RESPONSE OF DUCTILE REINFORCED CONCRETE FRAMES LOCATED IN ZONE C

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SYNOPSIS:

The results of theoretical prediction of the inelastic seismic response to selected severe earthquake motions of four prototype ductile frames are reported. The structures were proportioned in accordance with capacity design principles for loads corresponding with the loading requirements for seismic Zone C of New Zealand. The effects of P-delta secondary moments on seismic response are briefly reported. A three storey frame, in which factored gravity loads rather than specified lateral earthquake loads governed the proportioning of members, has been examined in detail. It was found that in all frames the intended hierarchy of the energy dissipating mechanisms could be maintained during the full El Centro excitation. The analysis predicted very satisfactory performance for all frames, with a few exceptions, with frames where members were provided with a minimum amount of reinforcement.

1. INTRODUCTION:

In a series of research projects, undertaken at the University of Canterbury and extending over some seven years, the elastoplastic dynamic response of ductile reinforced concrete multistorey frames to selected earthquake motions was studied. The primary aims of this program were to detect patterns of behaviour, development of energy dissipating mechanisms and to estimate ductility demands that are likely to be imposed on plastic hinges in beams and columns. For this purpose analytical techniques, developed at the University of Canterbury⁽¹⁾ and the United States⁽²⁾, were used. Certain results were then compared with findings obtained from experiments with reinforced concrete subassemblages. Such specimens were subjected to simulated cyclic seismic loading.

The experience gained during these studies enabled certain recommendations for the design of ductile reinforced concrete frames to be made⁽³⁾. The aim of these design recommendations was to encourage a desirable hierarchy in the development of energy dissipating mechanisms, required to ensure that the structure can survive the largest earthquake to be expected during its designated life. In this a strong column - weak beam structural system was used. Consequently, the design proposals attempted to quantify the requirements of capacity design philosophy⁽⁴⁾, as applied to ductile frames. In particular recommendations were made for the design of columns to enable them to absorb the flexural overstrength moment input from adjoining beams, while having reserve strength to accommodate bending moments, axial forces and shear forces that are magnified during the nonlinear dynamic response. Thereby

it is expected that in any storey a column sway mechanism is avoided and that the likelihood of yielding in columns at and above the first floor is minimized. As more information came to hand, the values of certain magnifying factors, suggested in earlier publications^(3,5,6), were improved.

In a detailed study the response of a 6, 12 and 18 storey frame, designed in accordance with capacity design principles⁽³⁾, was examined⁽⁶⁾. This work was subsequently extended by redesigning the 12 storey frame and by using two different computer programs for purposes of verification. The latter study⁽⁷⁾ indicated that frames, located in seismic Zone A of New Zealand⁽⁴⁾, and so designed, can be expected to follow closely the intended behaviour. Because Reference 7 summarizes the major steps in the capacity design of ductile frames and because this paper reports on the direct extension of previous studies, details of the design procedure are not restated here.

The aims of the last project in the series of this research program, to be reported here, were:

- (a) To study the earthquake response of prototype frames, designed to the provisions for New Zealand seismic Zone C⁽⁴⁾, to which the lowest seismic risk has been assigned. This should indicate whether the capacity design method provides a similar degree of aseismic protection than that previously observed⁽⁷⁾ for frames located in Zone A and designed for the highest assigned seismic risk.
- (b) To study the effect of P-delta secondary gravity moments on frames with greater flexibility and lower lateral load resisting capacity, located in seismic Zone C.

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- (c) To examine the response of a gravity load dominated frame in which, at a relatively high level of lateral load resistance, the formation of plastic hinges at the top and the bottom of interior columns was considered to be admissible^(5,8).

Previous studies^(6,7) indicated little difference in the response of 12 and 18 storey frames located in Zone A, both of which had fundamental periods larger than 2 seconds. For this reason an 18 storey frame was not selected for comparison in this project. One 12 storey and two 6 storey frames, designed to Zone C requirements were chosen for comparison with their counterparts located in Zone A. The fourth prototype frame was only 3 storeys high but, because of the large column spacing, factored gravity loads rather than code specified⁽⁴⁾ lateral seismic loads governed the proportioning of the beams. After a brief review of the design features of the four selected prototype frames their predicted response to different earthquake motions is described separately. Where appropriate comparisons with frames designed to Zone A requirements are made.

2. THE DESIGN AND PROPERTIES OF PROTOTYPE FRAMES:

2.1 Dimensions of Frames

Each prototype frame, shown in Fig. 1, was considered to resist gravity and earthquake loading corresponding with the tributary floor areas from one bay. One-way frame action was considered only. Earthquake resistance at right angles to the frames was assumed to be provided by independent shear walls. To reduce slab thickness, secondary beams were provided in all but the 6 storey A frames are⁽⁷⁾ identical with those used previously for frames located in Zone A.

To examine the effects of increased gravity load on strength, required for earthquake resistance, and on the possible contribution of P-delta moments to seismic response, the 6 storey frame B, with a somewhat artificial orientation of the secondary beams, as shown in Fig. 1, was chosen. At the interior supports of the long span beams of the 3 storey frame (Fig. 1) haunches were used.

Information relevant to design earthquake loading and the principal data for all members of the prototype frames, are assembled in Table I. The strength properties of all critical column and beam sections, together with other data, required for the time-history dynamic analysis, are fully given elsewhere⁽⁹⁾.

Table I also gives limits for the longitudinal reinforcement content used in the beams and columns for all frames. The reinforcement in the columns of the lower storeys generally were the minimum allowed by codes, i.e. 1.0%⁽¹⁰⁾ or 0.8%⁽⁵⁾. The larger content of reinforcement in

columns, shown in Table I, refers to upper floors where column sizes have been abruptly reduced.

2.2 Gravity Load Analysis

Isolated subframes of a typical floor were subjected to conventional elastic analyses, in order to derive gravity load induced bending moments along beams. In addition to the weight of the concrete, 1.4 kPa dead load was assumed for movable partitions, ceilings and floor finishes. The live load was taken as 2.5 kPa, except for the 3 storey frame, for which 4.0 kPa was allowed for retail occupancy.

