PRELIMINARY TESTS ON AN ENERGY ABSORBING ELEMENT
FOR BRACED STRUCTURES UNDER EARTHQUAKE LOADING
R.G. Tyler*

SYNOPSIS:

While quite slender cross bracing may suffice to maintain rigidity in a braced structure under static loading conditions, during an earthquake progressive slackness develops in the bracings, resulting from overstrain in tension, which gives rise to increasing horizontal movements and degradation. In this paper tests are described on devices, fabricated from round bars and located at the centre of the bracings, which allow repetitive overstrain to occur without the development of slack. It is postulated that, for a stable condition of cyclic overstrain, the device shall be of the same shape as the frame to be braced, and most usually fabricated in steel rod or bar. Successful tests were carried out on such a device in a square frame.

1. INTRODUCTION

Cross bracings are often employed in structures to resist in-plane shear forces. This has the particular advantage that, during load reversals, one member of a pair of cross bracings will always be in tension, thus allowing compression buckling to be ignored. Thus the bracings will often consist of quite slender steel bars. Sometimes a ring made of steel of rectangular cross section will be used at the centre of the braces (Fig. 1), as this allows adjustment of the bracing lengths to achieve the desired geometry. The ring could behave as an energy absorbing element, although for the usual design, with a ring of flat strip pierced to take the rods, it could have a short fatigue life.

While the usual design of cross bracings behaves satisfactorily under predictable loading conditions, during earthquakes severe cyclic loading will produce progressive overstrain in tension, which results in the development of slack. Under these conditions, energy absorption only occurs at the extremes of travel as further overstrain develops[1]. In contrast, the devices which were the subject of the tests described in this paper, promote a stable condition of cyclic overstrain.

A device of rectangular shape (Fig. 2) was first suggested by consulting engineers David Smith and Robert Henry of Auckland as a means of achieving energy absorption in a rectangular frame by overstain in bending at points a b c and d. Under dynamic loading the device distorts first to a parallelogram in one direction, and then in the other, and so on. Smith and Henry carried out static tests on two prototypes, one of which was used in a building construction. In general it was stipulated that the devices should be geometrically similar to the framework in which they were incorporated as the two parts of each diagonal are then co-linear. Because of their potential usefulness in structures such as towers the author decided to carry out tests at the Physics and Engineering Laboratory to prove stability under earthquake loading, as the

devices would be required to withstand rapid stress reversals.

Referring to the rectangular frame illustrated in Fig. 2, the diagonal members are held by single nuts on the inside of the inner frame, as shown for the square frame tested in Fig. 8, so that it is not possible for compression to develop in the diagonals. On application of the force Q, Fig. 2, tension develops in diagonal BD, and deformation of the inner frame causes the diagonal length ac to shorten more than the overall distance AC on the main frame, with the result that diagonal AC also goes into tension, producing self-straining forces which cause additional load at the jack in excess of that required to cause overstrain at points a b c and d. At the same time the shortening of diagonal ac requires an increasing force in diagonal BD to continue the plastic yielding at points a b c and d, which again throws additional load on the jack. As the inner frame is severely deformed, both effects call for large increases in the jacking load giving a resistance similar to a snubbing action. The two effects are considered in detail, for the square frame tested, in Appendix 2.

A device of this type will only perform satisfactorily where the bracings are loaded by dynamic horizontal forces oscillating about zero load. Should there be a permanent dead loading component then the effect of an earthquake will be to cause a steady movement in one direction on overstrain and a consequent locking up of the device in that direction. With no dead load forces present the device can finish up with a permanent set following an earthquake, as can any structure, and this can be taken out by jacking back to a central position. If the frame, without bracing, resists some horizontal loads, this resistance will provide a centering force for the braced structure and minimize permanent set.

2. TESTS ON A RING DEVICE

2.1 Test details

While it was thought that a ring type would probably suffer from the disadvantage

*Physics and Engineering Laboratory, DSIR, Lower Hutt, New Zealand.
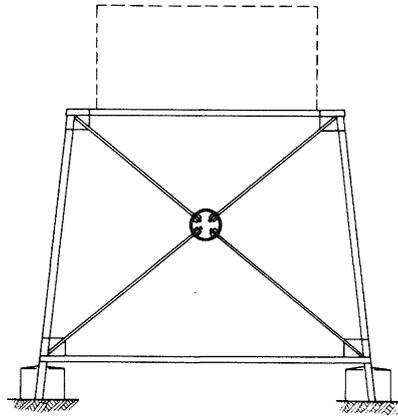
Fig. 1
Use of a ring to achieve
length adjustment in a
framed structure.

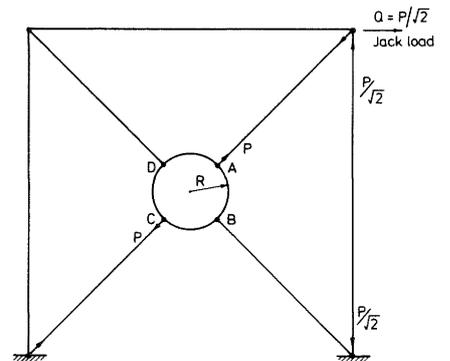Fig. 4 (right)
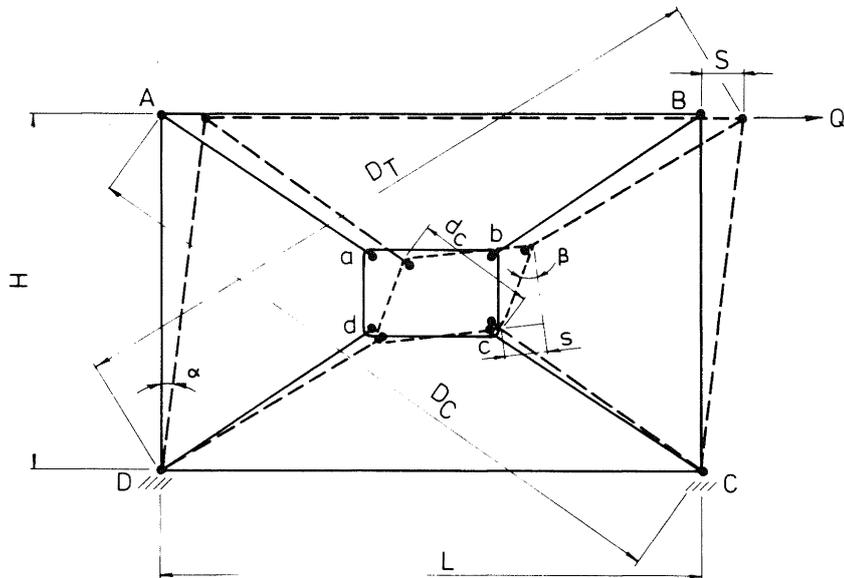Primary loading of a
circular ring device.

