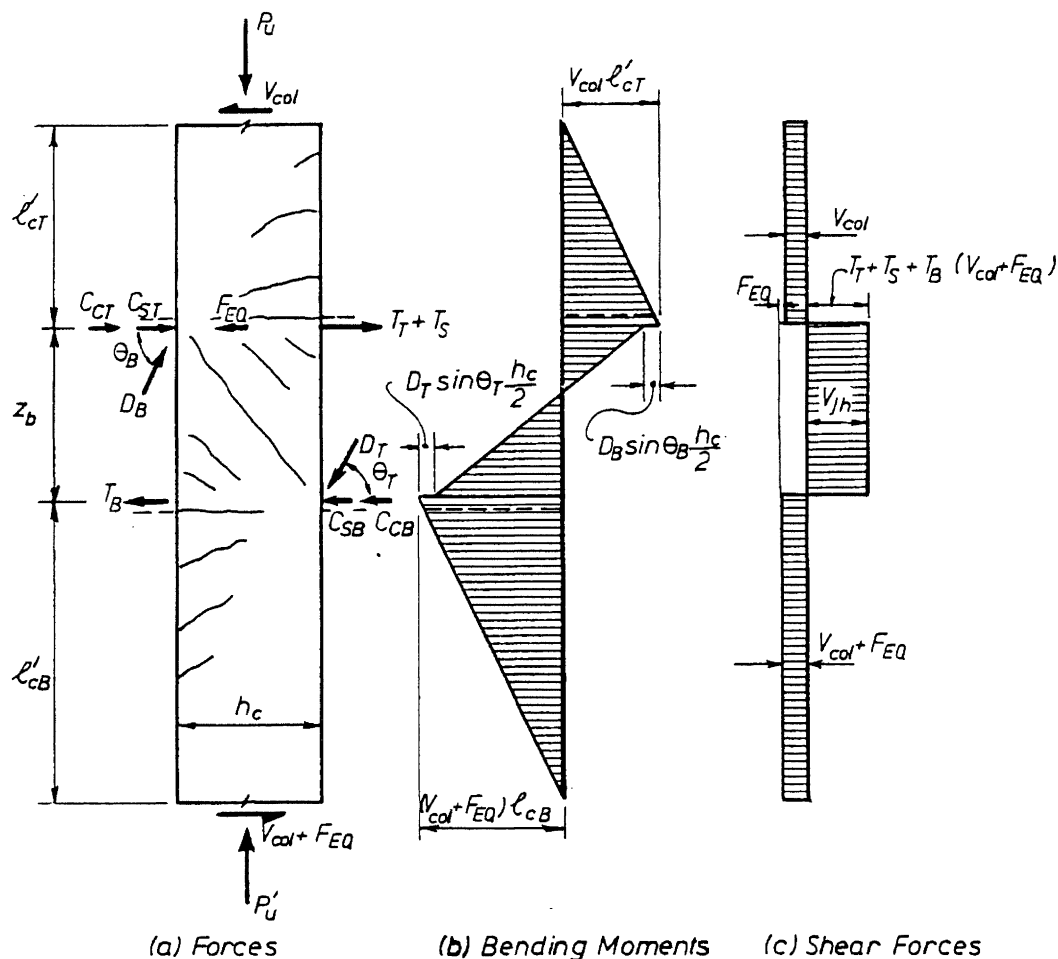


Figure 1: Forces acting upon a concrete column of a moment resisting frame subjected to earthquake loading.

The consideration of equilibrium of vertical forces in the joint panel results in an expression for the vertical joint shear force that is somewhat similar to that given by Eq. 1. However, since the columns are designed to remain elastic and have the longitudinal reinforcement distributed along the perimeter, a relatively lengthy procedure is often needed to estimate the vertical joint shear force. Instead, it has been proposed (Paulay *et al.*, 1978) that the vertical joint shear force be approximately determined by the following vectorial relationship:

$$V_{jv} = V_{jh} h_b / h_c \tag{2}$$

where  $h_b$  and  $h_c$  are the overall beam and column depths, respectively.

### BACKGROUND TO THE DESIGN RECOMMENDATIONS FOR BEAM-COLUMN JOINTS

The concrete diagonal strut and parallel truss model, first proposed by Park and Paulay (1975), has been consistently

used in New Zealand for obtaining equations for the seismic design of joints of moment resisting frames. This model is shown in Figure 2. According to this model a portion of the joint shear force can be transferred directly by the diagonal concrete strut without the need of any reinforcement. An additive truss mechanism, acting with a diagonal compression field parallel to the diagonal strut, transfers the remainder joint shear force, which is originated by bond forces in the beam and column reinforcement. Obviously, the truss mechanism requires vertical and horizontal joint reinforcement. According to the Park and Paulay model, the horizontal and vertical joint shear forces are resisted as the combination of the two mechanisms,

$$V_{jh} = V_{ch} + V_{sh} \quad \text{and} \quad V_{jv} = V_{cv} + V_{sv} \tag{3}$$

where  $V_{ch}$  and  $V_{cv}$  are the contribution to the shear resistance provided by the diagonal strut mechanism in the horizontal and vertical directions, respectively, and  $V_{sv}$  and  $V_{sh}$  are the contribution to the shear resistance provided by the truss mechanism in the horizontal and vertical directions, respectively.

The interplay between the diagonal strut and truss mechanisms in this model depends largely on the bond force distribution along the longitudinal reinforcement of the members framing into the joint. A lower bound approach was initially presented by Blakeley *et al.* (1979). In this approach the entire joint shear force resulting from forces associated with the development of the flexural overstrength in the beam plastic hinges was allocated to the truss mechanism. The concrete strut mechanism contributed to the joint shear transfer only if the column had an axial load above  $0.1 A_g f'_c$ , where  $A_g$  is the cross section area of the column and  $f'_c$  is the concrete cylinder compressive strength. This approach was

justified by Paulay *et al.* (1978) who suggested that after few load reversals involving yielding penetration of the beam longitudinal reinforcement, the bond forces would concentrate towards the centre of the joint and away from the strut mechanism. Such distribution is depicted in Fig.3 (a). In this model the presence of axial compression in the column, which increases the width of the strut, enabled the allocation of bond forces to be transferred through this mechanism. The proposal described by Blakeley *et al.* was incorporated with minor changes into the New Zealand Concrete Design Code, NZS 3101:1982 (SANZ, 1982).

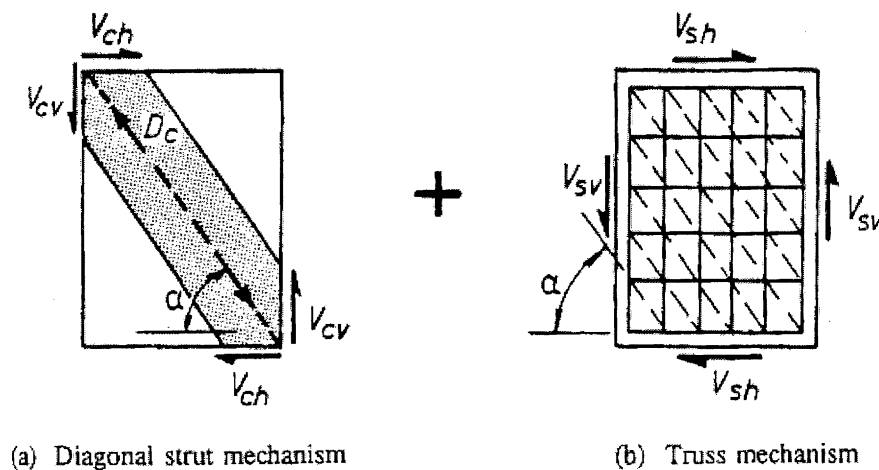


Figure 2: Diagonal strut and parallel angle truss mechanisms of joint shear transfer proposed by Park and Paulay (1975).

Experimental work conducted by Park and Dai (1988) showed that a significant reduction in the joint reinforcement from that required by the Concrete Design Code would not necessarily result in poor seismic performance. Park and Dai suggested that for column axial load levels below  $0.1 A_g f'_c$ , the concrete strut could be designed to carry about 40% of the joint shear force and 70% of the vertical shear force. It was reasoned that there were bond forces at the extremities of the beam longitudinal bars that were transferred by the diagonal strut mechanism.

Following the work of Park and Dai, Cheung *et al.* (1991) proposed a relaxation to the Concrete Design Code requirements by assuming the bond force distribution of the reinforcing bars anchored in the joint core could be represented by a trapezoid as shown in Figure 3 (b). Cheung *et al.* allocated the diagonal concrete strut a shear force equal to the bond forces integrated from the joint side to the neutral axis depth. These researchers also discussed the shear transfer mechanism of beam-column joints having a cast-in-place slab. Paulay and Priestley (1992) modified the bond force distribution proposed by Cheung *et al.* and suggested the model depicted in Figure 3 (c). The bond force distribution proposed by Paulay and Priestley allocates a greater

proportion of the joint shear force to the truss mechanism than if using the Cheung *et al.* bond force model. The work presented by Paulay and Priestley was incorporated with some modifications into the current New Zealand Concrete Structures Standard (SNZ, 1995). It can be inferred from the Standard that the shear force allocated to the truss mechanism in one-way interior beam-column joints is,

$$V_{sh} = \alpha_i \alpha_j T^* \quad (4)$$

where  $T^*$  is the maximum tensile yield force of top and bottom beam bars anchored in the joint; and  $\alpha_i = 1.4 - 1.6 N^* / A_g f'_c$  in joints of ductile frames or  $\alpha_i = 1.2 - 1.4 N^* / A_g f'_c$  in joints of frames designed for limited ductility response;  $\alpha_j$  is a joint shear intensity parameter defined as  $\alpha_j = 6 v_{jh} / f'_c$ ,  $\alpha_j$  shall not be taken greater than 1.2 nor less than 0.85; and  $v_{jh}$  is a joint shear stress index defined by,

$$v_{jh} = V_{jh} / (h_c b_j) \quad (5)$$

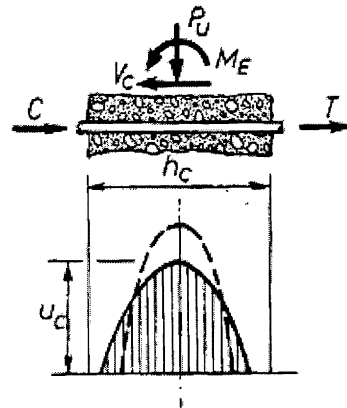
where  $b_j$  is an effective joint width defined as shown in Figure 4. To avoid a premature diagonal compression failure, the Concrete Structures Standard limits the joint shear stress to  $v_{jh} \leq 0.2 f'_c$ .