2.3 Lateral Load Analysis

The code specified⁽⁴⁾ equivalent lateral static load, corresponding with the total base shear given in Table I, was applied to each floor. With the aid of a standard computer program for elastic frame analysis, all internal actions resulting from the lateral load were obtained. Interstorey drift was also checked to ascertain that stipulated values⁽⁴⁾ were not exceeded.

2.4 Design of Members

Bending moments for the appropriately factored gravity loads and lateral seismic loads were superimposed for each beam. Subsequently moment redistribution^(5,11) for the continuous beams was carried out to optimize demands for flexural reinforcement, while also considering practical requirements for placement and detailing. The flexural overstrengths⁽⁵⁾ of all potential beam plastic hinges were determined to obtain maximum moment inputs into columns. These moments for columns were further amplified by dynamic magnification factors⁽⁷⁾, and were then combined with the upper and lower bound estimates⁽⁷⁾ of column axial forces, to determine the necessary longitudinal reinforcement. For most columns of the lower storeys of all frames, minimum requirements^(5,10) as stated in Table I, dictated the selection of column reinforcement. Consequently columns so reinforced possess flexural strength in excess of that required to resist the column design actions, particularly at ground floor level.

3. SELECTION OF EARTHQUAKE RECORDS:

As stated previously, one of the aims of this study was to examine the extent to which the desired behaviour of a frame, designed for low seismic risk, can be assured by the application of capacity design procedures. For this reason no attempt was made to utilise earthquake records considered to be representative of motions that might be expected in Zone C of New Zealand. Behaviour assuring survival, rather than damage control or repairability, were the consideration in the selection of records. Consequently in the analytical simulation all frames were subjected to the first 14 seconds of the North-South component of the El Centro May 1940 earthquake. Because of the very

TABLE I PRINCIPAL DATA FOR THE PROTOTYPE FRAMES STUDIED

DATA	12 Storeys	6 Storeys A	6 Storeys B	3 Storeys	
For overall dimensions see	Fig. 1(12)	Fig. 1(6A)	Fig. 1(6B)	Fig. 1(3)	
Total Seismic Weight (kN)	16990	2510	8540	10240	
Estimated Period ⁺ (sec)	2.09	0.99	1.07	0.69	
Computed Period (sec)	2.27	1.08	1.15	0.72	
Seismic Coefficient (-)	0.050	0.064	0.058	0.084	
Total Base Shear [*] (kN)	680	128	396	688	
Size of Main Beams (mm x mm)	1 - 2 floor 3 floor 4 - 6 floor 7 - 8 floor 9 - 10 floor 11 - 12 floor	750 x 400 750 x 400 750 x 400 700 x 400 650 x 400 600 x 400	500 x 250 500 x 250 500 x 250	750 x 400 750 x 400 750 x 400	750 x 550 ¹ 700 x 500 ²
Size of Exterior Columns (mm x mm)	1 - 3 floor 4 floor 5 - 6 floor 7 - 8 floor 9 - 10 floor 11 - 12 floor	725 x 500 625 x 500 625 x 500 575 x 500 525 x 500 500 x 500	400 x 400 400 x 400 400 x 400	575 x 500 575 x 500 500 x 500	650 x 650
Size of Interior Columns (mm x mm)	1 - 3 floor 4 floor 5 - 6 floor 7 - 8 floor 9 - 10 floor 11 - 12 floor	725 x 725 675 x 675 675 x 675 625 x 625 575 x 575 550 x 550	450 x 450 400 x 400 400 x 400	650 x 650 650 x 650 600 x 600	650 x 650
Longitudinal Reinforcement Content	Main Beams Columns	0.5 - 1.8% 1.0 - 2.0%	0.5 - 1.2% 1.0 - 1.3%	0.5 - 1.5% 1.0 - 1.9%	0.5 - 2.0% 0.8 - 2.0%
Slab Thickness (mm)	160	120	160	90	
Concrete Strength (MPa)	28	28	28	28	
Strength, Beam Bars (MPa)	275	275	275	275	
Strength, Column Bars (MPa)	380	380	380	380	
Acceleration Records Used	El Centro N-S Pacoima Dam Pacoima (Pδ)	El Centro N-S Pacoima Dam Parkfield (Pδ)	El Centro N-S Pacoima Dam Parkfield (Pδ)	El Centro N-S Pacoima Dam 60% Pacoima Dam	

* For all frames a structural type factor⁽⁴⁾ $S = 0.8$ and risk factor of $R = 1.0$ was used.

1 Haunch at column face = 1000 x 550.

2 Haunch at column face = 900 x 500.

+ Derived from requirements of the loading code⁽⁴⁾.

satisfactory performance of frames designed to Zone A requirements, and predicted in previous theoretical studies⁽⁷⁾, it was expected that the frames of this study would also exhibit behaviour during the El Centro excitation, which is considered to be attainable with detailing to New Zealand concrete code⁽⁵⁾ requirements.

To test whether the intended hierarchy in the formation of plastic hinges can be maintained throughout the frame even under the most extreme excitation, the first 14 seconds of the motions of the Pacoima Dam 1971 S14W and those of the Parkfield 1966 N65°E earthquake were also used. These excitations should be considered as being far in excess of and hence irrelevant to risk levels of Zone C. The earthquake records used for different analyses are shown for each frame in Table I.

These ground motions were also used in previous studies^(6,7) and hence the analyses enabled comparison of responses to be made for similar frames with different lateral load carrying capacities.

4. ANALYTICAL MODELLING:

The time-history analysis of these frames was carried out with a two-dimensional non-linear dynamic computer program developed by Sharpe and Carr⁽¹⁾, details of which are described elsewhere⁽⁹⁾.

As in previous studies, the response of preselected potential plastic hinges was simulated by a simple bilinear elasto-plastic moment-rotation relationship. For the end region of columns the conventional axial load-moment strength relationship⁽¹²⁾ was closely approximated by a cubic relationship^(7,9), applicable to the range of axial compression encountered in these frames during the 14 seconds of very severe excitation.

In computing stiffnesses allowance was made for the effects of cracking. In specifying the flexural capacity of potential plastic hinges, probable material properties were used with some compensation for strength overestimates that can occur during a time step used in the analysis.

The gravity load, assumed to be constant during the entire analysis, was taken as the dead and one third of the live load.