Fig. 2 (below)
Deformation of
rectangular device.

that the curved sections between the
loading points would tend to straighten,
it was decided nevertheless to investigate
a pair of round rings of mean diameter
610 mm, fabricated from 25 mm round
bar tested in a pin-jointed square frame,
Fig. 3, of 4 m side, in a horizontal
position, so that slackness was indicated
by sagging of the braces and rings.
The loading jack had a 200kN capacity with
a stroke of + 150 mm.   The rings were
formed from bright drawn mild steel, with
welds at points of zero moment between the
load points.   No heat treatment was
carried out.

Initially, for small displacements
and applied jack load Q, Fig. 4, the
loading applied to the ring is the
bracing load P, where P $=\sqrt{2}Q$, with
increasing deformation of the ring,
however, tension could develop in the
other diagonal as the circle stretched to
an ellipse.   In practice however, it
was found that after about 20 cycles at
a stroke of $\pm$ 100 mm, slackness developed
and the rings dropped to the floor.   The
hysteresis loop (Fig. 5a) indicates
the slack by the necking down at the centre.
The nuts were then tightened and testing
continued.   Further tightening was
needed periodically until failure occurred
in one ring of the pair after about 100
cycles of loading, the speed of loading
being up to 1 Hz.   The hysteresis loop
obtained later in the testing is shown
in Fig. 5b, when some stability had
developed;   the degree of deformation of
the ring on failure is shown in Fig. 6.

2.2 Ductility Characteristics

It is shown in Appendix 1 that the
bending moment at points A and C (Fig. 4)
under elastic conditions is greater than
that at points B and D for the loading
position shown.   Thus the ring yields
at A and C first with greater deformation
then at B and D.   On reversal of the
cycle yield will first occur at B and D
and the plastic deformation which has
already taken place at A and C will
not be corrected.   Thus a progressive
deformation takes place which caused the
device to drop to the floor.

The calculated jack load at a first
yield stress of 275 MPa was 6.1 kN
(Appendix 1) and the hysteresis loop,
Fig. 5, indicates that yield had occurred
when this load was reached.

In Appendix 1, the jack force Q_D at
zero displacement on the hysteresis
loops (Fig. 5b) is also calculated in the
manner perviously carried out for steel
damping devices[2,3].   The value of Q_D (4)
is used in design work[4].   The calculat-
ion is for a surface strain of $\pm 3\%$ at the
four points of plastic bending, when
the plastic yield stress f_p using the
conventional plastic theory of bending
is taken as 350 MPa, the increase in
stress above the yield value of 275 MPa
being attributable to work hardening.
The geometry of the ring was assumed to
be unchanged, i.e. differences would not
greatly modify the value of the plastic
moments.   The calculated value of Q_D
was 16.9 kN.   The loops shown in

Fig. 5 show that Q_D reached about 20 kN
at the maximum stroke of $\pm$ 140 mm, when
the loop shape indicated that the surface
strain would have been $\pm$ 2-3%.

2.3 Comments

In conclusion it can be said that
the circular device would probably work
reasonably well in a practical application
for a small number of cycles of loading
and this would be assisted if the diagonal
rods were pretensioned.   It is not,
however, to be preferred to the square
device tested later.

3.   TESTS ON A SQUARE DEVICE

3.1 Test details

As the 4 m square floor frame was
available from the tests on the circular
device, it was decided to proceed with
testing a device of square shape, with
a view to proceeding to devices of
rectangular shape later.

Accordingly, a square device was
made up from two bright drawn mild
steel rods, 25 mm diameter, having a
side L equal to 610 mm (Figs. 8-10).
Welds were arranged to occur at points
of zero bending moment and the corners
were bent hot, thus producing a normalising
effect similar to that in a hot-rolled
steel with a favourable characteristic
under repetitive overstrain[2].   The
saddles used for the circular device,
having a radius of curvature of
305 mm were used unmodified for the
square device.   This produced an
indentation at points NN (Fig. 8)
but was not otherwise detrimental.   Ideally
the saddles should be designed to the
curvature of the bars at the corner of
the square.

A set of hysteresis loops for
increasing shear displacement of the frame
up to $\pm$ 142 mm is shown in Fig. 11.   The
peaking up of the jack loading for
displacement above 100 mm is clearly
shown, thus indicating a locking-up
effect in the device.

3.2 Ductility characteristics

Up to the time of first yield at
points a, b, c and d (Fig. 7), when
deflections are not large, the loading
condition is as shown, with the jack
load Q producing a force P in a single
diagonal, with the other one unloaded.
For this condition the elastic moments
at the corners of the frame, both for
small and large deflections, are all
equal (Appendix 1) so that any overstrain
occurring at a corner resulting from
tension in one diagonal will be reversed
when tension develops in the other, ie.
there is a stable condition of repetitive
overstrain under dynamic loading.

The calculated load Q at first yield,
based on a yield stress of 275 MPa, is
5.58 kN (Appendix 1), while the hysteresis
loops (Fig. 11) indicate that yield had
occurred at 6 kN, which is fair agreement.

