Section D

COLUMN DESIGN

J.W. Butterworth* and K.C.F. Spring**

This paper is the result of deliberations of the Society's Study Group for the Seismic Design of STEEL STRUCTURES.

1. CONTENTS

2. INTRODUCTION

The following paper looks at the seismic design of columns in braced and unbraced structural steel frames. The design of such columns will be considered for the case where the column is to remain elastic and also for the less usual case in which hysteretic energy dissipation is to take place in the column.

In view of the small amount of research that has been conducted in New Zealand on steel column design, much of the following has been adopted from results of research done principally in the USA, Japan and Europe.

Unless noted otherwise, the column section that is implied throughout is an I or box section.

3. DUCTILITY

3.1 Available Ductility in Columns

The ability of a rolled steel column to dissipate energy in a stable flexural mode through a plastic hinge at one end has been demonstrated in a series of tests reported in (1). The stability of the process was shown to depend on a number of factors, particularly lateral torsional buckling, local buckling and the level of axial load. The effect of axial load is to amplify any tendency to lateral or local buckling leading to accelerated strength degradation.

Provided local buckling is prevented by suitable limits on the cross-section geometry (see section 6), it was found in the tests described in (1) that good ductility under inelastic cyclic loading was obtained when the axial load to squash load ratio P/P_y was less than 0.5. Strain-hardening reduces the area of steel needed to carry the axial load, resulting in a small increase in flexural capacity and better energy dissipation than might be expected. Axial shortening occurs due to accumulated strain and results in considerable energy dissipation. Unfortunately this dissipation is not helpful in counteracting seismic energy input.

Test results showed that with $P/P_y > 0.5$ sharp drops in column strengths occurred.

3.2 Ductility Demand in Columns

The usual strong column-weak beam design strategy forces plastic hinges to form predominantly in the beams where reliably ductile behaviour can be readily achieved. Capacity design procedures ensure that storey sway mechanisms should not occur and that column plastic hinges will be required to contribute only a very small proportion of the total ductility demand of the structure.

Columns which are expected to hinge, typically those at ground floor level, but possibly at other levels depending on vertical accelerations and mode of vibration, should have their P/P_y and slenderness ratios limited as suggested in section 3.2, have cross-sections complying with

---

* Senior Lecturer, School of Engineering, University of Auckland, New Zealand.
** Partner, Brickell Moss & Partners, Wellington.

section 6 and be laterally restrained in accordance with section 5.

Low rise structures with long span and gravity dominated beams will generally require columns of high flexural capacity which will have naturally low P/P_y ratios. The column hinging that is almost inevitable in such structures is unlikely to create a ductility demand that cannot be readily catered for.

3.3 Recommendations

The advisability of allowing plastic hinges to form in columns is also dependent on slenderness ratio and the curvature induced by the terminal bending moments. In particular, care must be taken to avoid hinges forming away from the ends at unrestrained interior points. Compliance with rule 10.5 of NZS 3404 should eliminate potentially unstable columns due to this effect. The following recommendations are therefore made:

Column ductility	P/P_y
Fully ductile	< 0.5 and $< \frac{1 + \beta - \lambda}{1 + \beta + \lambda}$
Limited ductile	< 0.7 and $< \frac{1 + \beta - \lambda}{1 + \beta + \lambda}$
Elastic	< 1.0

Figures 1a and 1b illustrate the bounds to the regions outside which plastic hinging should not be permitted to occur.

4. EFFECTIVE LENGTHS

4.1 Introduction

The concept of effective length was introduced into elastic column theory as a simple means of applying formulas developed for pin-ended columns to columns with other end conditions. In the case of an isolated column the effective length is readily understood to be the length of a pin-ended column which has the same critical load, cross-sectional and material properties as the column being studied.

Thus if KL is the effective length and P_{cr} is the critical load of the column,

$$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$$

When the column can not be effectively isolated from its parent structure the effective length is taken to mean the length of an equivalent pin-ended column whose Euler load equals the axial force in the real column at the moment when the structure as a whole reaches its critical load.

Rearranging the equation above

$$K = \sqrt{P_E/P_{cr}}$$

where P_E is the Euler load of the column and P_{cr} is the axial load in the column at the frame critical load.