A value of 8% of critical damping was specified for the first and the tenth and the first and the sixth modes of the vibration for the 12 and 6 storey frames respectively. Modal analysis of the three storey frame indicates that specifying 8% damping fractions for any normal mode lower than the 35th mode, forced supercritical damping fractions in the frame's highest natural modes of vibrations.

The program was also extended to include an option, the assessment of P-delta secondary gravity moments⁽¹³⁾.

This was achieved by modifying the lateral stiffness of the columns as a result of the column axial forces due to the gravitational dead and live loads. This enables the effect of P-delta moments on the computed response to be considered. Thereby previously obtained results^(6,7) for which P-delta effects were not yet taken into account in the analysis, could be reassessed.

5. THE RESPONSE OF THE 12 STOREY FRAME

5.1 Frame Characteristics

Even though the seismic design base shear was only 4% of the equivalent total mass, the design of most members was governed by the appropriate combination of gravity and earthquake load⁽⁴⁾, rather than by factored gravity loads only. Therefore this structure is considered as being an earthquake load dominated frame. Even though the maximum computed drift under the specified lateral static load⁽⁴⁾ was only 58% of that permitted by the code, i.e. 1% of storey height, member sizes could not have been reduced significantly without encountering construction difficulties, particularly with anchorage of reinforcement. Moreover, column sizes are the smallest possible, consistent with permissible axial load carrying capacities⁽⁵⁾, corresponding with the specified concrete strength.

5.2 Interstorey Drift and Lateral Displacements

The maximum interstorey drifts, encountered during the 14 seconds of the ground motions, are shown by corresponding envelopes marked (12) in Fig. 2. The drift in each storey is given in terms of its absolute value (mm) and as a multiple of the storey height h . The limiting storey drift specified by the New Zealand Design and Loading Code⁽⁴⁾, i.e. 1% of the storey height h , is a useful reference with which predicted drifts for various earthquake motions can be readily compared. Therefore this reference displacement, $h/100$, is distinctly shown alongside graphs where appropriate. It is thus seen in Fig. 2 that predicted drifts during the El Centro excitations were not critical. Separate studies⁽¹³⁾ have shown that a consideration of P-delta effects in the analysis made only insignificant and not necessarily adverse changes in the response of the 12 storey frame to the El Centro motions. As expected, a dramatic increase in the maximum drifts was predicted for the Pacoima Dam excitation. Because of the large lateral displacements it was expected that the influence of P-delta effects on response is likely to be more significant than in the case of the El Centro excitation. The outermost envelope curve for the 12 storey frame in Fig. 2 shows that P-delta secondary moments were responsible for the increase of the maximum drift in the second storey by approximately 60%. Therefore it was concluded that when for 12 storey frames the drifts are significantly in excess of $h/100$, P-delta effects should be

FIG. 1 PLANS AND ELEVATIONS OF PROTOTYPE FRAMES SHOWING PRINCIPAL DIMENSIONS.

FIG. 2 INTERSTOREY ENVELOPES OF PROTOTYPE FRAMES FOR SELECTED EARTHQUAKE MOTIONS

included in the analysis if meaningful approximations are to be achieved. The envelopes also indicate that P-delta effects, which are more fully discussed in Reference 13, are only critical in the lower half of the frame.

The displacements at roof level of the 12 storey frame, obtained by the three analyses are shown at the top of Fig. 3. The displacements may again be compared with the deflection of 1% of total frame height, H . Two dramatic increases in response to the Pacoima excitation, at approximately 3 and 8 seconds, are evident. Only after the first major inelastic drift, at 3.8 seconds, does the influence of P-delta moment become increasingly significant. The average permanent slope of the frame after 9 seconds of the Pacoima excitation is approximately 1:40. The displaced mass resulted in overturning moments at 14 seconds which exhausted 90% of the flexural strengths of the beams at the second floor. This indicates that a Zone C frame would be very close to collapse in an unlikely extreme event, such as the Pacoima excitation. Analytical predictions for such extreme displacements are, however, likely to be very crude. Neither strength or stiffness degradation nor strain hardening stiffness, with a partly compensating effect, have been considered in the analytical modelling.

It was not surprising that a comparison of the frame response with that of the 12 storey frame designed to Zone A requirements, involving an 8% increase in mass and a 61% increase in lateral load carrying capacity⁽⁷⁾ for the Zone A frame, indicated that for the Pacoima excitation the displacements for the Zone C frame were approximately 45% larger, while its stiffness was approximately 36% less.

The imposed permanent inelastic deformations during the Pacoima motions were particularly dramatic in the lower six storeys. This is seen in Fig. 4 which shows the predicted displacements for the 14 seconds duration for every second floor of this 12 storey frame.

5.3 Plastic Hinge Formation

The aims of the design procedure were to minimise the likelihood of column hinging with the exception of the base and roof level and to avoid storey mechanisms, involving the simultaneous hinging of both ends of all columns within a storey⁽⁷⁾. The required energy dissipation was to be achieved primarily through yielding in the beams.

The pattern of plastic hinge formation within the frame was evaluated by the analysis program at each time step. Hinge patterns at critical intervals of the response of these frames is reported elsewhere⁽⁹⁾.

At no instant during the El Centro ground motion was column hinging in the 12 storey frame predicted,

although considerable yielding in columns at ground level was considered to be acceptable and this was anticipated. At two brief instants 71% of the possible number of potential beam plastic hinges had yielded. The critical computed earthquake induced axial load in exterior columns was developed when beam plastic hinges formed simultaneously at 8 (67%) of the 12 floors. In terms of plastic hinge formation the computed response to the El Centro excitation indicated an extremely satisfactory behaviour of this frame, located in Zone C.

As can be expected (Figs. 3 and 4) inelastic deformation during the Pacoima excitation, where P-delta effects were also considered in the analysis, were much more dramatic. Up to 81% of the total number of plastic hinges developed simultaneously at one stage. Large axial loads in exterior columns were induced when beam hinges formed simultaneously in all 12 (100%) of the floors.

Fig. 5 shows that the envelopes for maximum induced axial compression during the response predicted values remarkably close to those obtained from the design, which are shown by cross shading. The minimum axial compression of the same columns, which is associated with earthquake induced axial tension, was underestimated in the design. This is only a reflection on the conservatism involved in the deliberate underestimation of factored dead load, as shown in Fig. 5. It should be noted that an underestimate of the minimum value of total compression load on the column leads to slightly larger demand for longitudinal reinforcement.

