SECTION C
SHEAR WALLS OF LIMITED DUCTILITY
L.M. Robinson*

ABSTRACT:

The design and detailing of earthquake resistant reinforced concrete shear walls of limited ductility designed by a modified strength design method are discussed. Suitable methods for the evaluation of actions and the determination of internal actions are advanced, having regard to energy dissipation and the consequences of heavy damage or of collapse. Discussion is restricted to uniform walls, but is extended to walls with openings, for which a suggested classification and treatment is presented, thus allowing for suitable design techniques for walls transitional between uniform walls and frames to be determined. Applications of the proposals are illustrated in an Appendix.

INTRODUCTION:

Preamble: Requirements of NZS 4203

NZS 4203 provides for a variety of structural type factors $S$ which are dependent principally on the anticipated mode of behaviour and the ability of the wall to dissipate significant amounts of seismic energy.

Cantilever walls are additionally categorised, in the same manner as ductile frames, according to the redundancy of the horizontal force resisting system. Accordingly a single ductile cantilever wall is assigned a structural type factor of $S=1.2$, while two or more parallel ductile cantilever walls are assigned a structural type factor of $S=1.0$.

To qualify as "ductile cantilever shear walls" walls must have a height to depth aspect ratio of at least 2.0. This provision infers a reservation that ductile flexural yielding, the energy dissipation mechanism generally sought, cannot be assured whenever this ratio is not greater than 2.0.

When the aspect ratio is 2.0, or less, walls must be designed for loadings derived from $S=1.6$. Walls designed at $S=1.6$ must be "suitably detailed" to ensure:

a) adequate confinement of concrete at potential hinges to provide limited ductile flexural yielding; and

b) that in walls with height to depth ratio less than or equal to 2, under earthquake attack, a distributed system of (shear) cracking of controlled width will form so as to preclude premature shear failure. No limit is placed on the height to depth ratio of such walls, and the requirements of capacity design are waived.

Frames of limited ductility are assigned a structural type factor $S=2.4$ unless the seismic design coefficient exceeds 0.36, in which case a smaller value is applied. Such frames must have vertical members at least 800 mm wide or beams at least 750 mm deep. When elastic response design is employed, using $S=6$, all detailing and dimensional limitations are waived. In either case, capacity design is not required.

General: Scope of Discussion

Walls of limited ductility might be defined as those which, at the maximum anticipated structural displacement, do not form flexurally ductile plastic hinges which would limit seismic actions. The total resistance of such walls may be governed by shear strength rather than by flexural strength.

It is clear that in spite of the designer's initial intentions, a flexurally ductile plastic hinge may not be able to form because of a gross overstrength in flexure brought about by material code(2) minimum reinforcement requirements, including vertical web reinforcement especially in squat walls, or constructional restraints such as the presence of walls framing in from other directions. Such an over-strength may occur regardless of the $S$ factor assigned to the element. It may not be appropriate then to detail the potential hinge region as though such a hinge would develop. Nor would it appear appropriate to pursue the philosophy of capacity design to the extent that for instance shear strength is matched to a very high flexural strength.

The waiver of capacity design requirements for all walls designed for $S=1.6$, however, appears to be dangerous, especially for walls of large aspect ratio, and is inconsistent with the general philosophy attaching to seismic resistance. Recognition must however be given to alternative mechanisms of energy dissipation such as shear related deformations, especially those in squat walls.

This paper attempts to introduce a suitable approach to such questions as:

- upper limits of design shear force;
- the degree to which controlled shear related deformations can be relied upon to limit seismic actions;
- the apportionment of shear resistance to the concrete;
- the degree of confinement required; and
- detailing. Discussion is restricted to those walls which may be proportioned and designed by strength methods alone. In

* Consulting Engineer, Dunedin
addition, the important issue of the transition of structural elements from uniform walls through walls with holes to frames is discussed and a suggested classification and treatment presented.

**DESIGN LIMITS FOR SEISMIC ACTIONS:**

**General Considerations**

Where ductile flexural hinges do not form, or the inelastic shear deformations are not then reliably limited by structural capacity, but may be by some upper bound on anticipated structural displacement. Such an upper bound might be based on an assumed elastic response design spectrum.

The design spectrum specified in NZS 4203 corresponds to between one-quarter and one-sixth peak response relating to systems with high strain capacity. It is not unduly conservative, especially for short period structures which are expected to form the bulk of structures under discussion, as the spectrum specified in NZS 4203 for short period structures is in the order of one-sixth of the elastic spectrum of El Centro type motions. It is expected however, that higher damping than customarily assumed will follow damage making the suggested limitation appropriate.

In an analytical investigation Murakami and Penzien[3] carried out nondeterministic response analyses using a stochastic model to represent the expected ground motion. Twenty each of five different types of artificial earthquake accelerograms were generated for computing nonlinear response spectra of structural models representing reinforced concrete buildings. The structural models were identified by hysteretic loops characteristic of member cross-sections subjected to extensive damage but at levels of deformation not involving yielding of reinforcement. These nonlinear elastic response spectra indicated that loads corresponding to $S=3.2$ for shear-failing systems or $S=2.4$ for systems with high strain capacity, represent the 85 per-centile level of response, when reinforcement does not yield, and where the damage criteria suggested by Umemura[4] are met.