4.2 Elastically Responding Structures

Effective length factors of columns in elastic structures may be determined most accurately by carrying out a stability analysis of the structure and substituting in the relationship above. Alternatively, there are many approximate methods, especially for regular framed structures, such as those set out in appendix E of NZS 3404 in the form of charts. Various refinements and corrections to these charts are available to allow for effects such as varying column sizes and loads, flexible connections and column base rotations (2,3,4). The appendix to this paper contains an example illustrating the conservatism of the chart approach for a structure with varying column loads across a storey.

Note that if a second order analysis is used to determine column actions then the effective length of each column need not be taken greater than unity, as to do so would effectively magnify the actions above their true values.

4.3 Inelastically Responding Structures

In the case of limited ductile or fully ductile frames there is a possibility of inelastic response under factored loading and a certainty of it during capacity design (although not necessarily in the columns). In either case it is necessary to ensure that elastic or elasto-plastic stability effects do not prejudice the ability of the structure to achieve the desired strength state or energy dissipating mechanisms.

Effective length factors are the same for both elastic and inelastic columns only when the column acts as an independent member with end conditions that may include hinged, free or fully fixed, with or without lateral sidesway restraint. When the column is part of a continuous frame, the relationship between its rotational stiffness and the stiffness of adjacent restraining members is variable with respect to both column load and the effective tangent modulus of the material. It follows that K cannot be determined a priori as in the elastic range for which column stiffness varies only with axial load.

Yura (5) has recommended the use of a modified value of stiffness ratio G

$$G = \frac{\Sigma(E_T I/L)_{columns}}{\Sigma(EI/L)_{beams}}$$

where E_T is the tangent modulus of the column material. The standard alignment charts are then used in an iterative procedure to obtain the inelastic K value for the column.

Le Messurier (6) has proposed an alternative and more accurate approach to determine K factors for columns in sway frames. His approach, in common with Yura's, requires no more than a first order analysis of the frame.