At four intervals of the Pacoima event, the largest of which was approximately 0.4 seconds duration, all three columns hinged at the ground level. As may be expected from the displacement response shown in Fig. 3, all four occasions involved hinge rotations in the same direction. This means that for the Pacoima excitation no alternating plastic rotations at these column hinges were predicted by the dynamic analysis. During two brief intervals of this extremely severe and, with respect to Zone C, entirely unrealistic shaking, plastic hinges formed at the tops of the columns at the 4th and the 5th floor. At no instant did plastic hinges form simultaneously at both ends of any column.

The dynamic magnification, by up to 80% of column design moments⁽⁷⁾, was recommended in anticipation of the radical departures of the bending moment patterns during the inelastic dynamic response of the frame from the pattern that was obtained for the code specified⁽⁴⁾ lateral static loading. Fig. 6 shows two cases for these higher mode effects on moment pattern and it also illustrates an instant when the design procedure adopted^(5,7) was insufficient to prevent the formation of a column hinge at the 5th floor. Subsequently it will be shown that such

FIG. 3 HORIZONTAL DISPLACEMENTS OF FRAMES AT ROOF LEVEL FOR SELECTED EARTHQUAKE MOTIONS

unexpected computed column hinges are associated with very moderate plastic rotations, i.e. ductility demand. Therefore it may be concluded that the design procedure^(4,7) fully achieved its aim in ensuring the desired hierarchy and the pattern in the formation of plastic hinges throughout the frame, even for the unrealistic extremes of the Pacoima excitation.

5.4 Column Bending Moments

Envelopes for peak column bending moments at the level of beam centre lines are shown for the interior column of the 12 storey frame in Fig. 7, together with those of the other frames. The dotted central regions represent moments from the initial elastic analysis for the required⁽⁴⁾ equivalent lateral static loading. These code load moments have been amplified in Fig. 7 by 25% so as to correspond with the probably flexural strength of the column if it had been proportioned to match exactly the static load requirements. The probable strength derived from the recommended⁽⁷⁾ capacity design procedure is represented by the horizontally hatched envelope. The slopes of the moment diagrams were arbitrarily chosen and hence have no relevance to shear forces.

It is seen in Fig. 7 that with the exception of the first and the top storey, the predicted column moments for the El Centro motions were generally well within the values indicated by the design envelopes. At ground floor and in the 12th storey much smaller design moments were used because in these regions the hinging of columns was anticipated. That column hinges did not form during the El Centro event was due to the excess flexural capacity provided at these levels by minimum (1%) longitudinal reinforcement. The El Centro envelope also shows, however, that induced moments consistently exceeded those derived from the equivalent static load only. The excellent predicted response of the column during the El Centro excitation justifies the recommendation that has been made that:

- (a) Where plastic hinges are anticipated to form, such as at the ground floor, full confining reinforcement, consistent with the maximum expected axial compression⁽⁵⁾, should be provided within the potential plastic hinge (end) region of the column.
- (b) Where, by the use of capacity procedures⁽⁷⁾, plastic hinges are not expected to develop, confining reinforcement in the end regions, one half of that required for case (a) above, should be sufficient, unless other requirements, such as shear strength, govern the design. In this context transient yielding, involving very small rotational ductility, should not be taken as plastic hinge formation.

As expected, the column moments predicted by the analysis for the Pacoima

excitation exceed the design values considerably, particularly in the lower half of the frame. However, with minimum or close to minimum column flexural reinforcement, moments in excess of the design values could be sustained. Ductility demands are discussed in Section 5.5.

Fig. 7 also shows that P-delta effects, considered in the analysis for the Pacoima motions, increased moment demands considerably in the 3rd to 7th storeys.

An aspect which required examination was the length of potential hinges in compression dominated columns over which confining reinforcement is required. Recent experimental studies⁽¹⁴⁾ showed that appropriately arranged confining reinforcement in rectangular columns will not only enhance the rotational ductility of the end region, but that it may also contribute to a very significant increase of the strength of the concrete within the confined core. Therefore when the section is subjected to large axial compression force, a very significant increase in the flexural resistance of the core will also occur. If the confined region extends only a short distance from the end of the column and when the significantly increased flexural overstrength of this confined region is developed, it is possible to overload the adjacent region, confined perhaps with nominal ties only, unless there is a rapid decrease of bending moments along the column. The development of excessive flexural overstrength at column ends may result in a brittle failure of the region adjacent to the end region of the column. This has been observed⁽¹⁴⁾. For this reason it is necessary to define the length of the potential plastic hinge region at the end of a column, not only in terms of the column dimensions, but also in terms of the moment gradient. This may be achieved conveniently by defining the length of the potential plastic hinge in the column in terms of the distance from the critical end section to the point of contraflexure. For this reason the movements of the point of zero moment along the first storey columns were also monitored during the analysis. It has been found that during the few and brief intervals when plastic hinges did form at the base, the point of contraflexure was always very close to the first floor (see Fig. 6) so that the entire first storey column was subjected to moments of the same sign. It is therefore recommended that a linear moment variation, from maximum at the base to zero at the centre line of the first floor beam, should be assumed when determining the amount and distribution of the necessary confining reinforcement at the end and at the adjacent regions of the first storey column.

5.5 Inelastic Rotation Demands

In order to evaluate ductility demands on members of the prototype frames during the selected ground motions, the maximum plastic hinge rotations predicted

FIG. 4 HORIZONTAL DISPLACEMENTS (IN MM) AT EVERY SECOND FLOOR OF THE 12 STOREY FRAME DURING THE PACOIMA EXCITATION WITH P-DELTA EFFECTS INCLUDED IN THE ANALYSIS

FIG. 5 AXIAL LOAD ENVELOPES FOR THE EXTERIOR COLUMNS OF THE TWELVE STOREY FRAME

FIG. 6 BENDING MOMENT PATTERNS AT TWO INSTANTS OF THE PACOIMA EXCITATION WITH P-DELTA EFFECTS INCLUDED IN THE ANALYSIS

12

6A

6B

3

FIG. 7 COLUMN BENDING MOMENT ENVELOPES

by the analysis have been assembled in Fig. 8. Interconnected solid dots represent the maximum "observed" beam hinge rotations. Isolated solid dots with a single extended line refer to column hinges which were expected to develop, while isolated circles represent plastic rotation at column hinges which were not anticipated.