The Concrete Structures Standard also recommends that

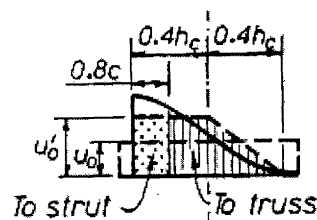
vertical joint reinforcement be provided in order to sustain the diagonal compression field required for equilibrium in the truss mechanism. It can be inferred from the Standard that the vertical joint shear force,  $V_{sv}$ , carried by the truss mechanism

$$\text{is: } V_{sv} = \alpha_v V_{jh} h_b / h_c \quad (6)$$

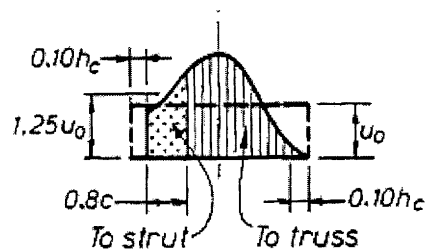
where  $\alpha_v = 0.7 / (1 + N^* / (A_g f_c))$ .



(a) Paulay et al (1978)



(b) Cheung et al (1991)



(c) Paulay and Priestley (1992)

Figure 3: Bond force distribution models proposed for longitudinal bars of beams forming plastic hinges at the joint faces.

The requirements for vertical joint shear reinforcement in the Standard are the same for joints of frames designed for limited ductility of fully ductile response.

#### ANALYTICAL MODELING

From the historical development of the diagonal strut and parallel angle model described in previous section, it can be

concluded that the major source of uncertainty in this model results from the apportioning of the joint shear force to each of the mechanisms. Restrepo *et al.* (1993) pointed out that the parallel angle truss and diagonal concrete strut model would not always result in equilibrium of internal forces and suggested that this model should mainly be used conceptually. They proposed the use of a variable-angle truss model to ensure equilibrium of internal forces in all cases.

The analytical model described in this paper is based on the

model proposed by Restrepo *et al.* (1993). The model is based on the lower bound theorem of plasticity and uses strut-and-tie

models to evaluate the internal force flow within a joint panel (Schläich *et al.*, 1987, Lin *et al.* 1997).

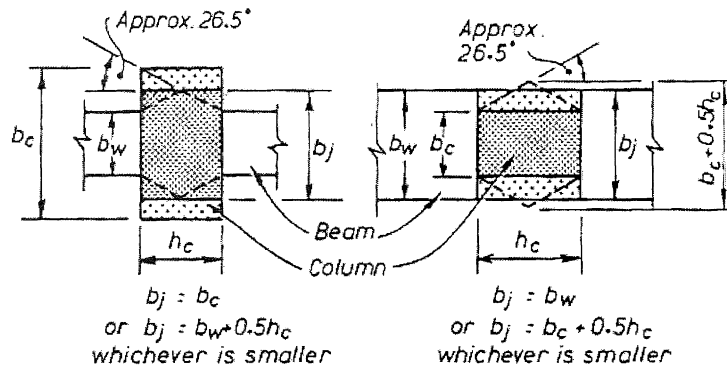


Figure 4: Definition of the joint effective width.

The model is based on the following assumptions:

1. The joint panel is in a plane-stress state,
2. the joint panel is cracked and a uni-axial compression stress field transfers the internal forces through the concrete,
3. all beams framing into the joint, including transverse beams if they exist, form plastic hinges at the joint faces,
4. the columns framing into the joint are loaded below their flexural strength,
5. the resultant force of the concrete compressive stress block resulting from flexure in the beam ( $C_c$ ) is in line with the centroid of beam compressive longitudinal reinforcement,
6. the beam longitudinal bars are adequately anchored in the joint region and, therefore, bond failure does not occur,
7. the column shear force is transferred into the joint through the column concrete compressive stress block,
8. the beam shear force is transferred into the joint as concentrated force acting at the face of the joint and at the level of the beam longitudinal reinforcement in compression,
9. bond forces in the longitudinal beam bars passing through the joint region develop in accordance with a prescribed bond stress law,
10. the centre of the joint panel is the critical region causing failure in the joint by crushing of the concrete
11. at failure the joint horizontal reinforcement is yields in tension.

A major difficulty in the establishment of the internal forces in the joint panel has been the selection of an appropriate bond

force distribution of the beam longitudinal bars that yield at the joint faces due to the development of plastic hinges. It is well known that the bond stress distribution of bars that yield in the plastic hinges and that are anchored in the joint is highly dependent on the cyclic load history. The profile used in this study was the one proposed by Restrepo *et al.* (1993). This model is shown in Figure 5. In this model it is assumed that bond stresses result from two different mechanisms. One mechanism is associated with shearing of the concrete between bar deformations, having a maximum value equal to  $2.2\sqrt{f'_c}$ . The other results from friction between the locally crushed concrete, which is associated with the concrete compressive strength  $f'_c$ . The latter mechanism only develops when bond slip occurs when the first mechanism is unable to provide full anchorage. The crushing mechanism acts only over the column concrete compression stress block.

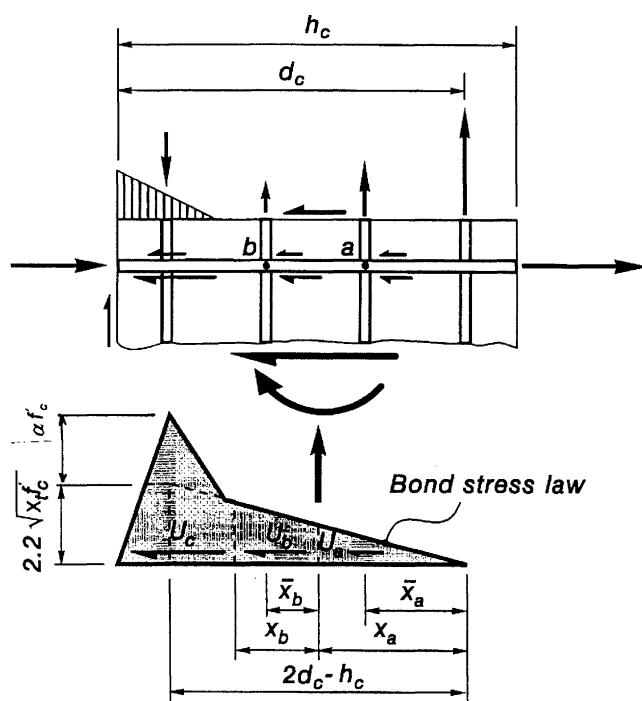
Observations made in laboratory tests justify the assumption that the joint reinforcement yields in tension prior to joint failure. Two exceptions to this assumption can be found: (i) if the yield force resulting from the horizontal joint reinforcement exceeds the horizontal shear force, some hoops will remain elastic, and (ii) plain round bar joint hoops placed next to the beam longitudinal reinforcement do not always yield in tension. In the analytical model described here it is assumed that the horizontal joint reinforcement has yielded and can be replaced with an equivalent uniformly distributed stress block acting at the vertical joint faces. The magnitude of the stress block is calculated from the yield force of horizontal joint hoops,  $V_{sh}$  divided by an effective joint depth. This effective joint depth is taken as 85% of the depth between the top and bottom longitudinal beam bars.

The column longitudinal bar forces can be obtained from a section moment- curvature analysis. It should be recognized though that the bar forces can differ, and in fact, can be larger than the forces obtained from the moment-curvature analysis, as a result of the presence of a diagonal compression stress field in the column.

Figure 6 shows schematically the forces acting in an interior beam-column joint panel in accordance with the assumptions made for the model. The diagonal compression stress field in

the joint panel is modeled with three main strut types. Strut type CC, see Figure 6, runs from corner to corner and is balanced by forces resulting from the column compressive stress blocks. This strut type is similar to the diagonal concrete strut mechanism postulated by Park and Paulay (1975). Strut type TT transfers compressive forces from the nodes formed at the intersection between the beam and interior column longitudinal bars and is equilibrated at the other end by a node within the equivalent joint hoop stress block. This strut type represents the truss model postulated by

Park and Paulay, except that the inclination of this strut may be different is solely dictated by equilibrium. The last strut type, strut CT shown in Figure 6, carries compressive forces from the column compressive stress block to a node within the equivalent joint hoop stress block and is equilibrated at the other end by a node within the equivalent joint hoop stress block. Bond forces in the beam longitudinal bars are allocated initially at the nodes where beam bars meet column interior bars based on a prescribed bond force distribution.



#### Discrete potential bond forces

$$U_a = 2.2 \sqrt{x_a} f_c \frac{(x_a)^2}{2d_c - h_c} \frac{S_o}{2}$$

$$U_b = 2.2 \sqrt{x_b} f_c \frac{(x_a + x_b)^2}{2d_c - h_c} \frac{S_o}{2} - U_a$$

Figure 5: Bond stress distribution proposed by Restrepo et al. (1993).

A horizontal joint hoop force resulting from the equivalent stress block balances the discrete bond force at the node at the intersection between the beam and interior column longitudinal bars. Once the magnitude of this force is determined, the position of the node along the column exterior bars can readily be found. The vertical component of the force in the strut needs to be carried by the column interior bar providing that the total force in the bar is less than its yield force. Also the vertical component of the force in this strut is balanced by the column exterior bars. This force is a discrete representation of the bond force sustained by the column exterior bars.

The position and forces in the struts close to the joint center are determined from equilibrium requirements. In some

particular cases interior nodes located along the column interior bars may be required for equilibrium. Figure 7 depicts strut details of an example of joint analysis.

An arbitrary uni-axial compressive stress index,  $f_{c,s}$ , is established as the force carried by the central strut,  $S_o$ , divided by half the distance between the struts at either side of the strut,  $W_s$ , and by the joint width  $b_j$ , see Figure 6. This stress does not necessarily represent the maximum uni-axial compressive stress in the joint but is taken as index to predict the strength of a joint, as experimental work done on the past nearly has always showed that joint failure occurs by crushing of the concrete at the joint center [Paulay and Priestley 1992].

Once the beam longitudinal bar bond forces are allocated to the nodes along these bars the strut-and-tie model used to

represent the internal force flow becomes not only statically determinate but also uniquely defined. One of the disadvantages of this model is that the stress  $f_{c,s}$  is sensitive to the number of struts chosen to represent the diagonal

compression stress field in the joint panel. Therefore, in a parametric analysis the number of struts should be kept reasonably constant to avoid a bias in the analytical results.