The trend of these results may also be deduced from examination of linear response spectra drawn for systems possessing high viscous damping. Equivalent viscous damping in the range 10-25% of critical were identified in the study for those systems exhibiting load-deflection hysteretic loops typical of systems failing in a flexural mode and displaying stiffness degradation.\(^{(3)}\)

**Inelastic Shear Deformations**

In addition to the above considerations it is recognized that, particularly in squat walls, some dissipation of energy may occur due to processes other than ductile flexural yielding, such as inelastic shear deformation. Quantification of the dissipation occurring in such processes is difficult, however, and there appears to be no reliable method currently available which would allow the calculating the resulting degree of attenuation of seismic actions to be made with any precision. Further the total seismic displacement ductility required of short period structures to a given earthquake than long period structures having yield strengths less than two. It should also be noted that short period structures are more likely to be subjected to a greater number of yield excursions in a given earthquake than long period structures having yield strengths of similar fractions of spectral values. Consequently the cumulative ductility demand, which has some relevance to damage potential is high.

Because of the severe stiffness and strength degradation associated with inelastic shear deformation\(^{(4)}\), and the unreliability of this phenomenon in evaluation of response, the concept has been ignored, but it is recognized that if displacement response corresponding to $S > 3.2$ is encountered in structures having yield strengths less than two, this source of ductility and damping may prove decisive in avoiding sudden failure.

**ANALYSIS:**

**Structural Type Factors**

**Uniform walls (walls without openings)**

Where uniform walls are proportioned and designed according to the strength method alone it is suggested that flexural strength be based on a structural type factor equal to 1.6, for all values of the aspect ratio, in recognition of the poorer anticipated performance of these walls. At such high loading, and because wall-like elements, particularly the squatter types, tend to form sub-structures prior to complete failure the question of
redundancy would appear to have little relevance, and therefore the value of 1.6 may be used to design structures in which horizontal restraint is provided by a single wall orientated in the direction of loading.

Where uniform walls of various aspect ratio are present, which is not uncommon in buildings of the type for which the strength method will find greatest application, the use of a constant $S$ factor obviates any complication arising from the variations in $S$ factors dependent on aspect ratio, in the overall analysis of the structure.

Walls with openings –

All structural elements in reinforced concrete rely on truss-like mechanisms for the efficient resistance of at least that part of the imposed shear characterised by $(v-v')$, especially when the elements are subjected to reversing cyclic loading. Therefore detailing should be such as to allow of the formation and maintenance of such viable truss mechanisms.

Such mechanisms are more difficult to enhance where structural elements are penetrated by holes, not only because the holes may interrupt any necessary diagonal force field but also because of unavoidable stress concentration surrounding the holes.

Whether the holes are such to produce overall frame-like action or to endue a wall to behave essentially as a wall with penetrations, depends on the relative sizes of the holes and their distribution.

It is to be appreciated that there is a gradual transition from uniform walls with holes, through deep-membered frames, to frames of more usual proportions. It is accordingly difficult to introduce comprehensive guidelines to allow an exhaustive categorisation.

Furthermore, the overall mode of action may be quite distinctly different in the inelastic range from that revealed by elastic analyses in the pre-yield range. In general, behaviour prior to yield will be controlled by stiffness, but following yield, behaviour will be more directly related to relative strengths of component members especially when yielding occurs throughout the structure.

For a given loading pattern, elastic analysis will produce a set of design actions and these will allow of a reasonable estimate of required strength, and the distribution of required strength throughout the structure, to be made. For the same loading pattern, the manner in which the structure will collapse can be established by the identification of a critical collapse mechanism. Thus any preferred collapse mechanism can be chosen, and suitable design techniques employed to ensure that such a mechanism may occur.

In the real earthquake conditions, difficulties are presented by the varieties of loading patterns which may occur during seismic excitation; a critical condition which may for instance lead to undesirable failure such as flexural failure of columns, may develop. Conditions may even be such that the preferred mechanism, or indeed any mechanism at all if particular members are to be absolutely safeguarded against failure, cannot physically form.

Structures of limited ductility not proportioned according to the principles of capacity design present additional problems because, unless a capacity design approach is followed, identification of a collapse mechanism, even with a correctly assumed loading pattern, cannot be made. Where design loadings are significant fractions of those derived from a elastic response analysis, application of capacity design principles becomes less important. Attendant on this consideration is the implication that structures which are particularly vulnerable to undesirable failure mechanisms should be designed to a higher equivalent static loading than those which are more tolerant to such conditions.

Such structures are those which are "frame-like", and the required increased strength may be measured by higher $S$-factors, and by shear amplification where appropriate.

The following suggestions are made with reference to walls with openings, designed by the strength method. As with uniform walls, capacity design procedures should be applied wherever practicable.

$$P_o = \frac{A_o}{A_w}$$ (C-1)

To be useful in describing the vulnerability of walls of limited ductility to undesirable failure mechanisms the appropriate areas $A_o$ and $A_w$ need to be suitably defined.

It is suggested that $A_o$ should be the aggregate of all openings in a storey, and, for irregularly shaped holes, should be based on the smallest rectangular area with vertical sides which can encompass the hole. Such a rectangle may also envelope other holes in the neighbourhood.

$A_w$ should be measured in the same storey in which $A_o$ is measured, and therefore might be considered to be greater than $P_o^2$ that is a square panel might be taken as a basis.

Examples of the application of these suggestions are shown in figure 1.
a) For all these walls $p_o \geq 0.4$, $S = 2.4$

Fig. 1. PROPOSED STRUCTURAL TYPE FACTOR $S$ FOR WALLS WITH OPENINGS

Fig. 2. SECONDARY ELEMENTS.
For \( p_o > 0.4 \), Muto has suggested that the action of the walls with holes partakes largely of the action of deep membered frames. Thus when \( p_o > 0.4 \), \( S=2.4 \) might be considered appropriate.