The solid heavy line at 35×10^{-3} radians (2 degrees) indicate the amount of plastic hinge rotation that has been attained in test beams^(15,16). This large rotation has been achieved with little or no strength loss after 6 load reversals. It may be expected that such performance is assured by satisfying the detailing requirements of the New Zealand Draft Concrete Design Code⁽⁵⁾.

In recent experimental work with near full size rectangular column specimens⁽¹⁷⁾, detailed in accordance with proposed code⁽⁵⁾ requirements, a displacement ductility of 6 was easily achieved without significant loss of flexural capacity after 6 inelastic load reversals. These test columns carried axial compression loads in the range of $0.15f'_c A_g$ to $0.55f'_c A_g$ with good ductility exhibited at all levels. (f'_c is the measured compressive strength of the concrete used in the tests and A_g is the gross concrete sectional area of the column.) The maximum plastic hinge rotations observed in these columns were of the order of 25×10^{-3} radians (1.4 degrees).

As Fig. 8 shows, in the 12 storey frame very modest and approximately uniform beam hinge rotations were required during the El Centro excitations, while column hinges did not form. However, as Fig. 7 shows, moments in the first storey column were well in excess of the design values and the base was close to yielding.

Plastic hinge rotations computed for the Pacoima response show the following distinct features:

- (a) Neglecting P-delta effects in the analysis, when large inelastic inter-storey drift can occur, will lead to gross underestimation of the plastic rotation demand in both beams and columns.
- (b) The analysis which neglected P-delta effects suggested that the structure was taxed to its limit during the Pacoima excitation. The inclusion of P-delta moments revealed, however, that inelastic deformations for both groups, the beams and the column bases, were well in excess of the capability of these members. The collapse of the real structure may well have been imminent.
- (c) Rotation demand in the columns at the 4th and 5th floor were small during the unrealistic Pacoima excitations. It may be said the intended energy dissipating

mechanisms in the 12 storey Zone C frame could be maintained for the most severe disturbance, i.e. a shaking with a return period in excess of 1000 years.

5.6 Column Shear Forces

Envelopes for maximum shear forces across selected columns of the prototype frames are compared in Fig. 9. For example for the interior column of the 12 storey frame, the stepped line with the half circles represents the shear forces derived from the elastic analysis for the code⁽⁴⁾ specified lateral loading. To make a meaningful comparison with shear predicted by the time-history analyses the code shear forces were amplified by 21% so as to correspond with the probable strength of the materials used. The diagonally hatched line represents the design shear, V_{col} , based on ideal strength as derived for upper storeys, from the simple relationship⁽⁵⁾

$$V_{col} = 1.4\phi_o V_{code} \quad (1)$$

where ϕ_o is the larger of the overstrength factors, relevant to the beam or beams at the top or the bottom of the column in a storey, and V_{code} is the shear force derived from code⁽⁴⁾ loading. Eq. (1) is based on the estimated maximum value of moment gradient along each column. This depends on the moment input from the beams at their flexural overstrength rather than on the flexural capacity of the column itself. In the first storey, however, the column shear is based on the estimated magnitude of the flexural overstrength of the column base section as built, as well as on the moment input from first floor beams. This logical step accounts for the radically increased design shear forces across first storey columns, the flexural strength of which happened to be much larger than that required for code loading (see Fig. 7).

To indicate the relative magnitude of these shear forces, the minimum contribution of the concrete mechanisms in the column, $V_c^* = 0.17bd\sqrt{f'_c}$ for use as a reference, is shown by the shaded area. In this f'_c is the probable compression strength of the concrete used and b and d are the width and effective depth of the column section. This contribution of the concrete is a minimum value everywhere where plastic hinges are not expected to develop, because the beneficial effect of axial compression to shear strength has not been included. A comparison of the various envelopes for the predicted shear forces encountered during the dynamic responses, with the shaded area, giving V_o^* , indicates that with the exception of the first storey, the shear reinforcement required in columns is less than 50% of that required to resist the total design shear. Hence for the upper storeys, confining and column tie reinforcement requirements⁽⁵⁾ are likely to be more critical than those for shear strength.

Because of the undesirable characteristics of a shear failure the design procedure developed^(5,7) for shear is more conservative and this is reflected in Fig. 9. Only for the Pacoima response, where P-delta effects were also considered, did the predicted shear forces exceed the design value in two storeys of the 12 storey frame. Here the third storey shear is not likely to be critical because at no stage was column yielding in this storey indicated.

It may be said that the design procedure for shear in the 12 storey frame appears to be overly conservative. However, it should be noted that this design approach is not likely to govern the amount of transverse reinforcement to be used in the columns.

5.7 A Comparison of Responses

A comparison of two 12 storey frames, one designed to Zone A^(6,7) and the other to Zone C loading requirements⁽⁹⁾, and subjected to identical ground excitations, shows the following features:

- (a) The displacement response to the El Centro excitations of both frames was very similar despite differences in both stiffness and strength (see Section 5.2). As expected, the Zone C frames was subjected to somewhat larger displacements. The same was observed for interstorey drifts.
- (b) Much larger differences in response were predicted for the Pacoima motions. The roof level permanent displacement at 14 seconds of the earthquake record in the Zone C frame was approximately 55% larger than that in the Zone A frame. In the analyses of Zone A frames⁽⁷⁾ and in the above comparison, P-delta effects were not considered. It is probable that the difference in the responses, including P-delta contributions, would be of the same order. Similar comparisons were found for the interstorey drifts predicted for the Pacoima earthquake record.
- (c) Because maximum lateral displacements during the predominantly inelastic response of a frame will directly affect plastic hinge rotation demands, the comparison of these for the two frames shows the same features as a comparison of deflections and interstorey drifts.
- (d) Because design moment and design shear force envelopes for upper storey columns are derived from the flexural overstrength of the beams, as detailed in each of the frames, rather than directly from the specified lateral static load, a comparison between respective envelopes for the two frames is

inappropriate. These envelopes simply quantify the aim of the capacity design philosophy, to give a high degree of protection to the columns against inelastic deformations. It is seen that, irrespective of the ground motions used in the analysis and the lateral load carrying capacity provided, this aim was achieved in both the Zone A and Zone C 12 storey frames.