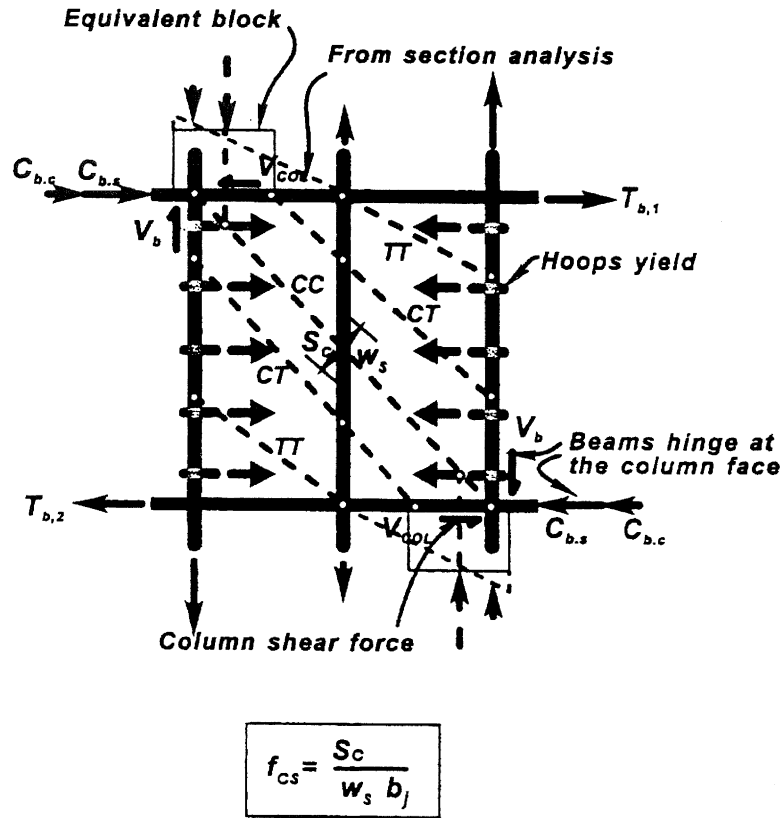


Figure 6: Modeling of the internal force flow in interior beam-column joints.

**PARAMETRIC STUDY**

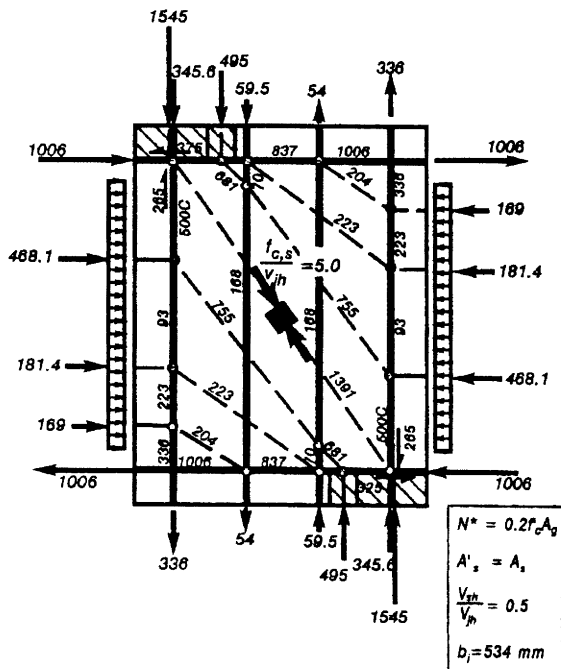
A parametric analysis was carry out to evaluate the influence that different variables have to the stress ratio  $f_{c,s} / v_{jh}$ . For a given joint shear stress ratio  $v_{jh} / f'_c$  an increase in the stress ratio  $f_{c,s} / v_{jh}$  due to a change in the parameter investigated gives an indication of the concentration of the compression stress field towards the centre of the joint panel. The presence of a slab was not accounted for in the joints investigated in the parametric analysis.

**Effect of Bond Force Distribution**

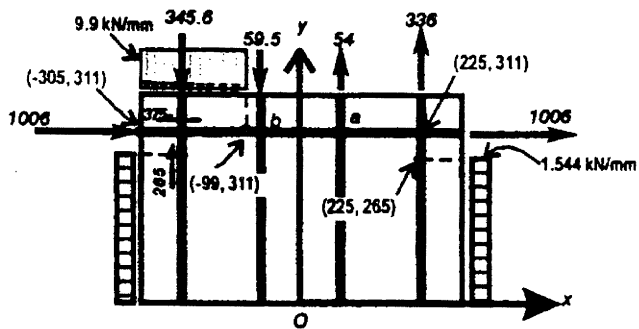
Figure 8 (a) and (b) evaluates the effect that the beam longitudinal bar bond force distribution has on the internal force flow and stress  $f_{c,s}$ . The two joints shown in Figure 8 are identical, except for the size of longitudinal beam bars. A comparison of the results shows that the stress ratio  $f_{c,s} / v_{jh}$  is very similar in both joints. The relative insensitivity to the

bond stress distribution on the stress ratio  $f_{c,s} / v_{jh}$  is caused by the presence of the strut type CT.

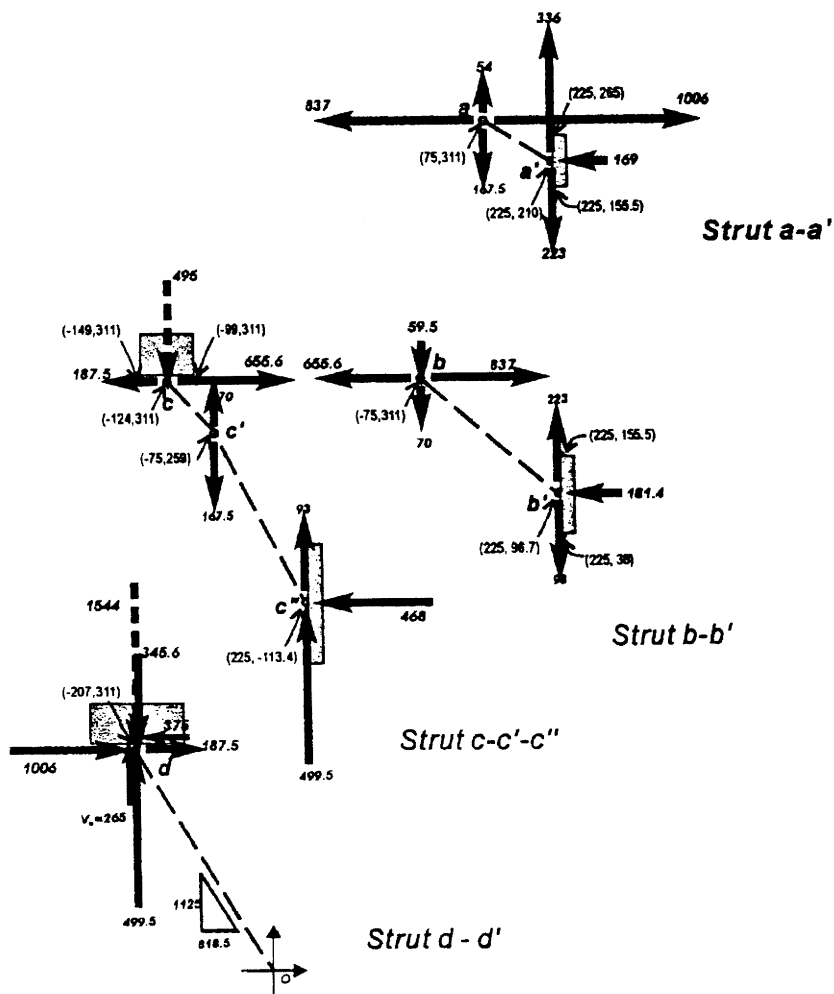
The results obtained from this analysis indicate that the bond force distribution does not seem to be an important variable associated with the strength of interior beam column joints. It should be noted, however, that this finding can not be extrapolated to those joints in which bond failure occurs prematurely.



(a). Actions and internal force flow.



(b). Node co-ordinates.



(c). Strut geometry and forces

Figure 7: Example of joint analyzed using strut-and-tie model.

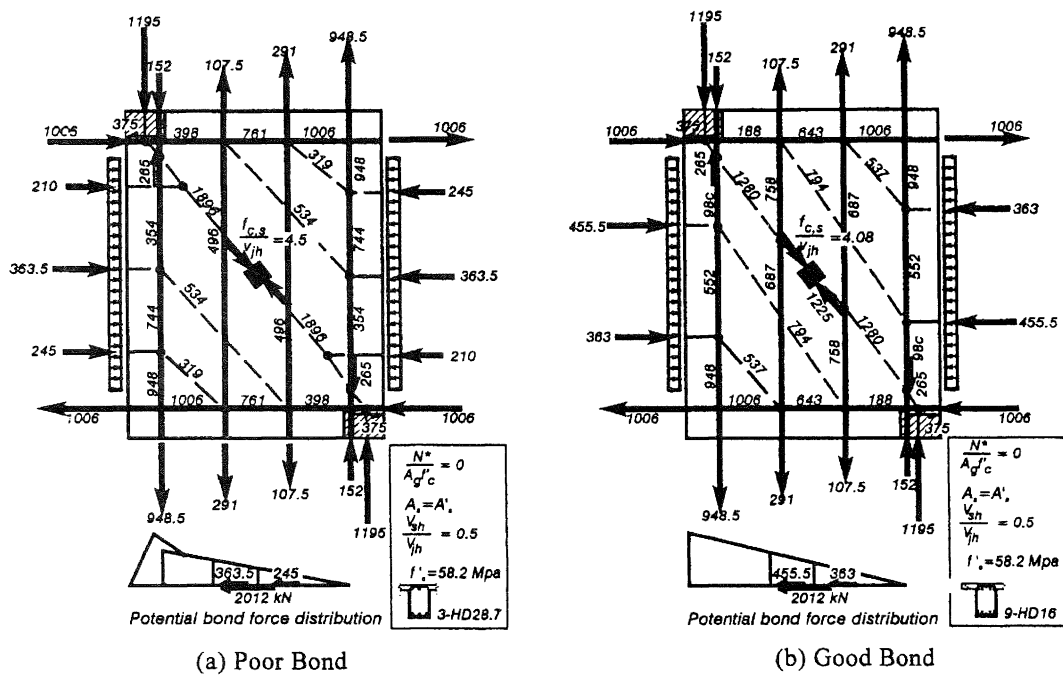


Figure 8: Comparison of strut-and-tie model analysis of joints having beam bars with poor and good anchorage conditions.