Wall action might be considered to predominate when \( p_o < 0.2 \), corresponds to the holes occupying about 4% of any storey. Then \( S=1.5 \) would appear to be relevant.

For intermediate cases \( S \) might be assumed equal to \( 0.8 + 4p_o \).

It is therefore proposed that the structural type factor to be applied to walls with holes, and which are exempt from capacity design, should be assessed in accordance with

\[
1.6 \leq S = 0.8 + 4p_o \leq 2.4 \quad (C-2)
\]

Where \( p_o > 0.4 \) the walls should be analysed and designed in accordance with the requirements for frames. Discussions of frames is beyond the scope of this paper.

Treatment of some elements as secondary by excluding them from the primary system should be permissible. It is therefore appropriate to allow of relaxation of the provisions of the foregoing where a penetrated wall can be rationalised into uniform walls by the exclusion of such elements. Generally this should be restricted to those elements by the removal of flexural continuity, but columns could be treated similarly. Examples of this approach are shown in figure 2.

Thus the increases in \( S \) factors or the restriction on \( p_o > 0.4 \) need not be applied where a penetrated wall can be rationalised into a series of uniform walls by the exclusion of such elements. Generally this should be restricted to those elements by the removal of flexural continuity, but columns could be treated similarly. Examples of this approach are shown in figure 2.

The effect of the resultant reactions on the primary structure should be allowed for.

Elastic response design procedure -

The structural type factor specified in NZS 4203 for structures designed by the elastic response design procedure, is \( S=6 \). Such a high figure was selected as appropriate by consideration of some special forms of structures, such as cross-braced tankstands constructed of structural steel, in which detailing for even limited ductility might not be appropriate or economically justified.

The section on Design Limits for Seismic Actions suggests an upper limit on design actions corresponding to \( S=4 \), on the basis of significant effective damping being achieved, and on the basis of analytical studies and non-linear response spectra derived from these studies(3). Such behaviour can be expected of normally detailed reinforced concrete structures.

Since detailing in accordance with generally acceptable practice for gravity load effects and wind loading will not usually be adequate for seismic situations unless the strength of the structure is high, the use of \( S=4 \) in reinforced concrete structures not designed for at least limited ductility is valid.

The appropriateness of \( 1.6 \leq S \leq 2.4 \) is dependent on the structure being able to sustain significant damage without a corresponding severe erosion of strength. Clearly this condition can only be met where appropriate detailing is employed.

The application of the elastic response design procedure for concrete wall structures will not be widespread. Its use will tend to be limited to structures possessing, by their nature and position, a high intrinsic strength, such as fire walls between adjoining residential units in apartment buildings.

Where the elastic response design procedure is employed, no special detailing or analysis will be required.

Mixed structures -

Common configurations and arrangements of wall elements in buildings will often mean that different values of \( S \) factors will be assigned to various resisting elements. It is appropriate that such suggestions be made as to how the overall analysis of the structure is to be performed.

The following suggestions are therefore offered to form a tentative basis for treatment of such mixed structures, pending further in-depth study of the wide range of mixed structures met with in practice. The suggestions should be used with caution, especially because of the manner in which torsional response may be affected.

(a) The analysis of the entire structure is performed using a value of \( S \) and of \( M \) equal to unity for all elements, and the loads on each load-resisting element are derived.

(b) Each load-resisting element in turn is designed for the loads found in (a) multiplied by the product \( SM \) relevant to it.

An alternative analysis procedure can also be performed in one step, by first modifying all stiffness terms of each element by multiplying by \( SM \) appropriate to that element, to derive equivalent stiffness terms for each element. These equivalent stiffnesses are then used in the analysis, in which case the loads appropriate to each element are determined directly. It may be shown that the loads thus determined are identical to those derived by the analysis performed under (a) and (b) above.

Analytical procedures -

The procedure to be followed in
analysis is common to walls of all types, and therefore the recommendation contained in Reference 9 should be followed. No differentiation between structures designed by the capacity design procedures or the strength method need be made in the analytical evaluation of design actions. It may be assumed that the design and detailing measures suggested later in this paper will permit sufficient ductile action to allow for redistribution of actions derived from the elastic analysis of the structure.

Where moment redistribution is employed, it may be assumed that the actions requiring amplification subsequently suggested are those associated with the redistribution of area. The apertures in these designated end-regions for S=1.6, compatible with flexural yielding, the additional moment redistribution may be carried out to reduce shear demands for instance, as much as to reduce flexural demands, and this suggestion allows for this possibility. Where gravity load effects are significant it will be necessary to deduce the seismic actions associated with the redistribution by more general analysis.

**Deflections -**

Recommendations for the calculation of deflections are presented by Paulay and Williams. Smaller degradation of stiffness due to inelastic deformations are to be anticipated in walls with higher intrinsic strengths; but it is likely that the greater number of stress reversals associated with shorter periods of vibration anticipated for walls of limited ductility, will generally have a roughly compensating contrary effect.

It is therefore recommended that the procedures suggested in Reference 9 be followed. Where the shear deformation components contribute significantly to total displacements it is suggested that these be computed separately and not be combined with flexural displacements, for instance by working with equivalent second moments of area. Other sources of displacement might also be separately computed in many instances.

The reason for computing these sources of deflection separately becomes particularly apparent in relation to such secondary elements as beams framing into squat shear wall structures, where shear deformations in the walls for instance may not contribute greatly to the actions induced in the beams.