6. THE RESPONSE OF THE SIX STOREY FRAMES

6.1 Frame Characteristics

Because the response of the two 6 storey frames (6A and 6B), the dimensions of which are shown in Fig. 1, was found to be very similar, their behaviour is described together in this section. Due to the short span of the beams in Frame 6A, lateral rather than gravity load requirements dominated the design of these beams. In contrast factored gravity loads on the beams of Frame 6B (Fig. 1) were much more significant. Consequently gravity moments alone controlled the proportioning of those beams, particularly in the upper floors. Hence the lateral load carrying capacity of Frame 6B was considerably in excess of that specified by the loading code⁽⁴⁾. Typically the ideal flexural capacity of the beams to resist lateral loading on Frame 6B was 37% and 118% in excess of the ideal strength at the 2nd and 5th floors respectively, required by code⁽⁴⁾ specified lateral loading. Because of reasons discussed in Section 7, the formation of plastic hinges at the 5th floor of Frame 6B was assigned to the interior columns rather than to the adjoining ends of the beams. The differences between the two 6 storey frames are highlighted by the ratio of the equivalent masses and the ratio of lateral static load carrying capacities for the 6B frame to those for the 6A frame. These ratios were 3.4 for masses and 4.6 for strengths respectively.

Only the earthquake dominated frame (6A) had a counterpart in the previous study⁽⁷⁾ of Zone A frames. A comparison of the performance of these frames is made in Section 6.7.

6.2 Interstorey Drift and Lateral Displacements

Envelopes for the maximum interstorey drifts, encountered in each of the frames during the El Centro, Parkfield and Pacoima excitations, are shown in Fig. 2. The similarity in displacement responses of the two frames is evident. During the El Centro excitations, maximum drifts of 1% of storey height were predicted. Surprisingly, there was little difference in drifts during the Pacoima or Parkfield excitations, irrespective of whether or not P-delta effects were included in the analysis. For reasons of economy in computations, the Pacoima analysis was not repeated with the inclusion of P-delta effects. The

excess strength of the 6B frame in the upper storeys resulted in some reduction of drift in those storeys.

As Fig. 3 shows, the displacement responses of the two frames were very similar throughout all four analyses. The influence of P-delta secondary contributions, in increasing displacements considered for the Parkfield record only by 25%, should be noted. During the most critical stages of the El Centro and Parkfield excitations, only 14% and 41% respectively of the sum of the strengths of the first floor beams of Frame 6A was required to resist moments due to P-delta effects. The permanent roof displacements in frame 6B at the end of the Pacoima and Parkfield records were only about 13% less than those encountered in frame 6A.

6.3 Plastic Hinge Formation

During the El Centro motions no column hinges were encountered in the upper storeys. At the ground floor, for a single period of approximately 0.10 seconds all three columns developed plastic hinges in both frames. The maximum number of hinges developed at any instant was 88% and 67% of the possible total in frames 6A and 6B respectively.

The patterns in the development of plastic hinges during the Parkfield excitation (with the inclusion of P-delta effects) were in both frames similar to those encountered during the El Centro motions. The only exception was that at several instants plastic column hinges were predicted for the Parkfield event, predominantly at the top end of the columns of the upper storeys. As will be seen subsequently, these hinges involved very modest ductility demand. The same feature was encountered with the 6 storey prototype frame located in Zone A. At no stage was a storey mechanism indicated in any of the storeys. It may be concluded that both frames fulfilled the criteria for the intended and desirable behaviour, even during the extreme and unrealistic Parkfield and Pacoima excitations. It is to be noted that dynamic magnification of the column design moments for the upper storeys was only by 44% to 49% as compared with 80% used for the 12 storey frame.

6.4 Column Bending Moments

As the bending moment envelopes for the interior columns of both of the 6 storey frames shown in Fig. 7, moments predicted for the El Centro motions are close to but within those obtained from capacity design procedures^(5,7). The exceptions are at ground floor and in the top storey. The minimum reinforcement content of 1% in most interior columns was in excess of that required by the design moment envelopes. For this reason much larger moments in all these columns could be developed during the Pacoima and Parkfield motions.

6.5 Inelastic Rotation Demands

The examination of predicted inelastic plastic hinge rotations provides the best assessment for frame behaviour and for the criticality of response during extreme excitations. These are presented, side by side, for the two 6 storey frames in Fig. 8. It is seen that during the El Centro motions modest ductility demands were imposed on beam hinges and that only limited yielding occurred at the base of the columns in both frames. However, beam plastic hinge rotations in Frame 6A were up to 80% larger than those predicted^(6,7) for its counterpart located in Zone A. Also, as Fig. 8 shows, they were considerably larger than those in the 12 storey Zone C frame.

Both predicted beam and column base hinge rotations for the Pacoima and Parkfield excitations are just within the limits considered to be attainable with standard⁽⁵⁾ detailing. It may be said that both of these frames would have a good chance of surviving, without collapse, even the extreme shaking of the chosen earthquakes. The same could not be said for the 12 storey frame because of the significantly more critical and detrimental influence of P-delta effects on inelastic dynamic response. These effects, considered for the analysis of the 6 storey frame for the Pacoima motions, were negligible in terms of ductility demands. The predicted plastic rotation demands for the Pacoima motions in Frame 6A were approximately 55% larger than those encountered in the equivalent Zone A frame.

It is of interest to note that in spite of the greater relative strength of Frame 6B with respect to Frame 6A (shown in Section 6.1 to be $4.6/3.4 = 1.35$), plastic hinge rotation demands were of the same order for both these Zone C frames. It was found, however, that the corresponding ratio of the maximum number of plastic hinges participating in significant energy dissipation, was (as shown in Section 6.3) only $67/88 = 0.76$. The product $1.35 \times 0.76 \approx 1$ suggests therefore that, on account of the lesser number of plastic hinges in Frame 6B, the plastic rotation demands for both frames had to be approximately the same.