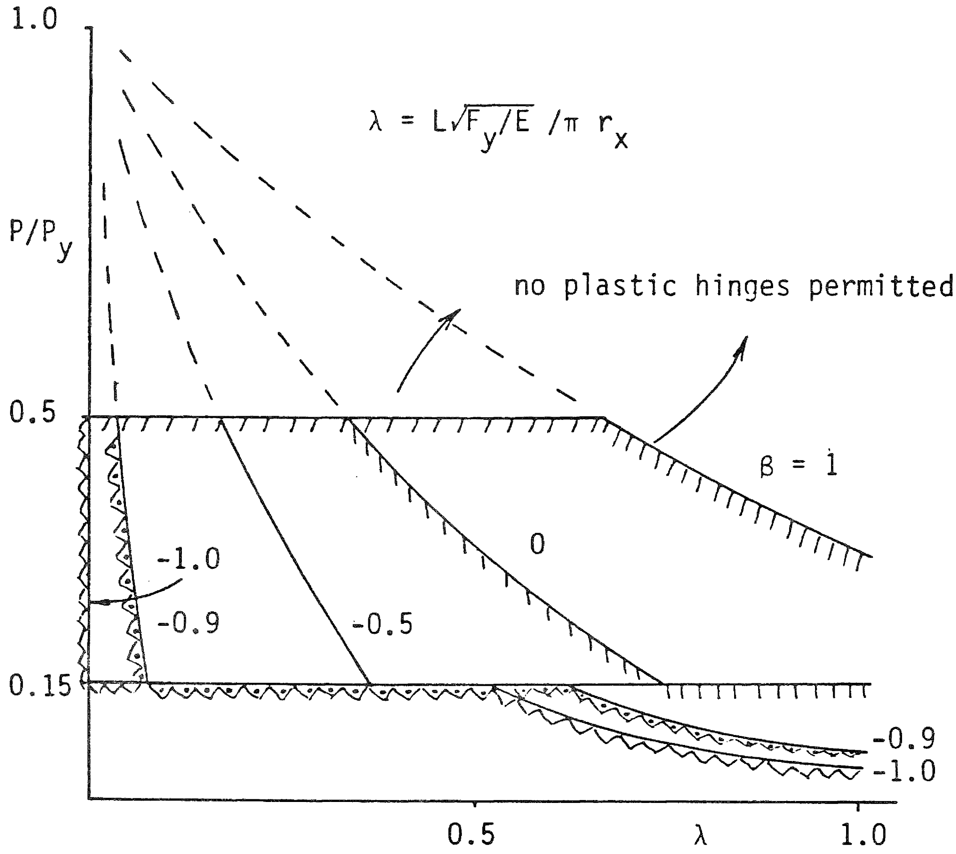


Figure 1a Fully Ductile

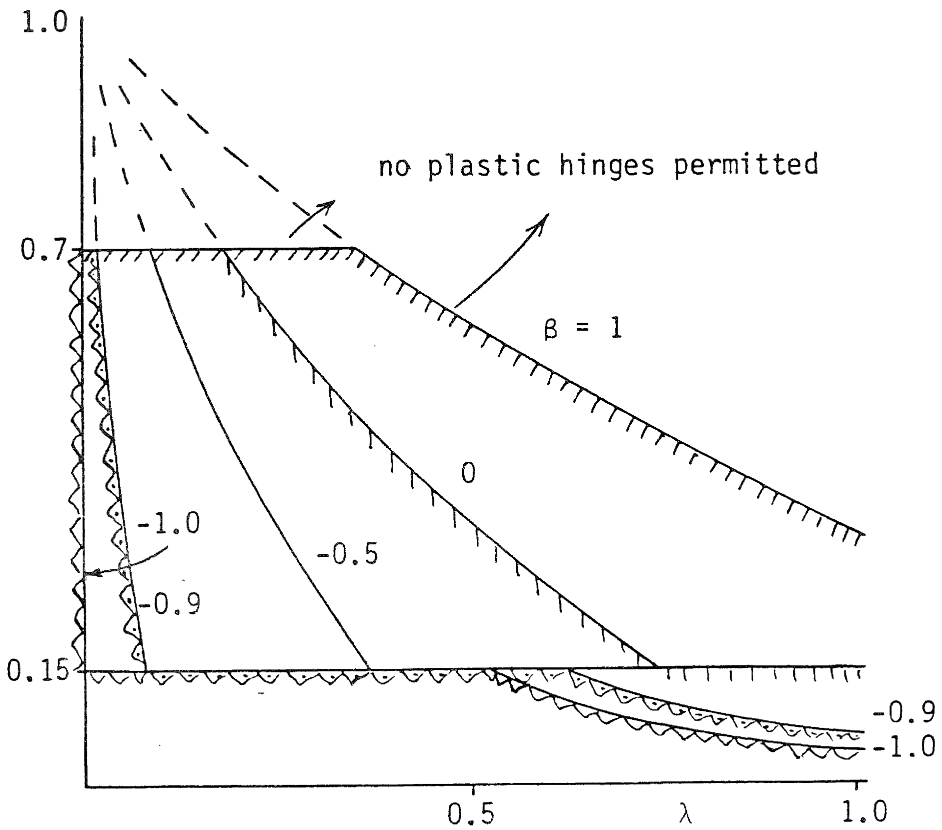


Figure 1b Limited Ductility

4.4 Effect of Drift Limitation

Interstorey drift limitation imposes a lateral shear stiffness constraint on a frame for the purpose of limiting damage to non-structural components. Teal (7) argues that any reasonable control of drift will also ensure structural stability (sidesway mode), even in the inelastic range, and that real problems of instability due to p-delta effects in seismic test frames are few and are connected only with large enough forces to cause extreme inelastic response.

This view is supported by the draft revision of the SEAOC code (8) in which it is proposed that frames designed to the code drift limit for seismic zones 3 and 4 may be considered "braced against joint translation". The effective length factor K for columns in the plane of seismic bending forces can therefore be taken as 1.0.

A comparison of the "average" building shear stiffnesses implied by various code drift limits for a ductile, moment resisting frame founded in non-flexible soils is presented in table 1. Shear stiffness is calculated in dimensionless form as

$$\text{shear stiffness} = Vh/Wd,$$

where V = seismic base shear, h = storey height, W = seismic weight, and d = interstorey drift.

Code	Period	1.2 secs	0.5 secs
NZS 4203	Zone A Zone C	7.5 7.5	15.0 15.0
UBC (9)	Zone 3 Zone 4	9.1 14.1	12.2 18.9
SEAOC (8)	Zone 3 Zone 4	8.3 11.1	14.9 19.9

Table 1. Drift limit-implied shear stiffnesses, (Vh/Wd)

It can be seen from the table that computed deformations under the seismic loadings of NZS 4203 would need to be increased by about 30 percent to give shear stiffnesses comparable with those of the UBC or draft SEAOC code provisions.