The calculation of the jack force Q_D
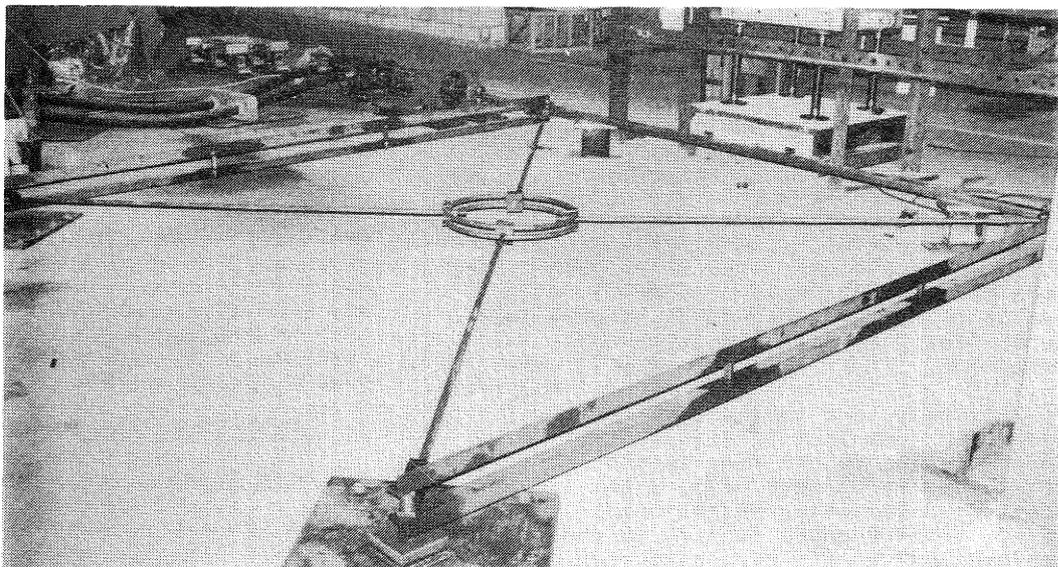
Fig. 3(a)   Round device in testing frame
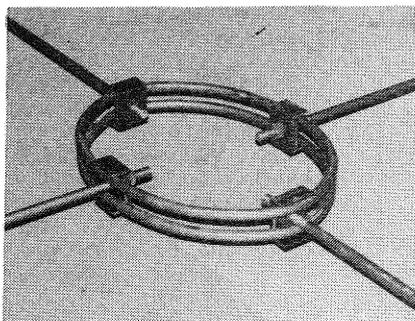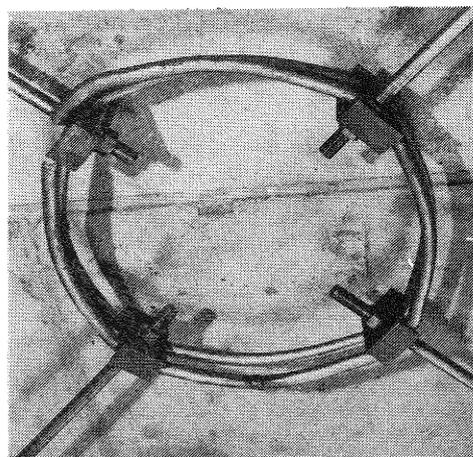   (Dynamic jack in top of picture)

Fig. 3(b)
Elastic loading of ring
device, top left to bottom
right.

Fig. 6
Failed ring device.

for zero displacement on the hysteresis loops follows that for circular section (Appendix 1) and on the assumption of small deflections, the value was found to be 13.2 kN at a plastic yield stress of 350 MPa and $\pm 3\%$ strain, which compares with a measured value of about 15 kN ± 142 mm of deflection, which is a fair agreement.

The overall ductility factor measured as the ratio of deflection at maximum stroke to deflection at first yield is about 10:1 (Fig. 11).

3.3 Locking-up effects

The peaking-up of the hysteresis loops is caused by the change in shape of device from a square to a rhombus (Fig. 2), when two separate effects occur:

(1) The incompatibility of the diagonal lengths of the outer and inner frames with increasing deflection causes self-straining forces which are calculated in Appendix 2. For a particular position of the jack these self-straining forces may be added to those causing plasticity by the principle of superposition. Resolution at the joints of the framework shows that it cannot withstand the self-straining forces without the additional forces Q' and R being imposed on the jack (Fig. 12). In the calculations the incompatibility is taken as a gap G in the shortened diagonal of the main frame for the particular geometry, a force X on the diagonal being necessary to close the gap by loading all the members by direct forces. The calculations show that the gap is closed mainly from the extensions of the 25 mm diameter diagonals of the main frame, which have comparatively small cross-sectional areas. Stiffness rates to close the gap G are tabulated in Table A2.2, for increasing displacements and plotted in Fig. 13, which shows the peaking-up effect evident in the measured loops.

(2) The force required to cause plasticity increases as the rhombus contracts in width because of the reduction in moment areas. The rhombus will have zero width at a frame displacement of 520 mm when the force required to cause plasticity will have risen to infinity. The force increases Q" arising from this effect are calculated in Appendix 2 and plotted in Fig. 13 based on an initial value Q_D = 15 kN taken from Fig. 11. The rise in force is seen to be a steady one with little peaking up by the time the maximum jack displacement is reached.

The total locking up effect caused by effects 1 and 2 is also plotted in Fig. 13. Evidently the peaking up is mainly caused by the incompatibility in the diagonal lengths, as, at the maximum stroke of 142 mm, effect 1 contributes about 7.2 kN, and effect 2, 2.2 kN. The observed increment in peaking up (Fig. 11) is about 7 kN.

It is pointed out in Appendix 2, that the reaction R at right angles to the jack axis will not be provided in the free condition of a structure under earthquake attack, so locking up will be provided by bending at the joints only, which may

lessen the effect. This could be investigated by a model test on a shaking table.