**Effect of Unequal Top and Bottom Beam Longitudinal Reinforcement**

It can be deduced from Eq. (4) that the horizontal joint reinforcement required by the Concrete Structures Standard (SNZ, 1995) is a function of maximum tensile force resulting from the beam longitudinal bars anchored in the joint core. On this basis, the same yield strength for top and bottom beam longitudinal reinforcement the joint shear reinforcement is dependent on the ratio  $A_s' / A_s$ , where  $A_s'$  is the area of beam reinforcement anchored in the joint and subjected to compression and  $A_s$  is the area of beam reinforcement anchored in the joint and subjected to tension. Take for example two identical joints subjected to the same horizontal joint shear force but having different  $A_s' / A_s$  ratios. According to the Concrete Structures Standard the joint with the smallest  $A_s' / A_s$  ratio requires a more horizontal joint reinforcement than the other joint. This is requirement is due to the bond force dependent characteristics of the diagonal strut and parallel angle truss model.

The three joints shown in Figure 9 were analysed. All the joints are identical except for the ratio  $A_s' / A_s$ . The joint shown in Figure 9 (a) has equal top and bottom beam bars, that is,  $A_s' / A_s = 1$  while those joints shown in Figure 9 (b) and (c) have  $A_s' / A_s$  ratios equal to 0.75 and 0.4, respectively. It can be observed that the direction of the internal force flow is dependent on the ratio  $A_s' / A_s$  ratio. However, and more importantly, the ratio  $f_{c,s} / v_{jh}$  is practically independent from the  $A_s' / A_s$  ratio. The relative insensitivity of the ratio  $f_{c,s} / v_{jh}$  to the ratio  $A_s' / A_s$  seems to suggest that the strength of an interior beam-column joint should not be made dependent on the  $A_s' / A_s$  ratio. Likewise, the demand for horizontal joint

reinforcement, should not depend on the ratio  $A_s' / A_s$  as is the case in the design of joints with the current Standard. An experimental corroboration of this finding is described elsewhere (Lin *et al.* 2000).

**Effect of the Horizontal Joint Reinforcement**

The influence of the joint horizontal reinforcement on the stress ratio  $f_{c,s} / v_{jh}$  was assessed by analysing three joints. The results of the analyses are presented in Figure 10. The only difference in the joints shown in Figure 10 is the ratio between the joint shear force carried by the horizontal joint reinforcement and the horizontal joint shear force,  $V_{sh} / V_{jh}$ . The joints shown in Figures 10 (a), (b) and (c) had  $V_{sh} / V_{jh} = 0, 0.5$  and  $1.0$ , respectively.

It can be observed in Figure 10 that the ratio  $f_{c,s} / v_{jh}$  is highly sensitive to the ratio  $V_{sh} / V_{jh}$ . This is because an increase in the horizontal joint shear force carried by the hoops enables the spreading of diagonal compression field in the joint panel, which results in a reduction of the compressive stresses at the centre of the panel. As a corollary, when the ratio  $V_{sh} / V_{jh}$  is low or nil the diagonal compression field is narrow and consists primarily on a corner-to-corner diagonal strut. Spreading of the compression field in these cases can only occur because of the presence of the column longitudinal reinforcement. The effect of the ratio  $V_{sh} / V_{jh}$  was corroborated experimentally and is described elsewhere (Lin *et al.* 2000).

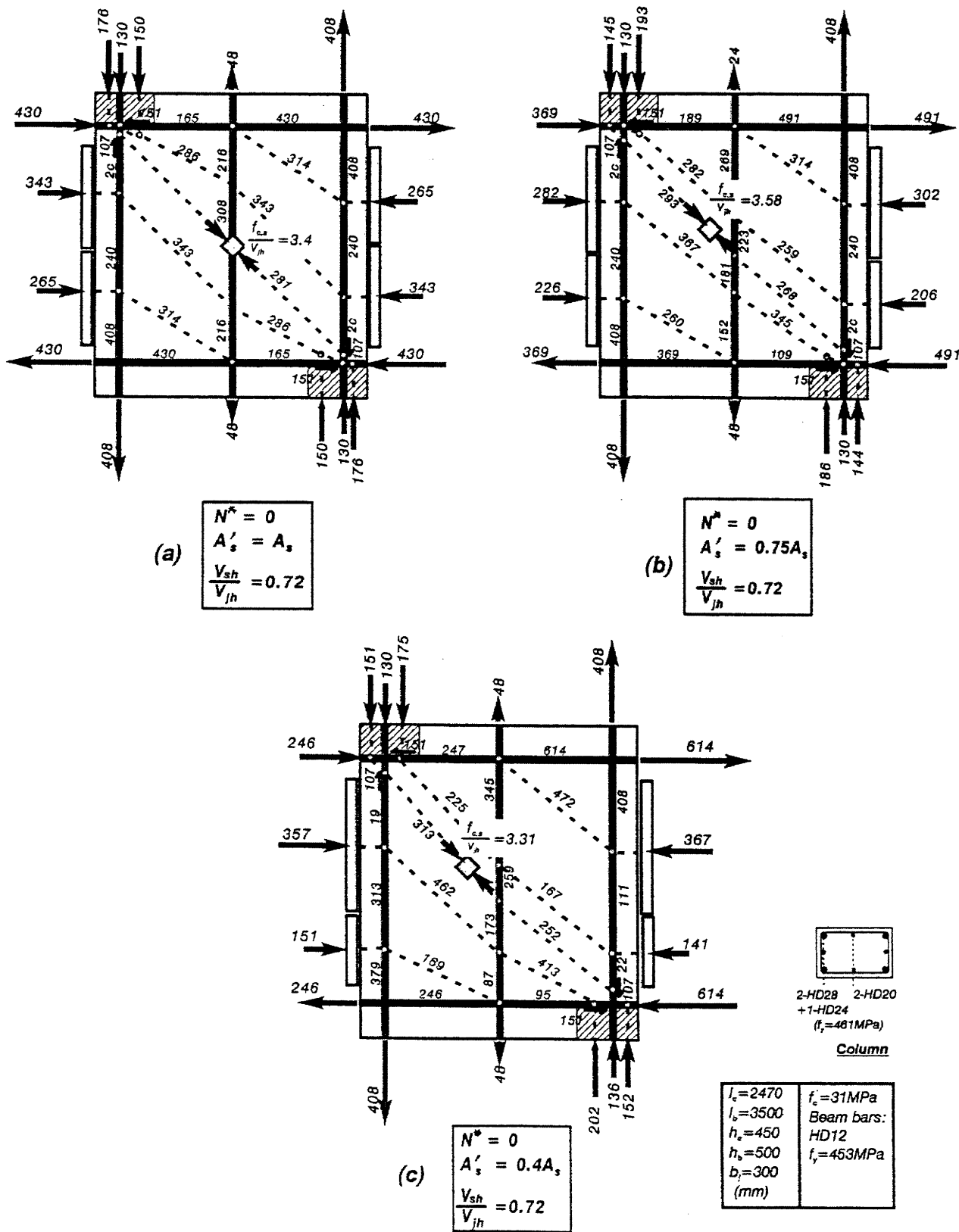


Figure 9: Comparison of internal force flow in joints with different  $A'_s/A_s$  ratios.

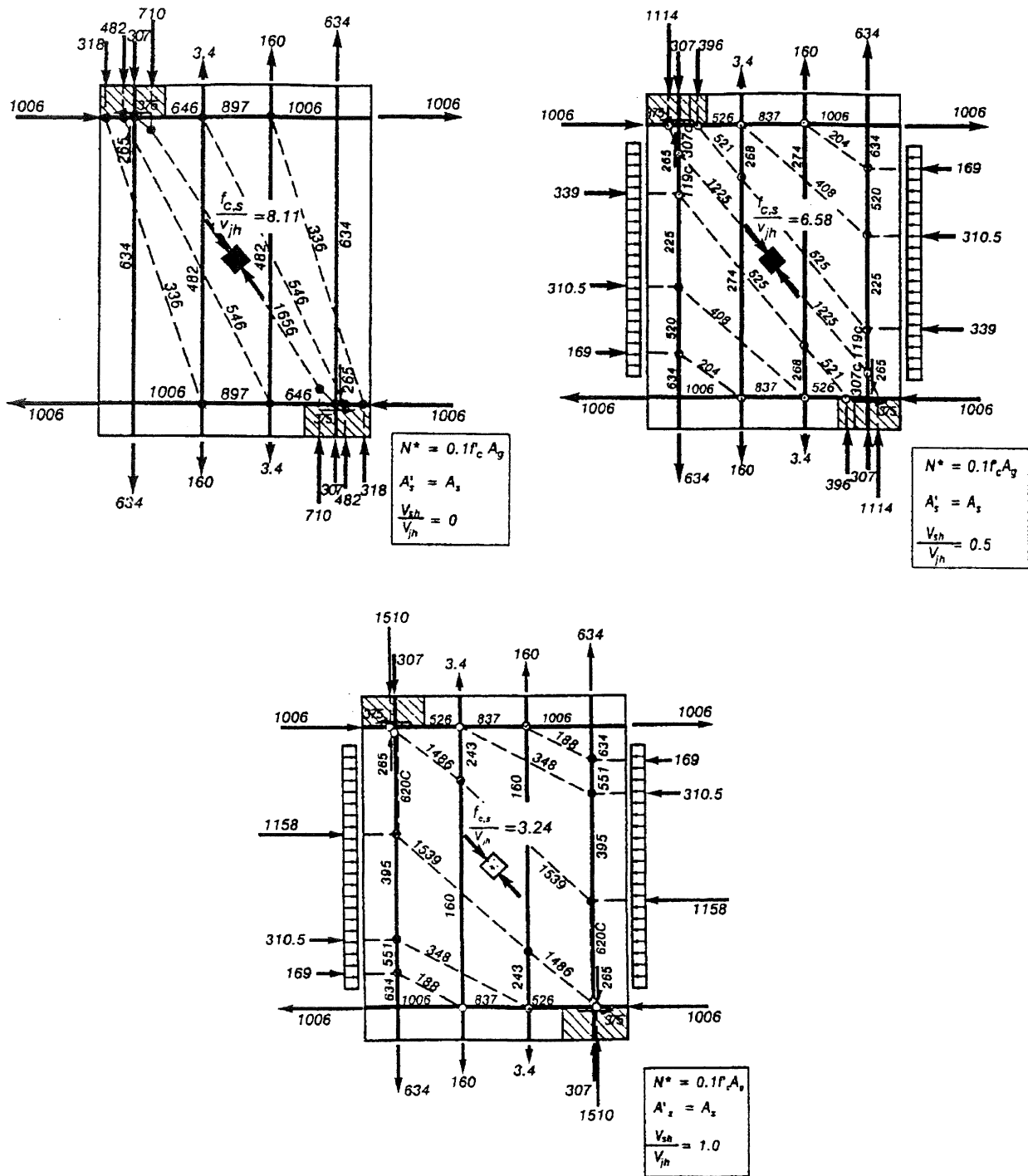


Figure 10: Comparison of internal force flow in joints with different amounts of horizontal joint reinforcement.