6.6 Column Shear Forces

Envelopes for the shear forces across the interior column of the two six storey frames in this study are also shown in Fig. 9. The bases and significance of the various envelopes have been outlined in Section 5.6 in connection with the 12 storey frame. It is seen again that the design shear envelopes consistently predicted ample margins of shear strength for the forces encountered during the El Centro excitations. The agreement may be considered to be satisfactory also for the Parkfield event. The shear forces are not critical. In Frame 6A the contribution of the concrete to the shear strength of the upper storey columns, is in excess of the shear demand. Hence with only nominal column transverse reinforcement

FIG. 8 ENVELOPES FOR MAXIMUM PLASTIC HINGE ROTATIONS

FIG. 9 ENVELOPES FOR SHEAR FORCES ACROSS COLUMNS

FIG. 10 PLASTIC HINGE FORMATION IN THE 3 STOREY FRAME AT CRITICAL INSTANTS OF THE REDUCED PACOIMA GROUND MOTIONS

shear failure should never occur in these columns.

6.7 A Comparison of Responses

The response of the corresponding 6 storey frames, designed to Zone A and Zone C load requirements respectively to the same ground motions was found to be very similar. As expected, the frame in Zone C responded more vigorously to all excitations. This is particularly evident in the larger demands for plastic deformations. Scaled down earthquake records were not applied in this project, but it is probable that the use of these in the analysis would have shown displacement response and hence damage control in Zone C frames comparable to or better than those predicted⁽⁷⁾ for the corresponding frame located in Zone A.

When comparing the performance of Frame 6A with its counterpart in Zone A⁽⁷⁾, it is again useful to form the ratio of quantities relevant to Zone A to those for Zone C. It is found that the approximate (Zone A to Zone C) ratios for equivalent masses, stiffnesses, lateral load carrying capacities and permanent displacements at the end of the Pacoima and Parkfield excitations, were 1.16, 2.17, 2.27 and 0.33 respectively. The ratio of absolute maximum displacements encountered by the two frames during the El Centro motions was 0.60.

7. THE RESPONSE OF THE THREE STOREY FRAME

7.1 Frame Characteristics

An economy consideration in the design for earthquake resistance of a gravity load dominated frame is the necessity of placing an upper limit on the level of required lateral load resistance. In the 3 storey frame chosen and shown in Fig. 1, the factored gravity loads governed the proportioning of the beams. To reduce weight and reinforcement content these beams were haunched at the interior supports. If the 'strong-column - weak beam' seismic design philosophy had been rigidly applied to ensure that column yielding, if any, occurred only after the formation of complete beam sway mechanisms, the column design shears and moments would have become unnecessarily large. Consequently partial beam sway mechanisms⁽¹⁸⁾ were adopted at all floors of this frame. To complete the sway mechanism hinging was admitted, in addition to the column base regions, at both ends of the interior columns. Moreover, to reduce further earthquake induced moment demands, which are associated with the yielding of the bottom beam reinforcement at exterior columns, the beam positive plastic hinges were relocated away from the face of the exterior columns.

The level of loading at which this partial beam sway mechanism is acceptable for this Class III frame, was considered^(5,18) to be equal to 3 times that specified by the loading code⁽⁴⁾. Consequently column hinging was admitted in the interior columns

only after the sum of the compatible ideal flexural capacities of all the beams attained a load corresponding with 3 times the code load demand. The exterior columns were designed, however, to resist 1.3 times the moments corresponding with the flexural overstrength of the adjacent beams. Thereby it was hoped to ensure that, if required, plastic hinges in both the beams and the interior columns, will be spread over the full height of the frame and that a column (soft) storey mechanism will be avoided. Moreover, it is hoped that the ductility demand in interior column hinges will be moderate. It will be appreciated, therefore, that the dependable lateral load carrying capacity of this prototype frame is approximately 2.7 times as much as that required by the loading code⁽⁴⁾. This high level of lateral load resistance is achieved by a negligible increase in reinforcement because, as stated above, factored gravity load demands controlled the strength of beams.

7.2 Interstorey Drift and Lateral Displacements

In spite of the considerable reserve strength of this frame with respect to lateral code loading, it responded vigorously to the El Centro excitation. As can be seen in Fig. 2 the reference drift limit of $h/100$ in the first storey was almost attained. This is an indication that the response spectrum, used in the New Zealand Design and Loading Code⁽⁴⁾, is more optimistic for low period structures. Drifts during the Pacoima excitation were of the same order as those exhibited by the other Zone C frames of this study. The dramatic reduction of drift during the response to the reduced Pacoima excitation should, however, be noted. This difference is attributed to the fact that a transient column storey mechanism developed only for the very short period of 0.02 seconds during the reduced Pacoima excitation, whereas column storey mechanisms occurred on two occasions for periods of 0.26 and 0.13 seconds respectively, during the full Pacoima ground motions.

As Fig. 3 shows, apart from instants at about 3 seconds of the record, no significant difference in the displacement response to the two Pacoima excitations were predicted. The permanent inelastic deflections at 14 seconds, relative to the total height of the frame, were smallest of all the frames studied. The frame responded mainly in its first mode for which the computed period was 0.72 seconds.

7.3 Plastic Hinge Formation

According to the analysis at no instant during the El Centro excitation did yielding occur in exterior columns above the base. Simultaneous hinging of all four column bases were predicted at five separate intervals of the 14 seconds record of the El Centro motions. This is in contrast with the response of

the 6 storey frames, in which only one such interval was encountered during the El Centro event, while column base hinging was not predicted at all for the 12 storey frame. This indicates once more the more vigorous response of low period frames, designed to the spectrum of the New Zealand loading code for El Centro excitations.

The maximum number of primary energy dissipating hinges developed during the El Centro motions was 70% of the possible total. Beam hinges adjacent to interior columns are termed secondary because they cannot significantly contribute to energy dissipation, which occurs mainly in the adjacent column hinge or hinges.

No transient column storey mechanisms were predicted during the El Centro ground motions.

In spite of the precautions taken in the design, transient hinges were predicted at the top of the first storey exterior columns during the reduced Pacoima excitation. Thereby a transient column storey mechanism of no significance developed. The most critical stages of plastic hinge developments, extending from 3.20 to 4.04 seconds of the reduced Pacoima event, are reproduced in Fig. 10. The solid circles represent plastic hinges developing with a storey sway to the right, and open circles with a sway to the left.