3.4 Life of the device

In the tests so far carried out, one set of bars failed after about 200 cycles, mostly at $\pm$ 100 mm stroke at frequencies up to 1 Hz. A further set was made up for demonstration purposes and so far this set has not failed after about 150 cycles at various stroke lengths. The nuts in the device were tightened with normal spanner to produce a preload prior to the start of testing and thereafter no further tightening was necessary. The device behaved consistently at fast rates of loading up to about 1 Hz and a cine film was prepared of its performance. Because the life was in excess of 100 cycles it is likely that the maximum value of surface strain was less than $\pm$ 3% at $\pm$ 100 mm stroke.

3.5 Comments

Evidently the device as fabricated would be best worked at a stroke of about $\pm$ 100 mm, before further deformation causes the loops to peak up. If a design life of 100 cycles is designed for as previously suggested[2,3] then the bar size could be increased slightly for the given geometrical configuration to give a surface strain of about $\pm$ 3%. Clearly all members should remain elastic within the working load, apart from the yield in bending at the corners of the device. If large deflections are contemplated then the forces in the members arising from secondary effects should be assessed to avoid overstrain in the diagonals.

Further work needs to be done to check the behaviour of rectangular devices and relate bar diameter to size of the device, and stroke, for the required strain of $\pm$ 3%. For instance, if a short-stroke energy absorber is needed for minimum interstorey drift, then the overall size of the device would need to be reduced to induce overstrain at small deflections of the frame.

4. CONCLUSIONS

The tests showed that circular devices were not ideal in that slack developed in the diagonals of the frame. The square device, on the other hand, performed consistently for up to 200 cycles at 1 Hz for maximum shear deformations of $\pm$ 142 mm in the 4 m frame, with no development of slack. If large deflections are contemplated in parctical applications, then the effect of the self-straining action should be assessed to avoid overstrain in the diagonal members.

As the use of such a device would appear to be an extremely reliable way of ensuring continuing frame action under earthquake attack, the work should be extended to rectangular devices in rectangular frames to establish their behaviour for various ratios of length of the sides; locking-up effects in practical structures should also be investigated.

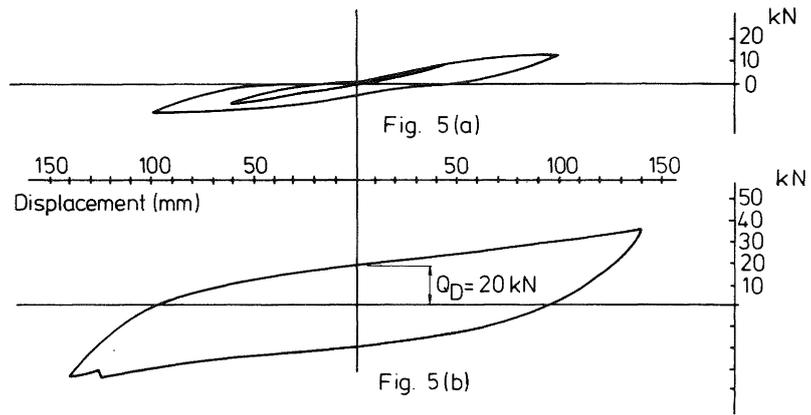Further work also needs to be carried
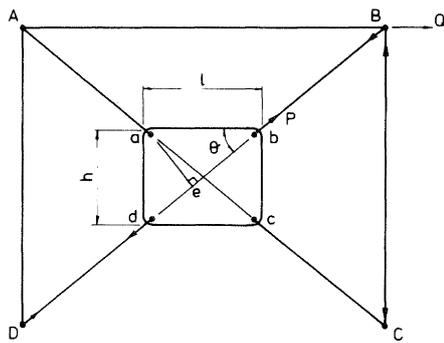
Fig. 5   Hysteresis loops for ring device.

Fig. 7 (above)
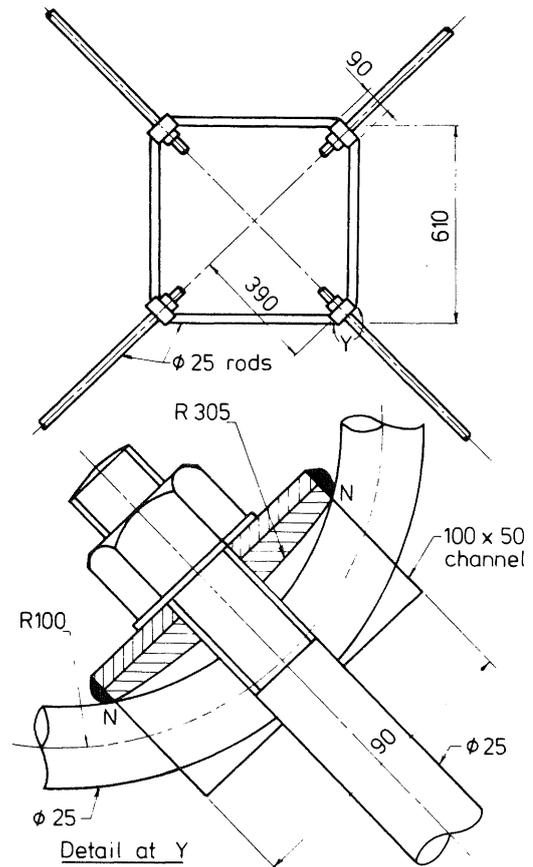Primary loading of a
rectangular device.

Fig. 8 (right)
Details of the square
device tested.

out to establish the relationship between bar diameter, size of rectangle or square and the frame distortion, for a surface strain of $\pm$ 3%, which will give about 100 cycles to failure.

Some work should also be carried out on devices formed from flat strip of rectangular cross section, perforated by a single hole at each corner to carry the diagonals, as there may be a simple way of reinforcing the holes to give a reasonable fatigue life; welding should be kept away from the zone of high strain however.

5.   REFERENCES

1.   Newmark N.M. and Rosenblueth E., "Fundamentals of Earthquake Engineering", Prentice-Hall Inc., N.J., USA, 1971, pp. 349-50.