**Effect of the Column Axial Load**

One of the most controversial issues in the design of interior-beam column joints is the effect that the column axial load ratio,  $N^* / (A_g f_c')$ , has on the strength of the joints. For example, in the Concrete Structures Standard (SNZ, 1955) the amount of joint reinforcement is made a function of the axial load, inferring that axial compression enhances the shear

strength of the joint. In contrast, the ACI-318 building code (ACI, 1999) ignores the effect of the column axial load.

The influence that the column axial load ratio,  $N^* / (A_g f_c')$ , has on the internal force flow in the joint panel and on the stress ratio  $f_{c,s} / v_{jh}$  was investigated with the four joints depicted in Figure 11. These joints are identical except for the column axial load ratio  $N^* / (A_g f_c')$ . The ratio  $N^* / (A_g f_c')$  was varied from 0 to 0.4.

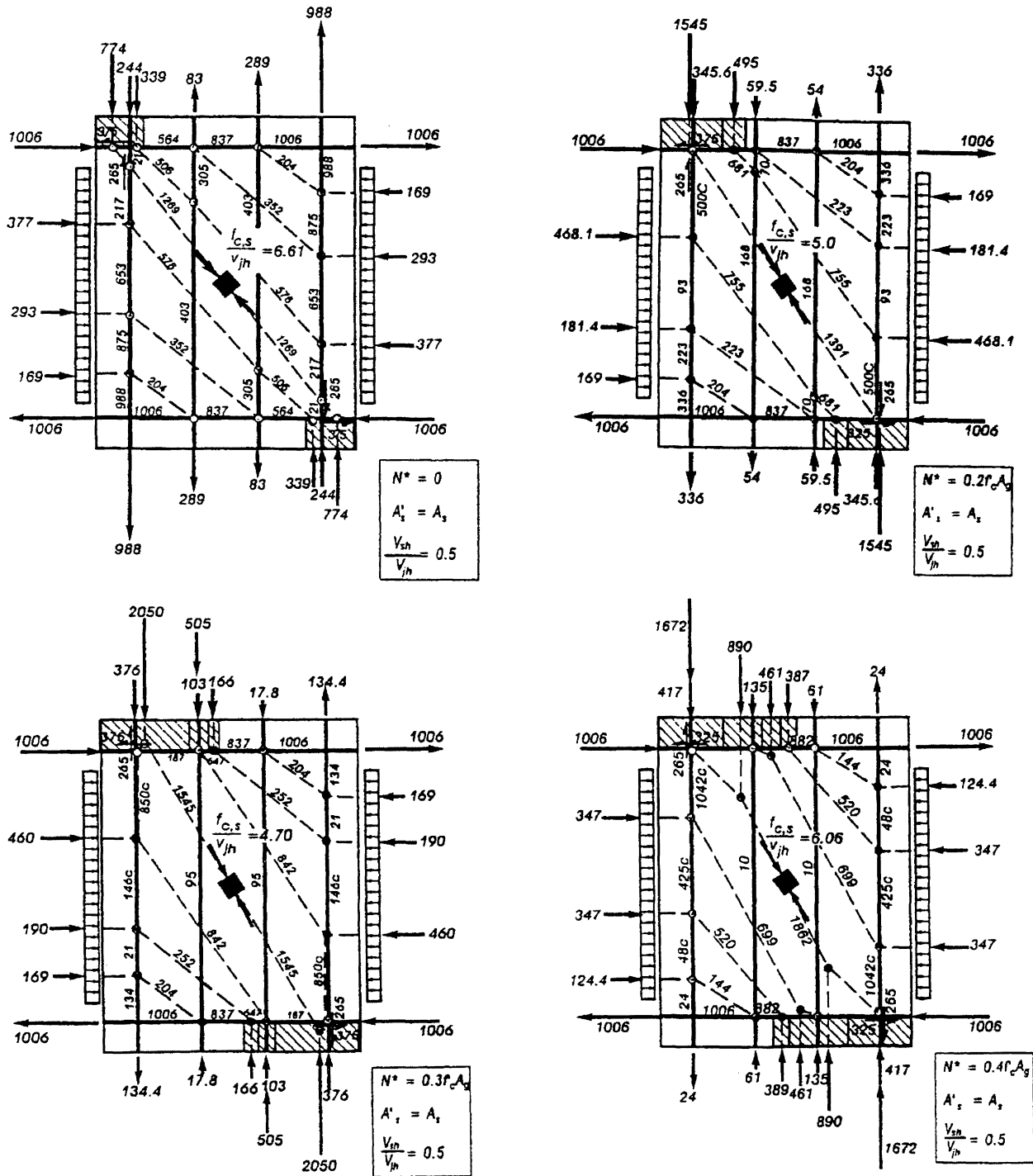


Figure 11: Comparison of internal force flow in joints with different column compressive loads.

The column axial load ratio  $N^* / (A_g f_c')$  was observed to significantly influence the stress ratio  $f_{cs} / v_{jh}$  and its effect was found to be coupled with the ratio  $V_{sh} / V_{jh}$ . At low axial load ratios, an increase in the column axial compression results in a decrease in the stress ratio  $f_{cs} / v_{jh}$ , suggesting that the diagonal compression stress field spreads as the axial compression increases. This is because of the depth of the column compression stress block increases with axial compression. Note also that the inclination of the struts increases with an increase in axial compression.

A striking result was obtained when the ratio  $N^* / (A_g f_c')$  was increased from 0.3 to 0.4 as the stress ratio  $f_{cs} / v_{jh}$  increased from 4.7 to 6.06. Further analyses were performed with the aim of understanding the reverse in the trend obtained for joints with lightly loaded columns. Fifteen identical beam column joints were studied (Lin *et al.*, 2000). The axial load ratio  $N^* / (A_g f_c')$  and the ratio  $V_{sh} / V_{jh}$  were varied in the study.

Figure 12 plots the stress ratios  $f_{cs} / v_{jh}$  obtained for each of the cases studied. At low column axial load ratios,  $0 \leq N^* / (A_g f_c') \leq 0.1$  the column axial load has little effect on the stress ratio  $f_{cs} / v_{jh}$ . At moderate levels of axial compression,  $0.1 \leq$

$N^* / (A_g f_c') \leq 0.3$ , an increase in axial load results in a decrease in the stress ratio  $f_{cs} / v_{jh}$ . In contrast, and as observed previously, at moderately high and high column axial load ratios,  $N^* / (A_g f_c') > 0.3$ , an increase in axial compression results in an increase in the stress ratio  $f_{cs} / v_{jh}$ , particularly when the ratio  $V_{sh} / V_{jh}$  is low. This trend indicates that for axial load ratios  $N^* / (A_g f_c') > 0.3$  the axial load is detrimental to the joint shear strength as the diagonal compression field is unable to spread out, thus, increasing the compressive stresses at the centre of the joint panel.

The detrimental effect on the joint shear strength caused when the axial loading exceeds  $N^* / (A_g f_c') > 0.3$  was experimentally demonstrated by Lin *et al.* (2000). In an experiment two identical units were tested. In one unit the horizontal joint reinforcement was designed following the recommendations given by the Concrete Structures Standard for joints of ductile frames. The second unit was designed in accordance with the model proposed in this paper, which required more horizontal joint reinforcement than the Concrete Structures Standard. Failure of the first unit occurred by crushing of the concrete at the centre of the joint panel at only a displacement ductility of  $\mu_\Delta = 4$ . The second unit performed satisfactorily.

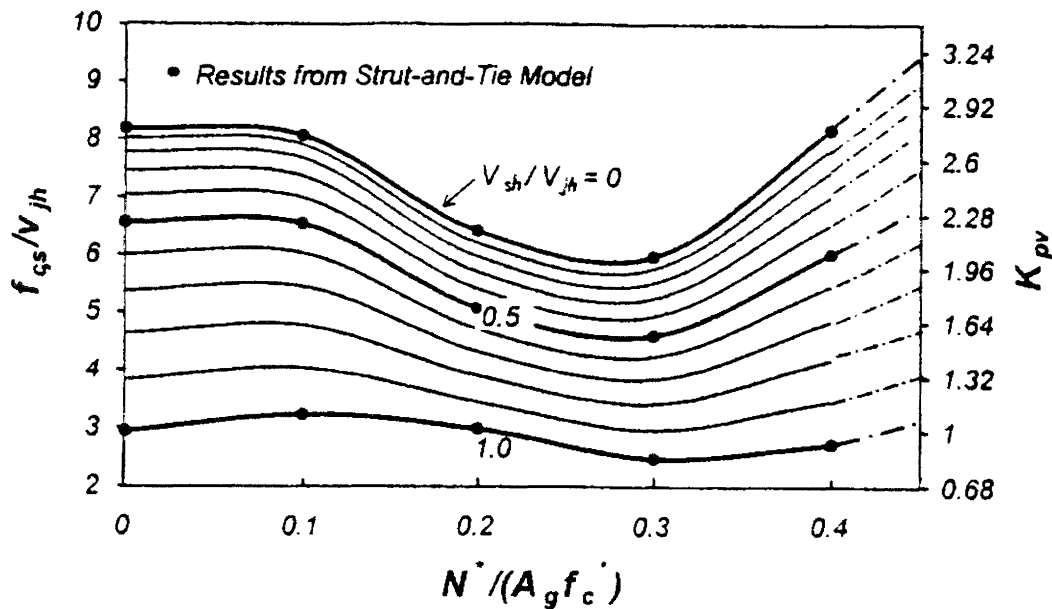


Figure 12: Influence of the combined ratios  $V_{sh} / V_{jh}$  and  $N^* / (A_g f_c')$  ratios on the stress ratio  $v_{jh} / f_c'$ .

### Synthesis

The parametric analysis described above indicates that the axial load level  $N^* / (A_g f_c')$  and the ratio  $V_{sh} / V_{jh}$  have the most significant influence on the stress ratio  $f_{cs} / v_{jh}$ . Moreover, the parametric analysis also showed that the  $N^* / (A_g f_c')$  and the ratio  $V_{sh} / V_{jh}$  have an interdependent influence on the stress ratio  $f_{cs} / v_{jh}$ .