4.5 Recommendations

Whilst greater availability of efficient and reliable second order analysis computer programs should result in a decline in the need for approximate methods of effective length determination, a short term need remains for simplified methods. Methods such as those of Yura (5), Wood (10) and Le Messurier (6) are suggested as amongst the best available for the inelastic cases, although further research is still needed if they are to be applied with confidence.

The adoption of K = 1 on the basis of meeting the drift criterion of NZS 4203 is recommended provided deformations are calculated on the basis of an accurate structural model (including joint and shear deformations, for example) and are multiplied by a factor of 1.3. Further research to support this approach is needed.

5. LATERAL BUCKLING

If a column has differing bending stiffnesses about each of its principal axis and bending moments are applied to the stronger of the two axes, then it may not be able to develop its in-plane rotational and bending capacities before failure occurs due to lateral torsional buckling.

In order to maintain in-plane rotation and moment capacities, sufficient bracing must be provided to prevent lateral deflection and twisting occurring.

5.1 Recommendations

5.1.1 Elastically responding structures

Lateral bracing should conform to the requirements of NZS 3404, chapter 5.

5.1.2 Limited and fully ductile structures

The provisions of section 10.9 of NZS 3404 are intended to ensure that a member does not buckle laterally before achieving the strains of up to the strain-hardening level (11). This should result in the section being able to sustain sufficient inelastic rotation to justify limited ductile categorisation.

For the fully ductile case it is suggested that the rotation ratio R be increased from the value of 10 used in plastic design to 24 in order to allow adequate rotational ductility.

If L_y denotes the length of column over which the compression flange is fully yielded, then the spacing of restraints to the critical flange is governed by whether L_y is greater or less than 640 αa, where $\alpha = 1.5/\sqrt{1 + R/8}$ and $a = r_y/\sqrt{F_y}$.

Section 10.9 of NZS 3404 allows L_y to be calculated as the length of column over which the bending moment $M \geq 0.5\ M_{pc}$. In the presence of axial load the 0.85 factor used may be unconservative as it corresponds to the yield moment in a member with zero axial force and a typical I-beam shape factor. When axial force is present the compression flange yields at a lower moment which is given by

$$M_{yc} = (0.85/1.18) M_{pc}$$
$$= 0.72\ M_{pc}$$

Allowing a small margin between first yield and full yielding of the flange leads to a suggested value of 0.75 M_{pc} as the basis for calculating L_y. For $P/P_y < 0.15$ the original value of 0.85 is satisfactory.

Table 2 sets out the appropriate restraint spacing for the fully and limited ductile cases.

6. LOCAL BUCKLING

The effects of local buckling on the behaviour of columns have been illustrated by a number of authors (12). In general the occurrence of local buckling in a section will reduce its strength and its capacity to absorb energy hysteretically. The strength and rotation capacity at a plastic hinge in a column are strongly dependent on the ability of the flanges and web to achieve strains of at least strain-hardening level prior to the initiation of local buckling.

Table 3 sets out the suggested limits on cross-section geometry which should ensure that the various member ductility levels are not impaired by local buckling. The values generally follow Section C (13) recommendations. For web geometry the axial load effects impose more restrictive values than flexural loading.

7. MOMENT-AXIAL LOAD INTERACTION

7.1 Introduction

There is currently a high level of research and discussion in the area of moment-axial load interaction in columns, especially in relation to the change to limit states or LRFD codes in several countries. Chen and Lui (14) review the current and proposed US design criteria and Chen and Atsuta (15) provide an extensive background to the problem.

The linear interaction equations provided in NZS 3404, section 14.5, although conservative in some situations are recommended to be retained until the next major revision of steel design code philosophy. The equations given in NZS 3404 contain a number of errors which have been corrected in the version given below.

7.2 Axially Loaded Members

The maximum load capacity of members not subjected to bending shall be

$$P_{ac} = \frac{A_s F_{ac}}{0.6}$$

where F_{ac} is determined from NZS 3404, chapter 6, for the appropriate effective length.