**DIMENSIONAL LIMITATIONS:**

As will be subsequently established, considerations of confinement in potential plastic hinge regions must ensure that the neutral axis depth will be less than \( \frac{1}{25} \). Therefore the dimensional limitations suggested in Reference 9 for walls proportioned according to capacity design procedures need not be complied with. In accordance with the lower limit suggested in Reference 10, it is recommended that the minimum thickness \( b \) be equal to \( \frac{h}{25} \), but it should not generally be less than 125 mm for practical reasons such as to facilitate concrete placement.

Where the length of the wall is less than four times the thickness of the web, dimensional limitations relevant to columns may be more appropriate, and it is suggested that in these cases such limits be observed.

**DESIGN FOR FLEXURE AND AXIAL LOADS**

**General Considerations**

The theory for the design of cross-sections subjected to flexure, with or without axial load, is well established, and may be applied directly to the design of walls particularly in view of the extensive and satisfactory testing of walls designed to existing code requirements. In this respect there is no need to take into account the deep-beam nature of squat walls; all reinforcement including web reinforcement required for temperature and shrinkage control or for shear may be assumed to contribute fully to the required flexural strength. The strength reduction factor used in design should be that customarily used in the design of members to resist gravity load or wind, and therefore will lie in the range 0.7 \( \leq \phi \leq 0.9 \), depending on the level of axial stress.

Because inelastic rotations will tend to concentrate at isolated localities within the wall, it is important to identify these critical locations and to detail them accordingly. While the use of \( S=1.6 \), as a minimum, will reduce flexural ductility demand, it will not eliminate it, the level of demand remaining high. Thus, while detailing measures for confinement and for the prevention of buckling of principal flexural reinforcement can be somewhat relaxed below those obtaining in walls designed to be fully ductile, certain minimum requirements remain.

Away from regions of potential flexural yielding, the additional requirements, above those employed in non-seismic applications, need not be met. In order to reduce the likelihood of flexural hinging away from the identified hinging regions, flexural strength should be suitably increased. For this purpose a margin of about 50 percent appears to be suitable and realistic. This is compatible with a flexural overstrength factor of 1.5 in the designated end-regions, and for \( S=1.6 \) walls, corresponds to designing regions outside of the designated end-regions for \( S=2.4 \), compatible with the flexural strength found appropriate in Reference 3.

A margin of 50 percent will not safeguard all regions away from yielding should the designated hinge regions be conservatively designed, pointing to the need to use realistic, and minimum reinforcement in the hinge localities, and to locate the potential hinges in areas where strength and performance can be reliably predicted. Nor will such a margin necessarily safeguard against yielding due to higher mode dynamic effects, but these effects are not likely to produce high ductility demands because of the small...
component of total energy associated with the higher modes of vibration, it is in structures possessing a short fundamental period.

Potential plastic hinge zone -

The critical section of the wall will normally be located at the base. The assurance that yielding will not occur to any great degree elsewhere, and the height to which the potential hinge region will extend up the wall, will depend on the design and detailing at levels above the base. To this end the following suggestions are made:

(a) Where the suggestions concerning the termination of flexural reinforcement in the section 'Termination of flexural reinforcement (b)' are not adopted, the height of the end-region should be assumed to be the full height of the wall.

(b) Where the suggestions concerning the 'Termination of flexural reinforcement (b)' are followed, with consequent adoption of the increased concrete shear resistance and restriction of confinement detailing, the height of the end-region may be assumed to be greater than the horizontal length of the wall, or one-sixth the height of the wall.

Termination of flexural reinforcement -

(a) In general, the flexural reinforcement may be terminated in accordance with the bending moments derived from the application of loads specified in the loadings code (1), and in accordance with the anchorage provisions of the materials code (2,10).

(b) However, to take advantage of concessions proposed for confinement and for concrete shear resistance outside of the end-region defined in 'Potential Plastic Hinge Zone', it is recommended that the flexural reinforcement not be terminated unless the continuing bars provide a dependable moment of resistance of at least 1.5 times the moment derived from the loadings code, taking 1.6 \( \leq S \leq 2.4 \). In many cases this requires reasonably small extensions of flexural reinforcement above cut-off points required in accordance with (a), and the anchorage provisions of the code (2,10) need still be met.

The suggestions of 'Potential Plastic Hinge Zone' and 'Termination of Flexural Reinforcement' are illustrated in figure 3.

Confinement in the end-region -

Since capacity design is not required, it is inappropriate to enforce calculation of overstrength flexural capacity. Without this any rule relating to neutral axis depth as in Reference 9 is also inappropriate.

It is doubtful that complex analytical procedures will shed much light on the true position, and, in any event, especially in view of the approximations used elsewhere, simple procedures are to be preferred. Furthermore it is preferable for other reasons, such as in ensuring that lateral instability is not a critical design criterion, to restrict the neutral axis depth to the critical value or less, particularly in view of the relative ease with which this objective can be accomplished. Several simplifying assumptions can be made, in particular assumptions of compressive failures, effective depths, efficacy of flanges if present, and the like.