It can be inferred from Figure 12 that the stress ratio  $f_{cs} / f_c'$  is approximately equal to 0.3 in a joint with  $V_{sh} / V_{jh} = 1$  and  $N^* = 0$ , when  $v_{jh} = 0.1 f_c'$ . Furthermore, the stress ratio  $f_{cs} / f_c'$  in a beam-column joint with  $V_{sh} / V_{jh} = 0$  and  $N^* / (A_g f_c') = 0.4$  is approximately 2.8 times the stress ratio  $f_{cs} / f_c'$  of a joint with  $V_{sh} / V_{jh} = 1$  and  $N^* / (A_g f_c') = 0$  if the shear stress ratio  $v_{jh} / f_c'$  is the same in both joints.

With the assumption that crushing of the concrete at the center of joint panel leads to failure, it may be reasonable to say that a joint with  $V_{sh} / V_{jh} = 1$  and  $N^* / (A_g f'_c) = 0$  can sustain approximately 2.8 times the stress ratio  $v_{jh} / f'_c$  of a joint with  $V_{sh} / V_{jh} = 0$  and  $N^* / (A_g f'_c) = 0.4$  if the same ratio  $f_{cs} / f'_c$  is to be attained. According to this rationale it is possible to relate the shear stress ratio  $v_{jh} / f'_c$  of a joint with given values of  $N^* / (A_g f'_c)$  and  $V_{sh} / V_{jh}$  to the shear stress ratio  $v_{jh,e} / f'_c$  of an equivalent joint with  $N^* / (A_g f'_c) = 0$  and  $V_{sh} / V_{jh} = 1$ , so that both joints have equal stress ratios  $f_{cs} / f'_c$ . This transformation can be achieved using factor  $K_{pv}$  shown in the vertical axis at the right hand side of Figure 12. Factor  $K_{pv}$  is defined by:

$$K_{pv} = (v_{jh,e} / f'_c) / (v_{jh} / f'_c) \quad (7)$$

### DATABASE REDUCTION

Data obtained from cyclic reversed load tests on one-way interior beam-column joint assemblies was collected. The database excluded tests in which beam-column joints failed prior to yielding of the beam longitudinal reinforcement. This type of failure is undesirable and should be precluded by limiting the maximum permitted joint shear stress. Tests in beam-column joints reinforced with hoops in which the stress-strain relationship did not show a well-defined yield plateau were also excluded. This is because joints with this type of reinforcement has shown improved joint behavior (Lin *et al.*, 2000). Furthermore, joint assemblies incorporating lateral beams that were not loaded into the inelastic range were not considered in the database.

The joint shear stress ratio  $v_{jh,e} / f'_c$  of the beam column assemblies in the database was calculated based on the measured material properties and measured lateral force over-strength and then transformed using Eq. (7) into the equivalent joint stress ratio  $v_{jh,e} / f'_c$  and using the  $K_{pv}$  factors derived from Figure 12. The ultimate lateral displacement of the test units was defined as equal to the displacement associated with 10 percent strength degradation measured in the lateral force-lateral displacement response envelope. It is noted that the 10 percent degradation criterion is different to the 20 percent adopted in New Zealand (Park, 1989). This is because a majority of the test results available in the database were not loaded beyond the peak load at which 20 percent degradation occurred. The usefulness of the database would have been very limited if the normally accepted 20 percent strength degradation concept had been adopted in this investigation. Table 1 summarizes the main parameters characterizing the database.

Degradation of the joint shear strength is expected to occur as a result of imposed reversed cycles and ductility in the beam plastic hinges, primarily due to the "tearing" effect that well anchored deformed bars have on the joint panel. This was quantified though the rotational ductility capacity,  $\mu_\theta$ , of the test units. The rotational ductility is defined similarly to the displacement ductility but excludes the column elastic components of lateral deformation, that is,

$$\mu_\theta = \frac{\Delta_u - \Delta_c}{\Delta_y - \Delta_c} \quad (8)$$

where  $\Delta_u$  is the ultimate lateral displacement measured in accordance with the prescribed failure criterion,  $\Delta_y$  is the yield displacement defined by Park (1989) and  $\Delta_c$  is the component of the yield displacement due to the elastic column displacements. Details of the method used to calculate the  $\Delta_c$  can be found elsewhere (Lin *et al.*, 2000).

The rotational ductility in Eq. (8) can also be expressed in terms of the displacement ductility  $\mu_\Delta = \Delta_u / \Delta_y$ , as shown in Eq. (9)

$$\mu_\theta = \frac{\mu_\Delta - \frac{\Delta_c}{\Delta_y}}{1 - \frac{\Delta_c}{\Delta_y}} \quad (9)$$

It should be noted that the difference between both ductility definitions is that the rotational ductility does not consider the bias imposed by the elastic component of the column displacement (Lin *et al.*, 1997, Lin *et al.*, 2000).

The rotational ductility factor was chosen as a base for assessing existing test results because it is anticipated that joint behaviour depends on the rotational ductility of the adjoining beams and itself, rather than on the displacement ductility achieved by the whole frame assembly.

A bilinear trend with strong correlation can be observed in Figure 13. Beam-column joint failures occur after beam flexural yielding if the equivalent joint shear stress ratio exceeds  $v_{jh,e} / f'_c = 0.3$ . For smaller equivalent joint shear stress ratios failure takes place elsewhere. The relationship between the stress ratio  $f_{cs} / v_{jh}$  and the rotational ductility  $\mu_\theta$  in assemblies failing in the joint has a physical explanation. Yielding of the deformed beam longitudinal reinforcement anchored in the joint penetrates gradually as the ductility imposed in the beam plastic hinges increases. Also, the horizontal joint reinforcement begins to yield as the tensile stresses carried by the concrete diminish and the internal forces are sustaining a uni-axial diagonal compression field. Yielding of the horizontal reinforcement with a well-defined yield plateau becomes unrestricted and the tensile strains grow with every large amplitude cycle. A consequence of unrestricted yielding is dilation of the concrete in the plane of the joint, which leads to the reduction in the strength of the diagonal compression field (Vecchio and Collins, 1986).

### DESIGN RECOMMENDATIONS

#### Horizontal Joint Reinforcement

The simple failure criteria given by the bi-linear relationship plotted in Figure 13 can be used to develop charts for the design of the horizontal joint reinforcement in interior beam-column joints of one-way frames. Design recommendations can be given in terms of a ductility-based design or displacement based design approach (Lin *et al.*, 2000). For example, in ductility based design of frames designed to form beam sway mechanisms, rotational ductility demands of approximately 7.7 and 3.7 may be expected for fully ductile or limited ductility response, respectively when the column contribution to the yield displacement is 20%. The

Table 1: Reduced Data of Tests in the Database.

Researcher	Test	$f_c'$ MPa	$N' / (A_g f_c')$	$V_{sh} / V_j$ <sub>h</sub> <sub>s</sub>	$v_{jh} / f_c'$ !	$v_{jh,e} / f_c'$	Measured Ductility Capacity, $\mu_0$		
							Joint Failure	Other Failure Modes	Beam Hinging Failure
Beckingsale <i>et al.</i> (1980)	B11	35.9	0.043	1.000	0.140	0.140			10.0
	B12	34.6	0.045	1.000	0.147	0.147			10.0
	B13	31.4	0.442	0.879	0.155	0.227			7.4
Birss <i>et al.</i> (1978)	B1	27.9	0.053	0.463	0.220	0.502	6.6		
	B2	31.5	0.439	0.139	0.197	0.596	4.4		
Cheung <i>et al.</i> (1991)	1D-1	40.8	0.000	0.639	0.119	0.230			12.0
Durrani and Wight (1982)	X1	34.3	0.055	0.332	0.204	0.513	4.5		
	X2	33.7	0.056	0.485	0.213	0.473	5.2		
	X3	31.0	0.053	0.437	0.171	0.404	6.3		
Dai and Park (1987)	U1	45.6	0.000	0.911	0.076	0.095			12.3
	U2	36.0	0.000	0.831	0.132	0.192		7.2	
	U3	36.2	0.000	0.450	0.094	0.217			15.2
Lawrence and Beattie (1993)	HSC	83.2	0.000	0.595	0.079	0.160		5.2	
Milburn and Park (1982)	U1	41.3	0.100	0.843	0.209	0.309	8.2		
Menheit and Jirsa (1997)	II	41.8	0.254	0.128	0.261	0.496	# 2.6		
	VI	36.8	0.483	0.124	0.307	0.956	2.6		
	XII	35.7	0.300	0.388	0.420	0.690	3.9		
Otani <i>et al.</i> (1985)	C3	25.6	0.077	0.848	0.198	0.294		9.6	
Priestley (1975)	IBC	48.5	0.030	1.000	0.080	0.080			11.5
Restrepo <i>et al.</i> (1993)	U6	44.0	0.000	0.897	0.090	0.119			12.9
Stevenson <i>et al.</i> (1980)	U1	34.0	0.237	0.346	0.253	0.442	4.8		
Teraoka <i>et al.</i> (1994)	NO 43	54.0	0.200	0.279	0.109	0.213			10.2
	NO 47	54.0	0.200	0.183	0.166	0.335	7.0		
Viwanatepa <i>et al.</i> (1979)	BC3	31.1	0.361	0.312	0.149	0.320		4.7	
Xin <i>et al.</i> (1992)	X1	30.9	0.000	0.642	0.167	0.327	8.2		
	X2	40.8	0.000	0.645	0.097	0.191		9.7	
	X3	42.5	0.000	0.663	0.128	0.244		8.9	
	X4	47.2	0.000	0.742	0.083	0.140		9.0	
	X5	60.7	0.000	0.513	0.124	0.276		9.0	
	X6	59.3	0.000	0.554	0.118	0.250		8.6	
Lin <i>et al.</i> (2000)	Unit 1	33.3	0.430	0.461	0.166	0.400	6.1		
	Unit 2	33.3	0.430	0.720	0.173	0.302		7.3	
	Unit 3	37.0	0.100	0.462	0.111	0.246		9.7	
	Unit 4	37.0	0.100	0.462	0.115	0.255		9.3	
	Unit 8	51.7	0.100	0.623	0.169	0.331	7.8		

Note : # Beam Bars did not yield until failure.

<sup>5</sup> Yield force of Joint hoops immediately adjacent to the top and bottom beam bars is discounted when calculating  $V_{sh}$ .

<sup>1</sup> Calculated based on measured overstrength.

equivalent joint shear stress ratios corresponding to these ductility levels are 0.3 and 0.52 when using the 95 percent confidence limit line shown in Figure 13. The equivalent joint shear stress ratios are substituted into Eq. (7) to get the associated values of  $K_{pv}$  and then the required  $V_{sh}/V_{jh}$  ratios can be found from the curves in Figure 12.