2.   Tyler R.G. "Tapered Steel Cantilever Energy Absorbers", Bulletin of the New Zealand National Society for Earthquake Engineering, Vol 11, No. 4, December 1978, pp. 282-294.

3.   Tyler R.G., "A Tenacious Base Isolation System Using Round Steel Bars", Bulletin of the New Zealand National Society for Earthquake Engineering, Vol. 11, No. 4, December 1978, pp. 273-281.

4.   Park R. and Blakeley R.W.G., "Seismic Design of Bridges" RRU Bulletin No. 43, National Roads Board, Wellington.

7.   NOTATION

M =   bending moment

P =   load in a diagonal (Figs. 4 and 7) to cause bending of the central energy absorbing device

R =   radius of circular device

d =   circular bar diameter

z =   section modulus in elastic bending

f_y =   yield stress in bending for mild steel taken as 275 MPa

Q =   load at jack to cause shear deformation of the outer frame

M_y =   moment at first yield of outer fibres in bending

θ =   angle between side l of rectangular device and the adjacent diagonal (Fig. 7)

L =   length of main frame

H =   height of main frame

l =   length of inner rectangular device

h =   height of inner rectangular device

S =   shear deformation of main frame

s =   shear deformation of inner rectangular device

D_O =   original diagonal length of outer frame = $\sqrt{L^2 + H^2}$

D_T =   extended diagonal length of outer frame = $\sqrt{L^2 + H^2 + 2LS}$

D_C =   shortened diagonal length of outer frame = $\sqrt{L^2 + H^2 - 2LS}$

d_O =   original diagonal length of inner frame = $\sqrt{l^2 + h^2}$

d_T =   extended diagonal length of the inner frame on the assumption that the whole of the extension of the outer frame diagonal is imposed on the inner, i.e. $d_T - d_O = D_T - D_O$

d_C =   shortened diagonal length of inner frame for the given value of d_T, ie $d_C = \sqrt{l^2 + h^2 - 2ls}$

s =   shear deflection of inner frame for the calculated value of d_T. Evidently $d_T = \sqrt{l^2 + h^2 + 2ls}$ and the value of s is calculated from $d_T - d_O = D_T - D_O$

i.e. $s = \dfrac{(D_T - D_O + d_O)^2 - (l^2 + h^2)}{2l}$

G =   incompatibility in shortened diagonal lengths between outer and inner rectangles. For a given shear deflection the inner rectangle diagonal length shortens more than the outer i.e.

$G = (d_O - d_C) - (D_O - D_C)$

α =   angular distortion of outer frame such that $\sin\alpha = S/H$

β =   angular distortion of inner frame such that $\sin\beta = s/h$

X =   force in shortened diagonal of main frame arising from closure of gap G.

t =   associated tensile forces in links of inner frame to close the gap G, assuming a pin-jointed framework

V =   corresponding tensile force in extended diagonal

U =   corresponding compressive force in outer frame members

Q =   jack force

Q_D =   jack force at zero displacement on the hysteresis loop, used in design work[4]

Q' =   additional jack force required to overcome self-straining action of frame
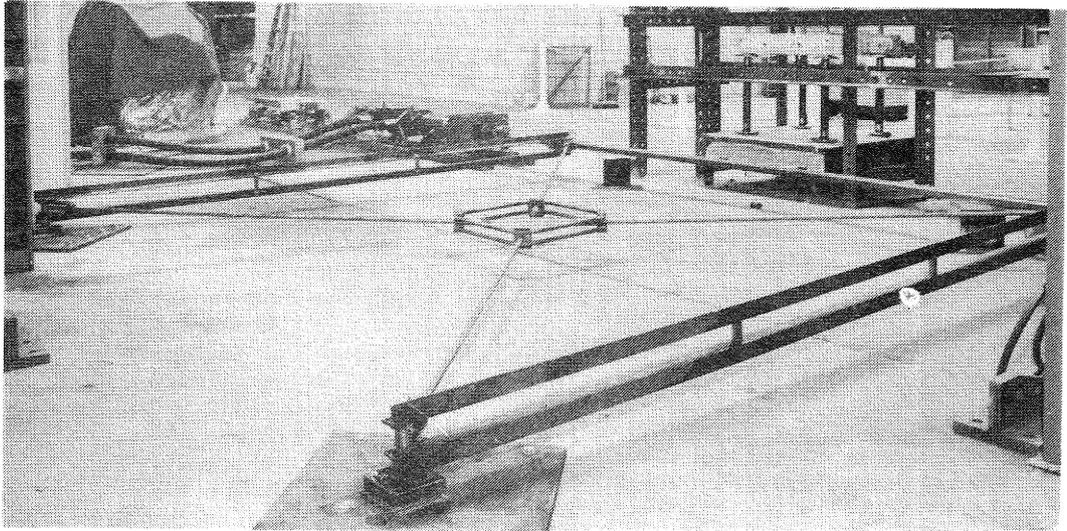
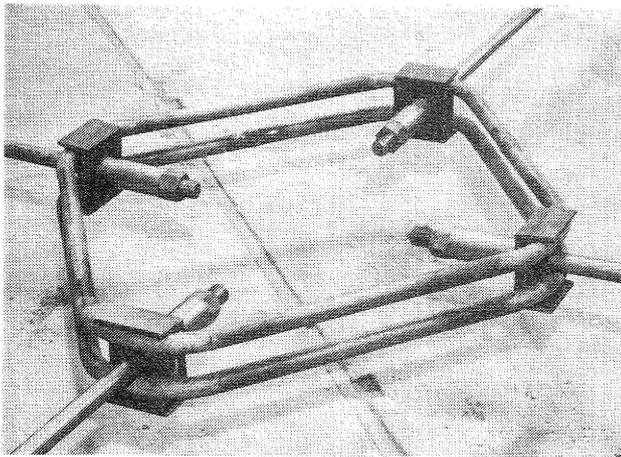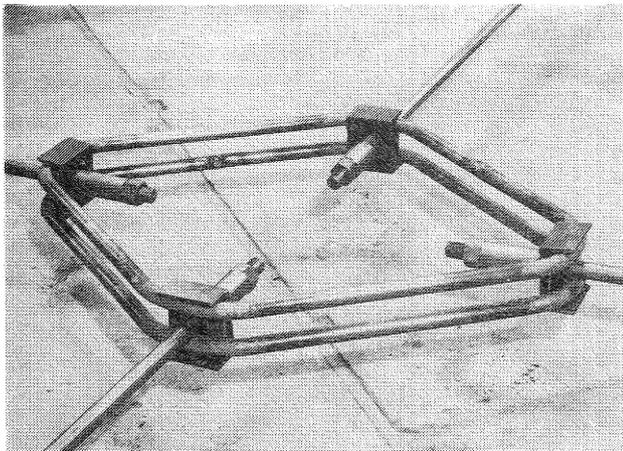Fig. 9   Square device in test frame.