A simplified approach based on the recommendations of References 9 and 12 is as follows, as applying to the end regions only:

(a) Where the value of \( \gamma \) calculated in accordance with equation (C-3) exceeds 1.0, that region defined by \( A^* \) should be confined as for columns but with reduced confining steel in accordance with (b) and (c)

\[
\gamma = \frac{M^* + 0.3P_0u_{aw}}{0.6 f_c' A_g'w} \leq 3.0 \quad \text{(C-3)}
\]

where \( M^* \) is the moment demand derived from code (1) loading referred to the mid-depth axis of the wall cross-section, \( P_0 \) is the axial load acting simultaneously with \( M^* \) and \( A_g \) is the area of concrete bounded by the compression edge of the section and by a line parallel to and located 0.2\( w \) therefrom.

The definition of \( M^* \) thus, avoids algebraic complexity, but may require minor transformation because design moments will generally be related to the centroidal axis, which may not coincide with the mid-depth axis of the wall section.

(b) The quantity of confinement reinforcement, \( A_h' \), consisting of hoops and/or supplementary cross-ties, in each set spaced at \( s_h \), is given by

\[
A_{sh} = R_c \left( 0.02 s_h w_f \frac{f_c'}{f_{yh}} \right) \quad \text{(C-4)}
\]

(c) The reduction factor, \( R_c \), is dependent on the value of the moment and axial load on the section, as measured by \( \gamma \), and on the quantity of compressive reinforcement within \( A_g \), given by \( A_h \), expressed as the ratio \( A_h/A_g = \rho' \)

\[
R_c = \frac{\gamma - 1}{1 + \rho'm} \leq 1.0 \quad \text{(C-5)}
\]

in which \( m \) equals \( f_{c'}/0.85f_{c'}^* \). In all cases the relation between \( \gamma \), \( \rho' \) and \( m \) should be such that \( R_c \) does not exceed unity.

In no case should \( \gamma \) exceed 3.0. When \( \gamma \) exceeds 1.0, walls with a single layer of reinforcement should
Fig. 3. THE INFLUENCE of TERMINATION of FLEXURAL REINFORCEMENT on HEIGHT of END REGION
Figure 4 defines the relevant quantities. It is easily established that confinement will seldom be required for squat walls; it will tend to be required in tall walls or in walls which are subjected to high axial load - walls which are 'column like'. Even where the possibility of the need for confinement is indicated through computing $\gamma > 1$, it is likely that the provision of a small ratio of compression reinforcement $p^*$ will enable $R_c$ to be made zero, thus obviating confinement.

Where confining reinforcement is required it should be satisfactorily arranged as suggested in 'Lateral tying of longitudinal reinforcement' and 'Spacing of confinement reinforcement'.

For rectangular sections, or T or L shaped sections in which the flange is in tension, the foregoing equations can be greatly simplified. The resulting criteria are shown in figure 5.

**DESIGN FOR SHEAR:**

**Design Shear Force -**

The performance of structures exposed to recent damaging earthquakes in Japan, and the analysis relating to the observed performance, suggest empirical rules appropriate to the level of shear resistance required of walls not specifically designed and detailed for flexural ductility. These observations and the corresponding approach appropriate to the review of structures are reported by Glogau (13) who draws comparisons with the existing requirements of NZS 4203 (1). In accordance with the principles discussed in "Design Limits for Seismic Actions", and as generally verified by this Japanese experience, the code derived shear forces need to be amplified to account for the limited amount of inelastic shear deformation which can be relied on to limit peak response.

The shear strength furnished should therefore be sufficient to resist loads corresponding to $S=3.2$. It is to be noted that the amplification is therefore required on the seismic component of the total shear only, so that the required ideal shear strength is given by

$$V_i \geq \frac{3.2}{5} \cdot V_e + V_d + 1.3V_{1R} / \phi \quad (C-6a)$$

or

$$V_i \geq \frac{3.2}{5} \cdot V_e + 0.9V_d / \phi \quad (C-6b)$$

in which $\phi$ should be taken as 0.85.

**Maximum Design Shear Stress**

It is great importance to avoid failures induced by diagonal compression, and therefore, in accordance with established practice, the design shear stress should not exceed $0.83\sqrt{f_c}$.

$$V_i = V_i / b_d \leq 0.83\sqrt{f_c}$$

in which $d$ should be assumed equal to $0.8d_w$.

**Shear Resisted by the Concrete**

The most important phenomenon responsible for the deterioration of the shear strength of the concrete in the end region of a shear wall, as measured by $v_c$, is the extent of flexural yielding during reversed cyclic seismic loading. The degradation is further accentuated by the number of occasions when such flexural inelastic excursions are encountered during an earthquake. It can therefore be expected that the value of $v_c$, to be relied upon in the design, will diminish with small values of $S$. As the flexural capacity of the critical wall section increases, as when larger values of $S$ are specified, both the demand for flexural yielding and the number of inelastic displacement excursions will be reduced. Consequently the contribution of the concrete to the shear strength of the end region will increase.

With the value of $S$ set at a minimum value of 1.6 it is to be expected that a significant fraction of $v_c$ specified in non-seismic applications may be considered furnished, especially since the shear strength, including the contribution from $v_c$, will be high, in accordance with 'Design Shear force'.

(a) Conservatively it may be assumed that in the end region the concrete affords one-half of the contribution specified for elements subjected to gravity loading only.

Where the minimum axial compressive stress $N/A$ on the wall, to be considered concurrently with earthquake actions, exceeds 2MPa, the value of $v_c$ need not be considered less than

$$v_c = 0.4 \cdot \sqrt{\frac{N_u}{A} - 2} \cdot \frac{f_c}{20}$$

(C-8)

These suggestions are shown in figure 6 for the value of $f_c = 20$MPa.