The design charts plotted in Figure 14 were generated following the procedure described above for different  $v_{jh}/f'_c$  ratios. There are three distinct regions in the charts depicted in

Figure 14. First, the amount of horizontal joint reinforcement is rather insensitive to axial compression when the column axial load ratio ranges between 0 and 0.1. Second, when the column axial load ratio increases from 0.1 to 0.3 the required quantity of horizontal joint reinforcement decreases. Third, the amount of reinforcement increases with an increase in axial load for column axial load ratios greater than 0.3, with large amounts apparently needed for joints with moderate to large stress ratios  $v_{jh}/f'_c$  when  $N^*/(A_g f'_c) > 0.4$ .

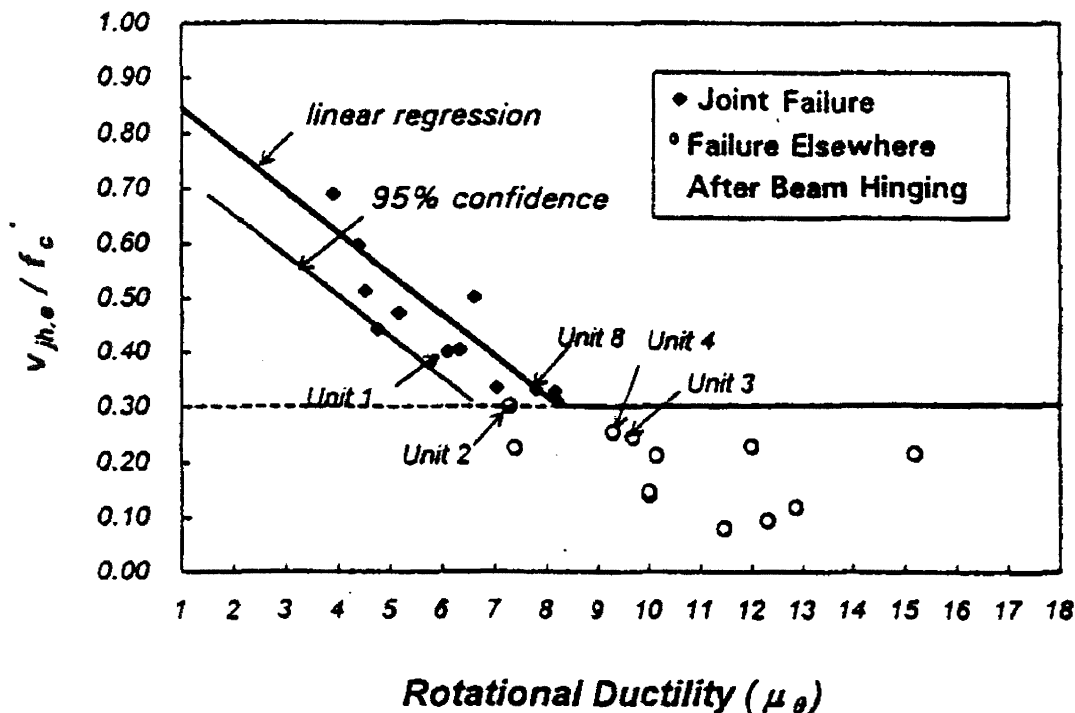


Figure 13: Plot of the equivalent joint stress ratio  $v_{jh,e}/f'_c$  versus  $\mu_\theta$  for a database of experimental results.

The trends obtained from the charts shown in Figure 14 are used below to propose that the horizontal joint shear is transferred by three additive and interdependent mechanisms,

$$V_{jh} = V_{ch} + V_{sh} + V_N \quad (10)$$

where  $V_N$  is the shear force carried by the column axial load.

For joints of frames designed using capacity design principles to ensure the development of a beam sideways mechanism, the fraction of the horizontal joint shear force carried by the concrete is,

$$V_{ch}/V_{jh} = 1 / \{ \alpha_\mu (v_{jh}/f'_c)^3 \} \leq 1 \quad (11)$$

where  $\alpha_\mu = 660$  for joints of frames designed for full ductile response, that is  $\mu_\Delta = 6$ , and  $\alpha_\mu = 140$  for joint of frames designed for limited ductility response, that is  $\mu_\Delta = 3$ .

The joint shear force carried by the column axial load can be

derived from the trends depicted in the design charts,

$$V_N/V_{jh} = 0 \text{ when } N^*/(A_g f'_c) \leq 0.1 \quad (12)$$

$$\text{When } 0.1 < N^*/(A_g f'_c) \leq 0.3 \\ V_N/V_{jh} = 1.6 \{ N^*/(A_g f'_c) - 0.1 \} \quad (13)$$

$$\text{When } N^*/(A_g f'_c) > 0.3 \\ V_N/V_{jh} = 1 - 2.3 \{ N^*/(A_g f'_c) \} \quad (14)$$

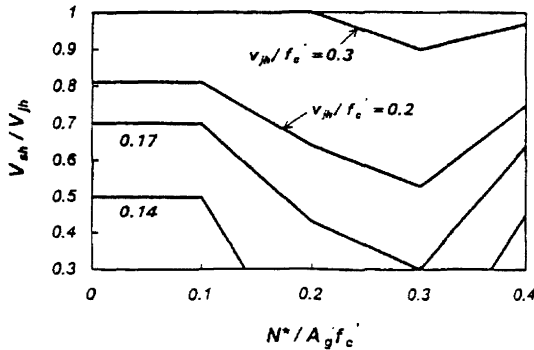
The force carried by the horizontal joint reinforcement  $V_{sh}$  is determined from Eq. 10 once  $V_{ch}/V_{jh}$  and  $V_N/V_{jh}$  have been found. Then, the amount of horizontal joint reinforcement  $A_{sh}$ , can be obtained as,

$$A_{sh} = V_{sh}/f_{yh} \quad (15)$$

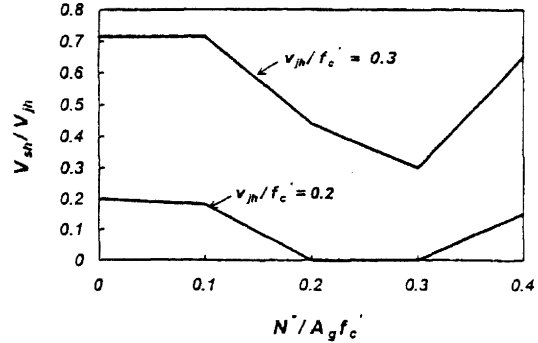
where  $f_{yh}$  is the yield strength of the horizontal joint reinforcement.

When using the approach by Eqs. (10) to (15) the following limit is recommended for the amount of joint shear reinforcement:

$$V_{sh}/V_{jh} \geq 0.4 \quad (16)$$



(a) Joints of Ductile Frames ( $\mu_s = 6$ )



(b) Joints of Limited Ductility Frames ( $\mu_s = 3$ )

Figure 14: Design charts for determining the horizontal joint reinforcement.

### Vertical joint reinforcement

Figure 15 compares the internal stress flow and the stress ratio  $f_{cs} / v_{jh}$  for two interior beam-column joints. The only difference in these joints is the presence or lack of interior column longitudinal bars. It can be seen that the inclination and magnitude of the forces carried by the struts and stress ratio  $f_{cs} / v_{jh}$  is barely modified by the presence of the column interior longitudinal bars. It appears that, in the presence of horizontal joint reinforcement, the presence of vertical joint reinforcement does not have significant influence on the joint strength. It should be noted, however, that the performance of a joint not having interior column bars is expected to be poor as a result of premature bond failure. This is because very large bond stresses, which are unlikely to develop and let alone be sustained, would be required to develop along the length of the bars clamped by the relatively small column compressive stress block.

It is the authors' opinion that, as long as horizontal joint reinforcement is provided to resist joint shear, the vertical joint reinforcement, in the form of column interior bars, is required for ensuring the anchorage of the beam longitudinal bars and not for strength purposes. For this reason the current provisions for the determining the vertical joint reinforcement in the Concrete Structures Standard (SNZ, 1995) seem adequate for design purposes.

### COMPARISON OF DESIGN APPROACHES

The horizontal joint reinforcement required by the proposed approach is compared with that required by the Concrete Structures Standard (SNZ, 1995) for interior beam-column joints of frames designed for full ductility and limited ductility

In addition, for joints of frames designed for full ductile response the joint shear stress should be limited to less than  $v_{jh} / f'_c \leq 0.25$  and for frames designed for limited ductility response the joint shear stress should be less than  $v_{jh} / f'_c \leq 0.3$ .

response. For clarity, the presence of a slab and the inertia force  $F_{EQ}$  were deliberately ignored. Furthermore, the column shear force,  $V_{col}$  was assumed equal to  $V_{jh} / 4$ .

Figure 16 (a) shows that, for joints of frames designed for limited ductility response, the proposed approach generally requires less horizontal joint reinforcement than that required by the Standard. It can be observed that a large difference in the amount of horizontal reinforcement required by the two approaches occurs in joints with  $v_{jh} / f'_c = 0.25$  when the columns are subjected to low or moderately low axial load levels.

A large difference in the amount of joint reinforcement between the two approaches is also obtained for joints of frames designed for fully ductile response when the ratio  $v_{jh} / f'_c = 0.14$ , and in particular when the ratio  $A'_s / A_s = 0.5$ , see Fig. 16 (b). The two approaches require similar amounts of horizontal reinforcement in joints of frames designed for fully ductile response when the ratio  $v_{jh} / f'_c = 0.25$  and when the ratio  $A'_s / A_s = 1$ .

### SUMMARY AND CONCLUSIONS

This paper reviewed the background to the current seismic design recommendations for interior beam-column joints of moment resisting frames in New Zealand. Historically, the provisions for the design of joints in New Zealand has been based on the diagonal concrete strut and parallel angle truss mechanisms postulated by Park and Paulay in 1975. The paper showed that the amount of horizontal joint reinforcement obtained from this model is sensitive to the bond force distribution model assumed.