7.3 Combined Axial Load and Moment

7.3.1 At a support

The effect of axial tension or compression on the moment capacity of the I-member shall be defined by

Bending about the major principal axis
(i) For $P/P_y < 0.15$, $M/M_p \leq 1.0$

(ii) For $P/P_y \geq 0.15$, $P/P_y + M/(1.18\,M_p) \leq 1.0$

Bending about the minor principal axis
(i) For $P/P_y < 0.4$, $M/M_p \leq 1.0$

(ii) For $P/P_y \geq 0.4$,

$$(P/P_y)^2 + M/(1.18\,M_p) \leq 1.0$$

Bending about both principal axes
(i) For $P/P_y < 0.15$, $M_x/M_{px} + M_y/M_{py} \leq 1.0$

(ii) For $P/P_y \geqslant 0.15$,

$$P/P_y + M_x/(1.18\,M_{px}) + M_y/(1.18\,M_{py}) \leq 1.0$$

7.3.2 Away from a support

The effect of axial compression on the moment capacity of a column shall be defined by

Bending about the major principal axis
(i) For $P/P_{ac} < 0.15$, $P/P_{ac} + M/M_{px} \leq 1.0$

(ii) For $P/P_{ac} \geq 0.15$,

$$P/P_{ac} + \frac{C_{mx}}{(1 - P/P_{ocx})M_{ox}} \leq 1.0$$

Bending about both principal axes
(i) For $P/P_{ac} < 0.15$,

$$P/P_{ac} + M_x/M_{ox} + M_y/M_{py} \leq 1.0$$

(ii) For $P/P_{ac} \geq 0.15$

$$P/P_{ac} + \frac{C_{mx}M_x}{\left(1-\frac{P}{P_{ocx}}\right)M_{ox}} + \frac{C_{my}M_y}{\left(1-\frac{P}{P_{ocy}}\right)M_{py}} \leq 1.0$$

8. SHEAR

In general shear is not a dominant action in the design of a column. However, should a column mechanism be critical in determining the strength of a structure, then it may be necessary to investigate the interaction of bending, shear and axial load. In such cases the interaction formula of Neal (16) is recommended:

$$M/M_p + (P/P_y)^2 + \frac{(V/V_p)^4}{[1-(P/P_y)^2]} \leq 1.0$$

9. NOTATION

A_s	Effective area of column cross-section
b	Outstand of flange beyond connection to web (I columns), or clear distance between sides of box column
d	Clear depth of column web
d	Interstorey drift
E	Elastic modulus in tension and compression
E_T	Tangent modulus in compression
F_{ac}	Maximum permissible compressive stress in a column in the absence of axial load
F_y	Yield stress of column material (in MPa units when used in dimensionally inconsistent expressions)
G	Ratio of inframing beam stiffnesses to column stiffness at beam-column joint
h	Storey height
I	Second moment of area about a principal axis
K	Factor by which column length must be multiplied to give effective length
L	Actual length of a column
L_y	Length of column over which compression flange is fully yielded
M	Bending moment in column

M_{ox} Calculated maximum moment capacity of column in absence of axial load (may be governed by lateral buckling or plasticity) about the major axis

M_p Fully plastic moment capacity of a column section in the absence of axial load

M_{pc} Fully plastic moment capacity of a column section in the presence of axial load

M_{yc} Moment to cause first yield in a column in the presence of axial load

P Axial load in column

P_E Euler load of pin-ended column

P_{oc} Elastic critical load of column with length = effective length

P_y Squash load of column (A_sF_y)

R Ratio of rotation at a plastic hinge to relative elastic rotation of far end of column segment containing the hinge

r Radius of gyration of cross-section about a principal axis

T Flange thickness

t Web thickness

V Seismic base shear force on a structure

V Shear force at plastic hinge in a column

V_p Shear force to cause plastic shear hinge in absence of axial load or bending moment

W Seismic weight of a structure

β Ratio of smaller column end moment to larger, measured in the same rotational direction

λ Normalised slenderness ratio $(L/r_x)\sqrt{F_y/E}$

10. REFERENCES

(1) Popov, Egor P., Bertero, Vitelmo V., and Chandramouli, S., "Hysteretic Behaviour of Steel Columns", EERC Report No. 75-11, September 1975.

(2) Chu, K.H., and Chow, H.L., "Effective Column Length in Unsymmetrical Frames", IABSE, Vol. 29-1, 1969.

(3) De Falco, F., and Marino, F.J., "Column Stability in Type 2 Construction", AISC Eng. J., Vol. 8, No. 2 (April 1966).

(4) Galambos, Theodore V., "Influence of Partial Base Fixity on Frame Stability", Trans. ASCE, Vol. 126, part 2, paper 3256, pp. 929-969.

(5) Yura, J.A., "The Effective Length of Columns in Unbraced Frames", AISC Eng. J., Vol. 8, No. 2(April 1971).