(b) Outside of the end-region, extensive flexural yielding is not expected. The design for shear in the wall may therefore proceed as for gravity load conditions. The requirements are specified in ACI318 (16).

**Design for Horizontal Shear Reinforcement**

The ratio of horizontal shear reinforcement $p_h$ may be determined as usual,

$$p_h = \frac{V - V_c}{V_{th}} \geq \frac{0.7}{\gamma_h}$$

(C-9)

in which $V_{th}$ depends on the location of the design section, either within or outside of the end-region.
Fig. 4. DEFINITION DIAGRAM FOR A TYPICAL SHEAR WALL CROSS-SECTION

Fig. 5. INTERACTION DIAGRAM for CONFINEMENT REQUIREMENTS IN RECTANGULAR WALL CROSS-SECTIONS.
Fig. 6. CONCRETE SHEAR CONTRIBUTION, $v_c$, in END-REGION
(Bracketted terms [ ] from ACI 318:77\(^{(10)}\))

\[ v_c = 0.5 \left[ 0.17 \sqrt{f'_c} \right] \]

\[ v_c = 0.5 \left[ 0.27 \sqrt{f'_c + N_u} \right] \]

\[ v_c = 0.5 \left[ 0.5 \sqrt{f'_c + 5A_g} \right] \]

\[ v_c = 0.5 \left[ 0.5 \sqrt{f'_c + 5A_g} \right] \]

$N_u \geq 0.1 f'_c A_g$

For $N_u > 0.1 f'_c A_g$

\[ f'_c = 20 \text{ MPa} \]

\[ S > 1.6 \]

Fig. 7. SHEAR WALL IN DESIGN EXAMPLE 1.
\[
D = 610 \text{kN} \\
L_R = 300 \text{kN} \\
V_{\text{code}} = 150 \text{kN}
\]

\[S = 1.6\]
\[f'_c = 20 \text{ MPa}\]
\[f_y = 275 \text{ MPa}\]

(a) Wall Details and Loading

(b) Ideal Interaction Diagram

Fig. 8. SHEAR WALL IN DESIGN EXAMPLE. 2.
(a) Wall Cross Section

(b) Ideal Interaction Diagrams

Fig. 9. SHEAR WALL OF DESIGN EXAMPLE. 3.
Design of Vertical Web Reinforcement

Reinforcement for diagonal tension -

Where failure is in a shear mode, considerations of equilibrium of internal truss mechanisms suggest that $\rho_n = \rho_v$, when aggregate interlock is ignored. Shear failure however is assumed to be suppressed by the foregoing suggestions, and therefore existing material code requirements are not considered appropriate. In conformity with established practice however a minimum ratio of reinforcement should be provided, and accordingly

$$\rho_n \geq \frac{0.7}{f_y}$$  \hspace{1cm} (C-10)

The adoption of this suggestion will do much to alleviate the problems associated with flexural overstrength in squat walls generated by the inclusion of excessive vertical web reinforcement, which makes the attainment of a desirable hierarchy of failure difficult. This reinforcement should be considered to contribute to the required flexural strength of the wall.

Reinforcement for sliding shear -

It is recommended that the suggestions of Reference 9 be followed, according to which the ratio of reinforcement crossing a construction joint at right angles is given by

$$\rho_{vf} > \left( \frac{N_i - N_u}{\bar{A}_g} \right) / f_y$$  \hspace{1cm} (C-11)

where $N_i$ is the minimum design compressive force of the wall. For tension, $N_u$ should be taken as negative.

Where the value of $\rho_{vf}$ exceeds the minimum value of $\rho_v$ suggested in 'Reinforcement for diagonal tension', it should be terminated as quickly as possible beyond the construction joint, or alternative means of inhibiting sliding should be utilised. For example, the use of diagonal wall reinforcement has been suggested in Reference 9.

DETAILS OF REINFORCEMENT:

Longitudinal Reinforcement

The ratio of longitudinal reinforcement over any part of the cross-section should not be less than $0.7/f_y$, nor more than $16/f_y$. In calculating the maximum reinforcement ratio, the gross area of the concrete may be taken as the square of the thickness of the wall at the locality, or as the product $b_w$, whichever is greater.

The diameter of bars used should not exceed one-eighth of the wall thickness at the bar locality. In regions within $A_g$, where confinement is required, the spacing between centres of bars should not exceed 200 mm.

Otherwise the longitudinal reinforcement may be placed in accordance with the code(10) requirements specified for walls resisting gravity or wind load.

Lateral Tying of Longitudinal Reinforcement

When the ratio of longitudinal reinforcement in walls with two layers of steel exceeds $3/f_y$, transverse reinforcement satisfying the requirements for gravity loaded columns(10) should be used, provided that in the end-regions the spacing of this reinforcement along the longitudinal bars should not exceed 10 times the longitudinal bar diameter nor the thickness of the wall.

Spacing of Confinement Reinforcement

The vertical spacing of hoops or supplementary cross-ties should not exceed 10 longitudinal bar diameters nor one-half the thickness of the wall. Spacing of cross-legs horizontally should not exceed the thickness of the wall, and should be so arranged that they engage longitudinal bars spaced not further apart between centres of 200 mm.

Ties provided under 'Lateral Tying of Longitudinal Reinforcement' may be assumed to contribute to the required confinement steel.