Fig. 10.
Square device deformed
in plastic yield.

(a)   Load top right to
        bottom left

(b)   Load left to right.

Q'' = additional jack force allowing for change in shape of the inner rectangle to a parallelogram

R = reaction at right angles to the axis of the jack carried by the shaft bearings

8. APPENDIX 1

DUCTILITY CHARACTERISTICS OF DEVICES

8.1 RING DEVICE

8.1.1 First yield characteristics

For the loading condition of tension across a diagonal Fig. 4, the bending moments under elastic conditions are a maximum at the loading points A and C:

$$M_A = M_C = \frac{PR}{\pi} = 0.3183 \, PR$$

for load P and mean radius R of rings, and shear effects neglected.

The moment at non-loaded connection points B and D is given by

$$M_B = M_D = PR \; (1/2 - 1/\pi) \; \text{i.e.} \; 0.1817 \, PR$$

As the ring will yield first at the position of maximum bending moment, i.e. at points A and C, this implies a progressive distortion in shape, since loading across the other diagonal at points B and D will first cause yield at B and D and will not entirely correct the plastic deformation which has already taken place at A and C.

For two round rods the section modulus Z is given by

$$Z = \frac{\pi \, d^3}{16} \quad \text{where } d = \text{bar diameter} = 25 \text{ mm}$$

i.e. $Z = 3.07 \times 10^{-6} \; m^3$.

For a yield stress, f_y, taken as 275 MPa, this gives a moment at first yield M_y as:

$$M_y = f_y Z = 3.07 \times 10^{-6} \times 275 \times 10^6 = \frac{PR}{\pi}$$

at the load point i.e. P = 8.69 kN for R = 305 mm.

This gives a jack load $Q = 8.69/\sqrt{2} = 6.1$ kN to cause first yield.

8.1.2 Force to achieve plasticity

The value of the force Q_D (Fig. 5b) at zero displacement of the hysteresis loops has been used previously in the design of energy dissipating devices[2,3,4]. In calculations a plastic yield stress of 350 MPa is used for a section in bending, the rise in stress above the initial yield value of 275 MPa being attributable to work hardening, as evidenced by the fattening of the hysteresis loops with increasing displacement. The surface strain is then approximately $\pm$ 3% and, for a black mild steel and the number of cycles to failure approximately 100.

Following the conventional plastic theory, as the stroke is increased hinges will develop at points A, B, C and D (Fig. 4) when the plastic moment M_p is given by:

$$M_p = Kf_p \quad \text{where } K = \text{plastic modulus} =$$

$\frac{4}{3} r^3$ for a round bar

$= 5.21 \times 10^{-6} \, m^3$ for the bars in the test

Also $M_p = \frac{PR}{4}$ for four hinge points, assuming changes in shape can be neglected.

i.e. $P = \frac{4Kf_p}{R}$, giving $Q_D = \frac{2\sqrt{2}Kf_p}{R}$

substituting $f_p = 350$ MPa gives $Q_D = 16.9$ kN.

8.2 SQUARE DEVICE

8.2.1 Yield characteristics

It may be shown that, neglecting shear effects, the elastic moments at the corners of the rectangular frame shown in Fig. 7 equal and are given by

$$M_A = M_B = M_C = M_D = \frac{P}{4}\ell\sin\theta \quad \text{for the load}$$

P in a diagonal as shown

As plasticity develops and the rectangle contracts to a parallelogram, the moments will be given by the same expression for the reducing value of θ. For a square of side L, and small deformations,

$$M_A = M_B = M_C = M_D = \frac{PL}{4\sqrt{2}}$$

The moment equality at the corners of the device ensures that overstrain in one direction is reversed on the next cycle, i.e. there is no progressive permanent set.

$$M_Y = Zf_y = \frac{PL}{4\sqrt{2}} \quad \text{for the moment } M_Y \text{ at first yield}$$

For fy at first yield = 275 MPa

P = 7.89 kN giving $Q = 7.89/\sqrt{2} = 5.58$ kN.

8.2.2 Force to achieve plasticity

Following the same argument as for the circular device:

$$M_P = Kf_p = \frac{P \, L}{4\sqrt{2}} \quad \text{with } K = 5.21 \times 10^{-6} \, m^3 \text{ as before and}$$

$f_p = 350$ MPa at ± 3% strain

giving P = 17.0 kN.