(6) Le Messurier, W.J., "A Practical Method of Second Order Analysis", AISC Eng. J., Vol. 14, No. 2 (1972).

(7) Teal, E.J., "Seismic Drift Control Criteria", AISC Eng. J., Vol. 12, No. 2(1975).

(8) SEAOC, "Recommended Lateral Force Requirements and Commentary", Draft Revision, 1984.

(9) "Uniform Building Code", International Conference of Building Officials, California, 1982.

(10) Wood, R.H., "Effective Lengths of Columns in Multi-storey Buildings", in three parts, Struct. Eng., Vol. 52, Nos. 7, 8 and 9, July, August, Sept., 1974 .

(11) Lay, M.G., "Source Book for the Australian Steel Structures Code - AS1250", AISC, 1975.

(12) Mitani, Isao, Makino, Minoru, Matsui, Chiaki, "Influence of Local Buckling on Cyclic Behaviour of Steel Beam-Columns", Proc. 6th World Conf. on Earthquake Eng., New Delhi, India, Vol. 3, 1977, pp. 3175-3180.

(13) Walpole, W.R. and Butcher, G.W., "Beam Design", Section C - NZNSEE Study Group on the Seismic Design of Steel Structures, Bull. NZNSEE, Vol. 18, No. 4, Dec. 1985.

(14) Chen, W.F., Lui, E.M., "Stability Design Criteria for Steel Members and Frames in the United States", J. Construct. Steel Research, 5, 1985.

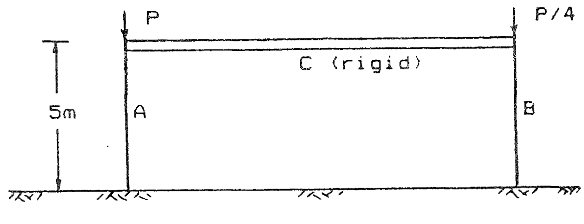
(15) Chen, W.F., Atsuta, T., "Theory of Beam-Columns", 2 Vols., McGraw-Hill, 1977.

(16) Neal, B.G., "The Effect of Shear and Normal Forces on the Fully Plastic Moment of an I Beam", J. Mech. Eng. Sci., Vol. 3, 1961.

11. APPENDIX

The following example illustrates the potential conservatism of the code chart approach to effective length determination.

Consider the frame shown below and assume that adequate out-of-plane restraint is provided. We wish to determine the effective length and column reliable loads as governed by in-plane frame buckling.



A and B

I : 4564000 mm^4

r_x : 88.1 mm

P_E : 3604 kN

A stability analysis gives the critical load of the frame as

$$P_{cr} = 1434 \text{ kN}$$

so $K_A = \sqrt{P_E/P_{cr}}$

$= 1.59$

and $K_B = \sqrt{P_E/0.25\ P_{cr}}$

$= 3.17$

For column A: $KL/r = 90$

giving $P_{reliable} = 1362$ kN

For column B: $KL/r = 180$

giving $P_{reliable} = 274$ kN

Column B governs, so maximum reliable load on frame is

$P = 4 \times 274$

$= \underline{1096\ \text{kN}}$

The code chart approach to this problem gives $K = 2$ for both columns.

The maximum reliable load in column A and column B is

$P_{reliable} = 632$ kN

Column A therefore governs now and the maximum reliable load on the frame is

$P = \underline{632\ \text{kN}}$

Table 2. Spacing of lateral restraints

	Fully ductile	Limited ductility
R	24	10
α	0.75	1.0
	$L_y < 480\ a$	$L_y < 640\ a$
placing of critical flange restraint	within or at one end of L_y	within or at one end of L_y
adjacent restraint	$\leq 720\ a$	$\leq 960\ a$
	$L_y \geq 480\ a$	$L_y \geq 640\ a$
spacing of critical flange restraints	$\leq 480\ a$	$\leq 640\ a$

Table 3. Section geometry limits

Member ductility	Fully ductile	Limited ductility	Elastic
b/T (I column flange)	$\leq 120/\sqrt{F_y}$	$\leq 136/\sqrt{F_y}$	$\leq 256/\sqrt{F_y}$
b/T (box flange)	$\leq 500/\sqrt{F_y}$	$\leq 512/\sqrt{F_y}$	$\leq 560/\sqrt{F_y}$
d/t (web)	$\leq 500/\sqrt{F_y}$	$\leq 512/\sqrt{F_y}$	$\leq 560/\sqrt{F_y}$