NOTATION:

- All lengths are in mm, areas in $\text{mm}^2$, forces in N, moments in N-mm, and stresses in MPa.
- $A_g$ = gross area of the cross-section
- $A_s^*$ = gross area of concrete located between the compressive edge of the section and a line $0.2b_w$ therefrom
- $A_o$ = aggregate elevational area of all openings in a storey
- $A_s^*$ = area of all vertical reinforcement contained within $A_g^*$
- $A_{sh}$ = total effective area of hoop bars and supplementary cross ties in direction under consideration within spacing $s_h^*$
- $A_w$ = elevational area of wall within $a_w$ storey, but not greater than $b_w^2$
- $b = \text{width of compression edge or face, or thickness of a member}$
- $b_w = \text{thickness of wall or web}$
- $c = \text{neutral axis depth}$
- $d = \text{effective depth, equal to the distance from the compressive edge to the centre of all reinforcement in tension}$
- $f_c = \text{specified compressive strength of concrete}$
- $f_y = \text{specified yield strength of reinforcement}$
\[ f_{yh} = \text{specified yield strength of hoop or supplementary cross-tie reinforcement, or of horizontal web reinforcement} \]
\[ h_w = \text{height of the wall} \]
\[ h_n = \text{clear height of wall within a storey} \]
\[ b_w = \text{horizontal length of wall parallel to the applied shear.} \]
\[ M = \text{material factor specified in NZS 4203(1)} \]
\[ M_i = \text{ideal moment of resistance} \]
\[ M_u = \text{moment demand resulting from loading combination U} \]
\[ M_u^* = M_{u} \text{ referred to mid-depth of wall cross-section} \]
\[ m = \frac{f_y}{0.85f'c} \]
\[ M_u = \text{design axial load acting together with } V_i, \text{ and due to loading U} \]
\[ P_u = \text{design axial load, due to loading U} \]
\[ \rho_o = \frac{A_o}{A_w} = \text{peripheral ratio of openings} \]
\[ R_c = \text{reduction factor for confinement} \]
\[ S = \text{structural type factor} \]
\[ s_h = \text{spacing of hoops and/or supplementary cross tie reinforcement sets} \]
\[ U = \text{design load combination specified in NZS 4203} \]
\[ V_d = \text{code specified dead load shear} \]
\[ V_e = \text{code specified earthquake shear demand} \]
\[ V_i = \text{ideal shear strength of section} \]
\[ V_{IR} = \text{code specified reduce live load shear} \]
\[ V_c = \text{ideal shear stress provided by the concrete} \]
\[ \nu_i = \text{minimum ideal shear stress required of the section} \]
\[ \nu = \text{shear stress due to } V_u \]
\[ \gamma = \text{strength parameter used for confinement criteria} \]
\[ \rho^* = \frac{A_f}{A_g} = \text{compressive reinforcement ratio} \]
\[ \rho_{vf} = \text{ratio of reinforcement crossing construction joint} \]
\[ \rho_h = \text{ratio of horizontal shear reinforcement area to the gross concrete area of a vertical section} \]
\[ p_n = \text{ratio of vertical shear reinforcement area to the gross concrete area of a horizontal section} \]
\[ \phi = \text{strength reduction factor} \]

Reinforcement Notation Used in Examples

- C = centrally placed
- D = deformed bar
- E.F = each face
- E.W = each way
- R = plain round bar

REFERENCES:


10. ACI Committee 318, "Building Code Requirements for Reinforced Concrete, (ACI 318-77)", American Concrete Institute, Detroit, 1977, 102 pp.


APPENDIX I

EXAMPLES OF THE STRENGTH DESIGN OF SHEAR WALLS OF LIMITED DUCTILITY:

Example No. 1: Design of a Squat Wall

Figure 7(a) shows the example wall and its design loadings. It will be assumed that the basement foundation wall is such as to preclude treatment of the wall as on a rocking foundation.

In accordance with NZS 4203, two loading combinations with the code specified lateral loading need be considered. Accordingly

\[ U = D + 1.3L_R \]

and \[ U = 0.9D \]

whence

\[ P_{u,\text{max}} = 330kN \]

\[ P_{u,\text{min}} = 180kN \]

Flexure -

The required ideal strength is determined in accordance with \( \phi \) calculated from

\[ 0.9 > \phi = 0.9 - 2P_u/f_c'Ag \]

so that the two conditions are:

(a) \( P_{u,\text{max}} \phi = 0.876 \)

\[ P_i = 377kN, M_i = 4.00 \text{ MNm} \]

(b) \( P_{u,\text{min}} \phi = 0.887 \)

\[ P_i = 203\text{ kN}, M_i = 3.95 \text{ MNm} \]

Minimum reinforcement of D16-400, C will be used (\( \rho = 0.0025 \)), and an additional D16 centred 50 mm from each end will be added to meet the minimum trim reinforcement requirements of the material code DZ 3101 (2mm" per mm wall thickness).

The web reinforcement contributes an ideal strength, in the presence of \( P \) equal to

\[ M_w = 4.44 \text{ MNm} \text{ (for} P_i = 377\text{ kN}) > 4.00 \text{ MNm} \]

and \[ M_w = 3.89 \text{ MNm} \text{ (for} P_i = 203\text{ kN}) > 3.95 \text{ MNm} \]

The added bars contribute an additional 380 kNm, so that the total ideal strength furnished is

\[ M_i = 4.82 \text{ MNm} \text{ (for} P_i = 377\text{ kN}) \]

and \[ M_i = 4.27 \text{ MNm} \text{ (for} P_i = 203\text{ kN}) \]

The strength demand is therefore met with minimum reinforcement.