Plastic hinge developments during the full Pacoima excitations were similar except that simultaneous base hinging and transient storey sway mechanisms were predicted more often and with considerably increased ductility demands.

7.4 Column Bending Moments

In Fig. 7 moment envelopes for an exterior column of this 3 storey frame are presented. As plastic hinges were expected for the interior columns, corresponding moments are of no special interest and hence envelopes for these columns are not reproduced.

Because the moment input into exterior columns depend greatly on the direction of the load and the corresponding flexural capacity of the exterior beams, the design moment envelopes are not symmetrical. Reinforcement was provided for the peak design moments and for practical reasons this was maintained throughout the column. As a consequence very considerable excess flexural capacity, such as at the base, was provided at various levels. In contrast to the taller frames of this study, the El Centro motions generally generated moments larger than designed for. As Table 1 shows the large flexural strength in the exterior columns was achieved by a reinforcement content of 2%. In contrast the reinforcement in the interior columns was only 0.8% of the gross concrete sectional area throughout the full height of the frame.

7.5 Inelastic Rotation Demands

The approximately linear variation

with the height of the frame of plastic hinge rotations in the beams is seen in Fig. 8 for all three earthquake records considered. During the El Centro event the predicted first floor beam plastic hinge rotations near the exterior columns were approximately twice as large as in the taller frames of this study. As expected, at the interior columns beam⁻³ hinge rotations did not exceed 2.5×10^{-3} radians. For the Pacoima motions plastic rotations in beams and columns were close to the limits attainable in reinforced concrete members.

Fig. 8 also shows that the maximum exterior column hinging, associated with transient storey mechanisms, i.e. hinges at the top of the first storey column, were moderate for the Pacoima and negligible for the reduced Pacoima motions. Plastic rotations at the intended hinges at the ends of the interior columns varied between 4×10^{-3} radians at the roof to 12×10^{-3} radians at the base during the reduced Pacoima motions. The corresponding plastic rotations were 8×10^{-3} and 30×10^{-3} radians for the 14 seconds of the full Pacoima excitation, during which some interior columns encountered full strength load reversals.

7.6 Column Shear Forces

The envelopes for the maximum shear forces predicted across an interior column of the 3 storey frame are shown in Fig. 9. It is seen that the proposed⁽⁷⁾ design shear forces would provide satisfactory protection against column shear failure for any of the ground motions used in the study.

7.7 Behaviour of the 3 Storey Frame

As no corresponding frame was previously studied, no comparison with a Zone A structure could be made.

The computer simulated response of this gravity load dominated frame to the El Centro and reduced Pacoima ground motions, both of which may be considered to represent excessive excitations for Zone C, was very satisfactory. The selected hierarchy of failure mechanisms was maintained throughout the excitations and satisfactory protection was provided for the exterior columns. Having provided lateral load carrying capacity approximately 170% in excess of that required by the loading code⁽⁴⁾, it was surprising to observe that the predicted response to the El Centro record was so vigorous. The comparatively larger plastic rotational demands are likely to be due to the formation of a lesser number of plastic hinges throughout the frame. This phenomenon required larger energy dissipation per plastic hinge. It may be assumed that storey column mechanisms, which would have developed had strong exterior columns not forced plastic hinge development throughout all three storeys, would have resulted in an even larger ductility demand on the much reduced number of column hinges. Because of the larger ductility demand for the same

excitation of this 3 storey frame and because of the larger number of load reversals in plastic hinges, the suggested design procedure for gravity load dominated frames^(5,8), involving the provision of lateral load carrying capacity up to three times that stipulated by the loading code⁽⁴⁾, does not appear to be unduly conservative.

8. SUMMARY AND CONCLUSIONS

To study the extent to which the behaviour of ductile reinforced concrete frames, designed to the lowest (Zone C) earthquake load requirements in New Zealand⁽⁴⁾, would match intended performance, the inelastic dynamic time-history analysis of four prototype structures, exposed to a number of severe ground excitations, was undertaken. The study also allowed a comparison to be made with the results obtained with identical analyses of similar frames designed to the most severe (Zone A) lateral load requirements. From the predictions of this analytical study it is concluded that:

- (a) The chosen hierarchy of ductile energy dissipating mechanisms was maintained for all frames exposed to the full El Centro excitations. For the six and twelve storey frames this feature was predicted even for the most extreme Pacoima motions.
- (b) The response to the El Centro excitation of the low rise gravity load dominated frames was more vigorous than that of the taller frames. This indicates that the New Zealand design spectrum⁽⁴⁾ is increasingly optimistic for low period structures, or conversely, that it is unduly conservative for long period structures.
- (c) The analyses indicated that code specified drift limits were not exceeded during the El Centro event. This suggests satisfactory damage control in these Zone C frames.
- (d) Predicted column actions for the El Centro event were comfortably within design values, and maximum plastic rotational demands for beams and for columns at ground level are considered to be easily attainable with standard⁽⁵⁾ detailing of these members.
- (e) P-delta effects were found to be critical for the 12 storey frame only and only for the extreme Pacoima excitation, an event irrelevant to risk expectation in Zone C of New Zealand. It may be concluded therefore that for reinforced concrete frames, designed to the strength and stiffness requirements of the New Zealand design codes^(4,5), P-delta effects need not be taken into account.

- (f) Envelopes for predicted drifts for all the prototype frames were found to be significantly different from those obtained from elastic analyses for lateral static loading.
- (g) Analyses indicated that, at least for the El Centro excitation, plastic hinge rotation demands increased in frames in which, because of the relative strength allocations, the number of plastic hinges mobilised was reduced.
- (h) As expected, the inelastic rotation demands and permanent inelastic drifts, caused by a given ground excitation, were found to be larger in the Zone C frames than in those designed to Zone A requirements.
- (i) The predicted overall performance of the frames studied indicate that existing loading and stiffness requirements⁽⁴⁾, when combined with capacity design procedures suggested for ductile frames⁽⁵⁾, are likely to result in a very high degree of seismic protection for frames situated in Zone C of New Zealand. From this study a relaxation in these requirements might appear to be warranted. However, such a move would not result in economic benefits as reinforcement, found to be necessary to provide the seismic resistance in members of these frames, was equal or close to that considered to be the acceptable minima.

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