This value would need to be increased slightly to allow for the curvature at the corners of the square, as the plastic bending occurred at the points Y (Fig. 8). At these points the dimension $L/\sqrt{2} = 610/\sqrt{2} = 431$ mm is reduced to approximately 390 mm. This would increase P in the ratio 431/390 i.e. P = 18.8 kN. Neglecting secondary effects gives: $Q_D = 18.8/\sqrt{2} = 13.3$kN.
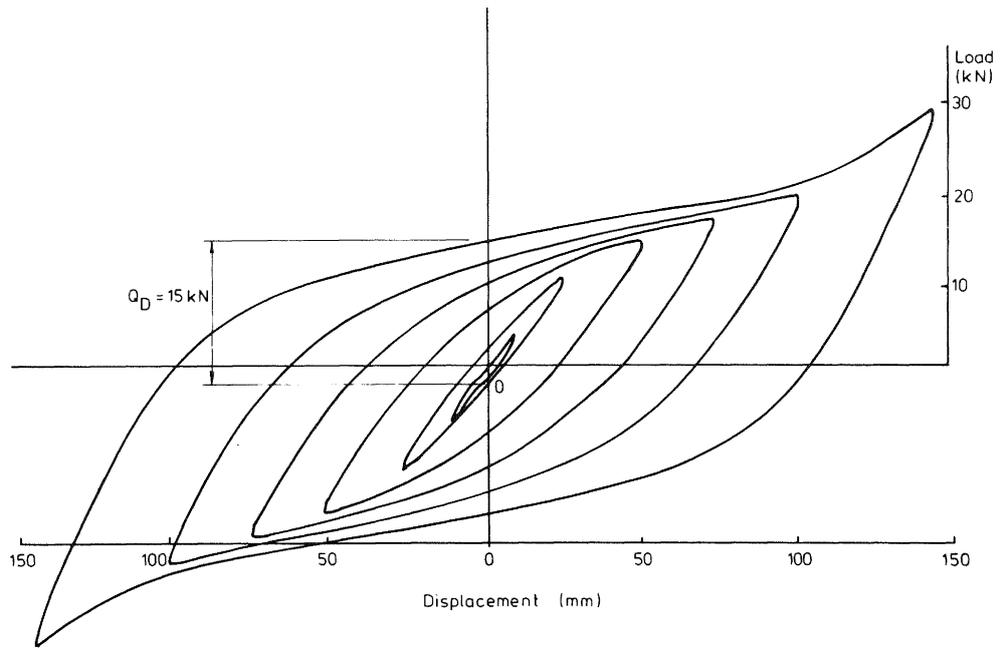
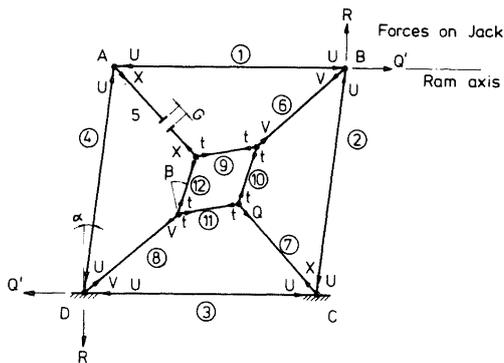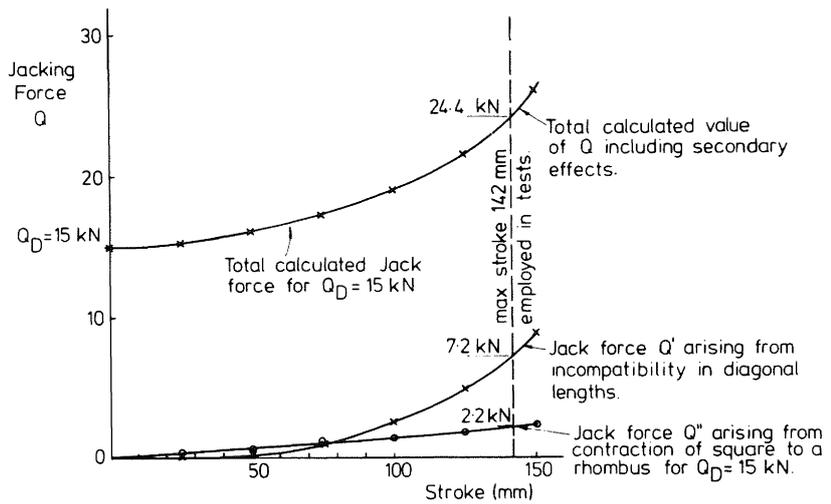Fig. 11.   Hysteresis loops for the square device.

Fig. 12 (left)
Self-straining forces on square
frame.

Fig. 13 (below)
Calculated jack forces including
secondary effects

9. <u>APPENDIX 2</u>

<u>CALCULATION OF ADDITIONAL JACK FORCES</u>
<u>ARISING FROM SECONDARY EFFECTS IN THE</u>
<u>SQUARE DEVICE</u>

9.1 <u>Calculation of additional jack</u>
<u>force Q' arising from incompatibility</u>
<u>of diagonal lengths</u>

9.1.1   General

On the assumption that the whole of
the extension of the diagonal carrying
the force V (Fig. 12) is imparted to the
diagonal of the inner square, then the
length of the opposite diagonal of the
inner rectangle shortens more than that
of the outer, giving rise to the set of
self-straining forces shown, the symmetry
being required for overall equilibrium.
It is convenient to consider that a gap G
in the diagonal member needs to be closed
to give the self-straining forces.   The
external forces Q' and R, along and at
right angles to the jack axis, respectively,
are generated, as incompatible forces
are induced within the frame.

The geometrical properties of the
loaded frame for shear deflection S up
to 150 mm are given in Table A2.1.
In Table A2.2, the force X required to
close the gap G, and the corresponding
forces U, t and V, in other members of
the frame are calculated.   The stiffness
to close the gap G is obtained by applying
unit virtual load at G for the pin-
jointed condition shown in Fig. 12,
changes in the bending condition at the
four corners of the inner square being
neglected, as the strain energy involved
would be small and only direct extensions
and compressions in the members can
accommodate the gap G for a given position
of the jack.   The greatest extensions
are in the four diagonals carrying the
forces X and Y, which consist of a single
25 mm dia. rod.

Values of Q' and R are calculated
at corners B and D from the resolved
components of the forces U and V in
Table A2.3.   The jack force Q' is
plotted against shear deflection in Fig. 12.
At the maximum stroke of 142 mm employed
in the tests the calculated additional
load is 7.2 kN.

Since the external force R at right
angles to the jack axis cannot be provided
in a practical framework resisting
earthquake attack any locking-up action
will be provided by bending in the joints
of the structure.   If the structure is
entirely pin-jointed, apart from the bents
of the inner rectangle, then it is likely
R dissipates in a change of geometry.
This aspect should be investigated by
shaking a model of an actual structure
on a shaking table.