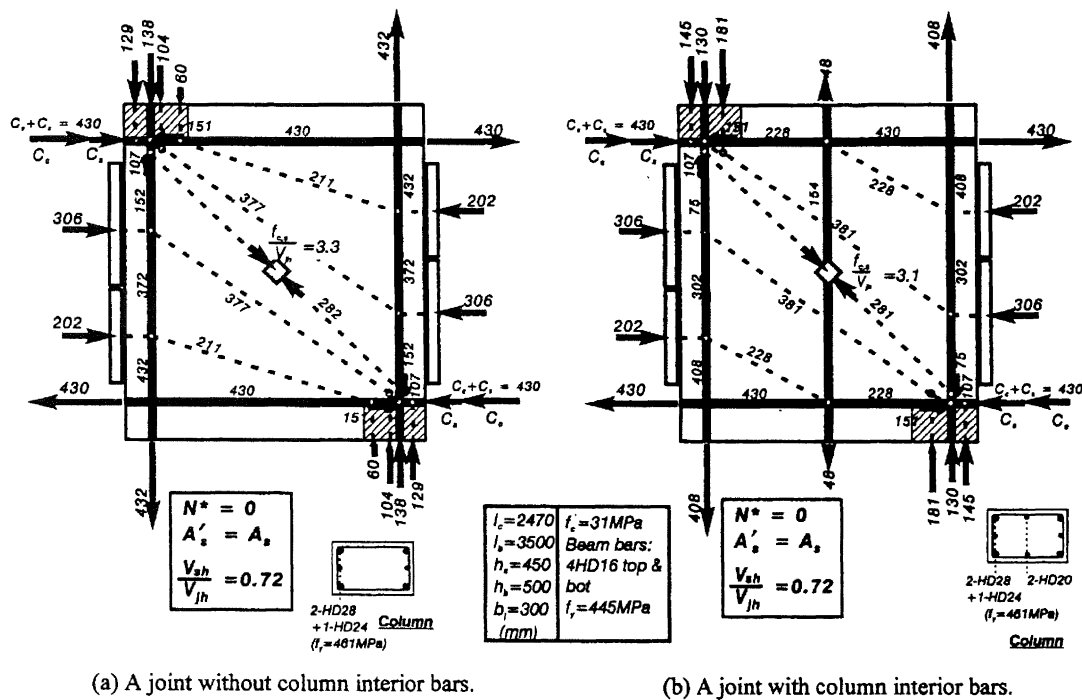


Figure 15: Comparison of the stress ratio  $v_{jh} / f_c'$  for joints without and with column interior bars.

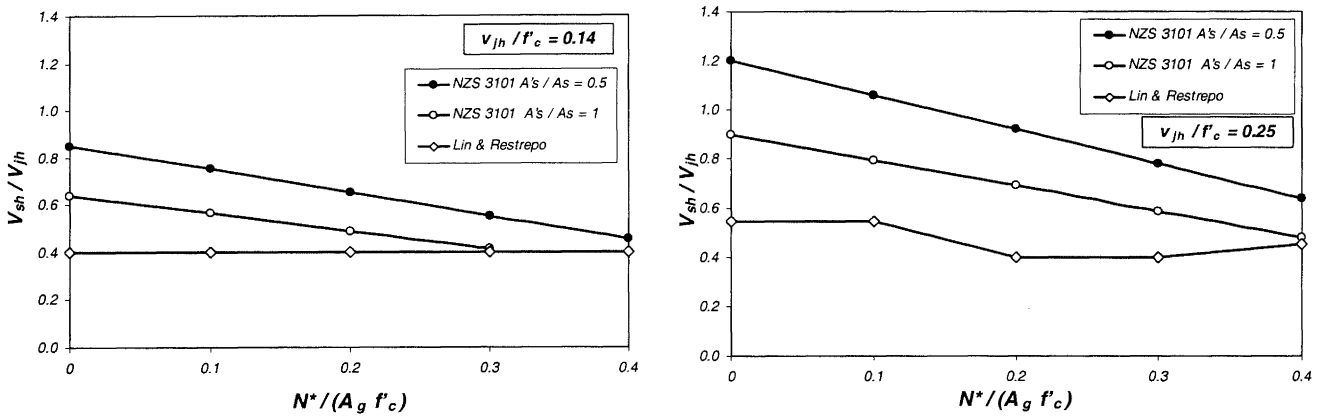
In an aim to further the understanding of the mechanisms of shear transfer in interior-beam column joints of reinforced concrete frames, this paper described a procedure for evaluating the shear strength of interior beam column joints. The procedure is based on the lower bound theorem of the theory of plasticity and uses struts and ties for obtaining the stresses acting on the diagonal compression field in the joint panel. The trends obtained from the analysis were calibrated using a database of test results. A comparison of the current design recommendations was made in light of the results obtained from the model. A simple three-component design equation was proposed in the paper.

The main conclusions derived from the paper are:

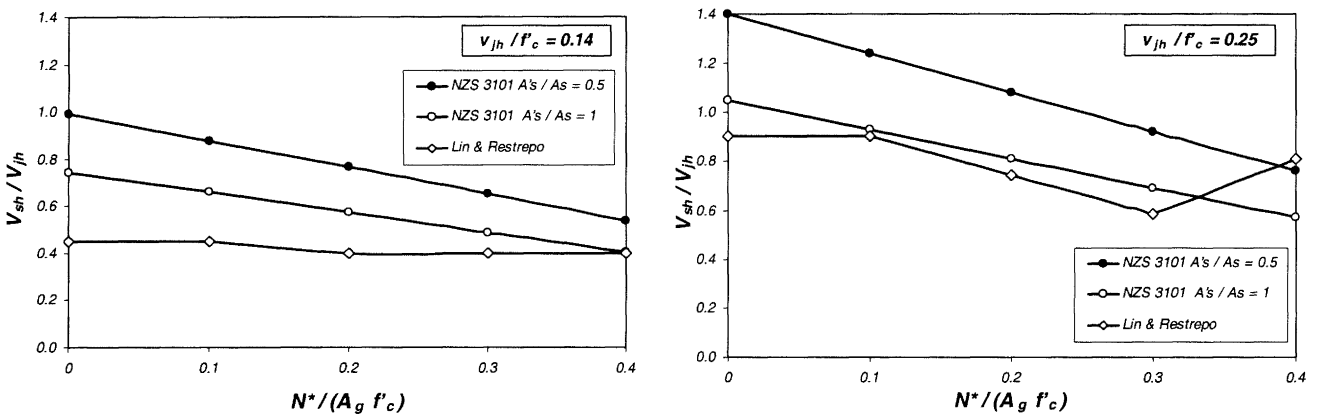
- (1) Based on the use of the lower bound theorem of the theory of plasticity, assuming that the joint reinforcement yields and applies an external stress to the joint panel, the internal force flow satisfying equilibrium can be determined with the use of a variable angle truss model. The magnitude of the internal forces can be determined using the strut-and-tie model.
- (2) The advantage of the strut-and-tie model analysis is that it can be used to conduct parametric analysis and observe behavioral trends that lead to the identification of variables that are most likely to affect the strength of joints.
- (3) Trends obtained from the parametric analysis of interior beam-column joints were used to reduce data available from past experimental work. The reduced data show a clear trend for establishing the shear strength of the

joints. In particular, it became evident that the ductility demand on the plastic hinges developing at the joint faces and the joint shear stresses significantly affect the joint shear strength.

- (4) The model predicted that column axial compression is beneficial to the joint shear strength if kept to levels below  $0.3 f_c' A_g$ . For axial compression above  $0.3 f_c' A_g$  the diagonal compression field tends to concentrate towards the diagonal of the joint, thus, becoming detrimental to the joint shear strength.
- (5) The trends observed were used to propose design recommendations for interior beam-column joints of one-way frames subject to cyclic loading.
- (6) The amount of horizontal joint reinforcement given by the proposed method was compared with that required by current Concrete Structures Standard, NZS 3101:1995. A comparison of the design requirements for determining the horizontal joint reinforcement indicates that, in general, the current design provisions for joints of ductile frames are conservative and could be relaxed.
- (7) The model also suggests that design provisions for determining the horizontal joint reinforcement in joints of frames designed for limited ductility could also be relaxed, providing that the joint shear stresses are kept below  $0.2 f_c'$ . Furthermore, it was proposed that the design of joints of frames designed for limited ductility response could be permitted if the joint shear stresses are less or equal to  $0.3 f_c'$ . Such joints, however, would require significant amounts of joint reinforcement in order to maintain the integrity of the diagonal compression stress field.



(a) Joints of frames designed for limited ductility response.



(b) Joints of frames designed for fully ductile response.

Figure 16: Comparison of approaches for determining the horizontal joint reinforcement in interior beam-column joints.

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**NOTATION**

$A_g$  = gross section area of a column  
 $A_s$  = area of longitudinal top beam bars

$A'_s$  = area of longitudinal bottom beam bars anchored in the joint core  
 $A_{sh}$  = area of horizontal joint shear reinforcement  
 $b_j$  = effective joint width  
 $C_{CT}$  = compressive force in the beam plastic hinge carried by the concrete  
 $C_{ST}$  = compressive force carried by the top beam reinforcing bars anchored in the joint core  
 $D_B$  = diagonal force carrying shear in a beam plastic hinge  
 $F_{EQ}$  = inertia force induced by the earthquake at

		the level of the diaphragm and that is carried by the column below the joint
$f'_c$	=	concrete compressive strength
$f_{c,s}$	=	average uni-axial compressive stress in the central strut of the joint
$f_{yh}$	=	tensile yield strength of horizontal joint reinforcement
$h_b$	=	overall beam depth
$h_c$	=	overall column depth in the direction of lateral loading
$K_{pv}$	=	ratio of $f_{cs}/v_{jh}$ with respect to that of the joint in which the applied column axial load is zero ( $N^*/A_g f'_c = 0$ ) and $V_{sh}/V_{jh} = 1$ ; a normalization factor to transform the joint shear stress ratio, $v_{jh,e}/f'_c$ , to $v_{jh,e}/f'_c$
$N^*$	=	column axial load
$S_c$	=	compressive force of the central strut in the joint
$T^*$	=	maximum tensile yield force of top and bottom beam bars with areas $A_s$ and $A'_s$ , respectively
$T_S$	=	tension force induced by the slab bars plus beam bars anchored outside the joint core
$T_T$	=	tension force induced by the top beam bars anchored in the joint core
$V_{col}$	=	column shear force
$V_{jh}$	=	nominal shear force across a joint
$V_{sh}$	=	joint shear force taken by the provided joint hoops
$V_N$	=	horizontal joint shear resistance due to column axial load
$V_{jv}$	=	vertical joint shear force
$v_{jh}$	=	nominal joint shear stress
$v_{jh,e}$	=	joint shear stress equivalent to a reference joint
$W_S$	=	width of the diagonal concrete central strut in the joint
$\alpha_i, \alpha_j, \alpha_v$	=	joint shear design parameters in the Concrete Structures Standard
$\alpha_u$	=	factor defining the contribution of the concrete mechanism in transferring horizontal joint shear
$\Delta_c$	=	elastic component displacement of the columns at $\Delta_y$
$\Delta_y$	=	ideal or reference yield displacement
$\Delta_u$	=	ultimate lateral displacement
$\theta_B$	=	inclination of the diagonal force carrying shear in a beam plastic hinge
$\mu_\Delta$	=	displacement ductility factor
$\mu_\theta$	=	rotational ductility factor

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