9.1.2   <u>Values calculated for the 4 m</u>
<u>square frame tested</u>

L = $H = 4$ m

ℓ = $h = 0.61$ m

D_o = 5.6569 m

D_T = $\sqrt{32 + 8S}$

D_C = $\sqrt{32 - 8S}$

d_o = 0.8627 m

d_t = $\sqrt{0.7442 - 1.22s}$

s = $((D_T - 4.7942)^2 - 0.7442)/1.22$

G = $D_C - d_c - 4.7942$

α = $\sin^{-1}(S/4)$

β = $\sin^{-1}(s/0.61)$

t = $X/2 \cos((90 - \beta)/s)$

V = $2t \cos((90 - \beta)/2)$

U = $X/2 \cos((90 + \alpha)/2)$

Q' = $V \cos((90 - \alpha)/2) - U(1 + \sin\alpha)$

R = $V \sin((90 - \alpha)/2) - U \cos\alpha$

9.2 <u>Calculation of additional jack force Q"</u>
<u>arising from contraction of the square</u>
<u>to a rhombus</u>

Values of the diagonal length d_c of
the absorber as the shear deformation of
the frame increases are given in Column 2
of Table A2.4.   The force required to
cause plasticity will be inversely
proportional to d_c and in column 3 the
percentage increase is given.   In column
4 the values of the jacking force Q" are
given based on an initial value of $Q_D = 15$ kN
at zero deflection.   These values are
plotted in Fig. 13.   As $d_c = 0$ when
S = 520 mm, Q" rises to   infinity
at that value.   From Fig. 13 at the
maximum value of shear deflection used
in the tests, i.e. S = 142 mm, the value
of Q" is approximately 2.2 kN.

9.3 <u>Calculation of total force Q based</u>
<u>on a value of $Q_D = 15$ kN</u>

Taking the experimental value of
$Q_D = 15$ kN then the total jack force Q
for a given displacement will be given
by Q = Q' + Q" as in column 6 of Table
A2.4.   This is shown plotted in Fig. 13.
The total value of Q at the peak
experimental displacement S = 142 mm is
shown to be 24.4 kN, with a peaking up
similar from that on the experimental
loop (Fig. 11).   The peak experimental
value was about 29.5 kN which compares
well with the calculated value of 24.4 kN
bearing in mind that, with no secondary
effects, there would be a normal rise in
the force value on the hysteresis loop
caused by work hardening.

TABLE A2.1   Geometrical properties of frame under shear deformation

1 Shear deflection of frame S (mm)	2 Frame Diagonal length D_T (m)	3 Frame Diagonal length D_C (m)	4 Shear deflection of absorber s (mm)	5 Absorber Diagonal length d_C (mm)	6 Incompatibility G (mm)	7 Angular distortion of frame α^o	8 Angular distortion of absorber β^o
0	5.6569	5.6569	0	862.67	0	0	0
25	5.6745	5.6391	25.19	844.67	0.23	0.3581	2.367
50	5.6921	5.6214	50.84	825.94	1.26	0.7162	4.781
75	5.7089	5.6043	75.80	807.29	2.81	1.0744	7.138
100	5.7271	5.5857	103.36	786.19	5.31	1.4324	9.755
125	5.7446	5.5678	130.38	764.94	8.66	1.7908	12.342
150	5.7619	5.5498	157.58	742.93	12.70	2.1491	14.971

TABLE A2.3   Calculation of jack forces Q' and to close the gap G

1 X	2 α^o	3 U (kN)	4 V (kN)	5 $\frac{90-\alpha}{2}$	6 $V \cos \frac{90-\alpha}{2}$ (kN)	7 $U \sin\alpha$ (kN)	8 Jack force Q' (kN)	9 $V \sin \frac{90-\alpha}{2}$ (kN)	10 $U \cos\alpha$ (kN)	11 Normal Reaction R (kN)
0	0	0	0	45	0	0	0	0	0	0
1.02	0.3581	0.723	1.063	44.821	0.754	0.005	0.026	0.749	0.723	0.026
5.40	0.7162	3.843	5.871	44.647	4.177	0.048	0.284	4.126	3.843	0.283
12.36	1.0744	8.823	14.031	44.463	10.014	0.165	1.026	9.828	8.821	1.007
21.86	1.4324	15.690	25.943	44.284	18.572	0.392	2.490	18.114	15.685	2.455
31.08	1.7908	22.330	38.611	44.105	27.725	0.698	4.697	26.872	22.319	4.553
45.75	2.1491	32.970	59.589	43.925	42.919	1.236	8.713	41.383	32.947	8.391

TABLE A2.2   Calculation of forces in members to close the gap G

S (mm)	U/X	t/X	V/X	Stiffness X/G kN/mm	G (mm)	X (kN)
0	.7071	.7071	1	4.808	0	0
25	.8093	.7220	1.0420	4.619	0.22	1.02
50	.7116	.7386	1.0872	4.427	1.22	5.40
75	.7138	.7564	1.1352	4.233	2.92	12.36
100	.7161	.7760	1.1868	4.033	5.42	21.86
125	.7184	.7974	1.2423	3.828	8.12	31.08
150	.7207	.8211	1.3025	3.625	12.62	45.75

TABLE A2.4   Calculation of total jack force required allowing for secondary effects

1 S (mm)	2 d_c (mm)	3 Force increase for plasticity %	4 Q" (kN) for $Q_D=15$ (kN)	5 Q' (kN) from Table A2.3	6 Total Q $=Q'+Q"+Q_D$ (kN)
0	863	0	0	0	15.0
25	845	2.2	0.3	0	15.3
50	826	4.5	0.7	0.3	16.0
75	806	7.0	1.1	1.0	17.1
100	786	9.8	1.5	2.5	19.0
125	766	12.7	1.9	4.7	21.6
150	743	16.2	2.4	8.7	26.1