Use D16-400, C plus 1-D16 each end

Confinement

From Section on 'Confinement in the End-Region'(a) and Eq(C-3)

\[ \gamma = M_u/0.3P_u/t_w = M_u + 0.3P_u/f_c'Ag \]

\[ = 0.20 < 1.00 \]

No confinement steel is needed

Shear

From 'Design Shear Force' and Eq(C-6), with \( S=1.6 \)

\[ V_i > 2 \times 1/0.85 = 2.35 \text{ MN} \]

\[ v_i = 2.10 \text{ MPa} < 0.83/f_c' = 3.71 \text{ MPa} \]

in accordance with 'Maximum Design Shear Stress'.

From 'Shear Resisted by the Concrete', or directly from Figure 6,

\[ v_c = 0.5(0.27/f_c' + N_u/4A_g) \]

\[ = 0.62 \text{ MPa} \]

From 'Design for Horizontal shear Reinforcement' and Eq(C-9)

\[ \rho_h = (2.10 - 0.62)/275 = 0.0054 \]

Use D16-175, C (0.0057)

The wall details are shown in figure 7(b).

Comparison

It is of interest to compare the above
results with those obtained from the application of the capacity design procedure of Section B.

It is to be noted that the vertical (flexural) reinforcement cannot be reduced. In accordance with NZS 4203 Amendment A.2, only the combination D and D + LR need be considered in capacity design. The heavier axial load will produce the larger flexural strength which is

\[ M_i = 4.58 \text{ MNm (for } P_i = 300 \text{ kN)} \]

and the neutral axis depth is found to be 400 mm.

\[ \phi_0 = 1.25 \times 4.20/3.5 = 1.64 \]

\[ \omega_v = 1.00 \text{ assumed for single storey building} \]

and \[ V_i > \omega_v \phi_0 V_u = 1.64 \times 1.00 \times 1.00 = 1.64 \text{MN} \]

because \( \phi = 1.00 \) is suggested,

and \[ V_i = 1.46 \text{ MPa} \]

\[ V_c = 0, \text{ assumed with } N_u < 0.1f' cA_q \]

and so \( c_h = 1.34/275 = 0.0053 \)

Thus the required horizontal reinforcement would be some 2% less by the capacity design procedure.

Since \( c = 400 \text{ mm} < c_0 = 0.1f' cS_N \) = 1,680 mm, capacity design (9) confirms that confinement would not be required.

**Example No. 2: Design of a Slender Wall**

Figure 8(a) shows the example wall and its design loadings. It is deliberately chosen to be column-like in its proportions.

**Flexure -**

The relevant load combinations are as in Ex. 1. At the base \( M = 150 \times 2.5 = 375 \text{ kNm} \) and is to be combined with either

\[ P_{u,\text{max}} = P_d + 1.3P_{AR} = 1,000 \text{ kN} \]

or \[ P_{u,\text{min}} = 0.9P_d = 549 \text{kN} \]

The strength reduction factor \( \psi = 0.7 \) because \( P_{u}/f' cA_q \) exceeds 0.10 for both loads. The smaller axial loads will control the required strength, and reference to an appropriate design chart with \( \phi = 0.7 \) indicates that 950 mm² at both ends of the wall will be satisfactory. The maximum bar diameter permitted is one-eighth the wall thickness, and 5 bars are a practical configuration, so 5-D16 will be used (1005 mm²). Interaction diagrams drawn for \( \phi = 1 \) are more useful and are now available. Figure 8(b) is such an interaction diagram drawn to include the 2-D12 at mid-depth. For this diagram the appropriate (i.e. ideal) strengths are given by

\[ M_i = 375/0.7 = 536 \text{ kNm} \]

with either

\[ P_{i,\text{max}} = 1000/0.7 = 1430 \text{kN} \]

or

\[ P_{i,\text{min}} = 549/0.7 = 785 \text{kN} \]

The loading combinations are marked on the interaction diagram thus *.* The section is seen to be satisfactory.

Use 10-D16 and 2D12

Away from the region of height 1.000m, reduced reinforcement is possible. As the shear reinforcement is quite heavy and is not confining, termination of flexural reinforcement will follow the suggestions of 'Termination of Flexural Reinforcement' (b), so that additional detailing is then limited to the end-region of height 1.000m. In practice no termination would be attempted except, from consideration of congestsment, near the top of the wall.

**Confinement**

From Eq(C-3)

\[ C = 375 \times 10^6 + 0.3 \times 1000 \times 10^3 \times 1000 \]

\[ 0.12 \times 0.7 \times 20 \times 200 \times 1000^2 \]

\[ = 2.01 \]

which is greater than 1.00 but less than 3.00.

From Eq(C-5), with \( r^* = (5 \times 201)/(200 \times 200) \) and \( m = 16.2, \)

\[ R_c = 2.01 - 1 = 0.43 \]

\[ 1 + 0.41 \]

This is less than unity, as required.

From Eq(C-4)

\[ A_{sh} = 0.43 (0.02 \times 1000 \times 1000 \times 20/275) \]

\[ = 625 \text{ mm}^2/\text{m} \]

Half of this will need to be by a supplementary cross-tie to uniformly confine the concrete. (Stirrup reinforcement alone is not satisfactory). These cross-ties will engage the inner D16 bars, so the maximum spacing will be 160 mm, in accordance with 'Spacing of Confinement Reinforcement'.

Use R6 -100 cross-ties (332 mm²/m)

**Shear**

From 'Design Shear Force' and Eq(C-6), with \( S=1.6, \)

\[ V_i > 2x 150/0.85 = 353 \text{kN} \]

whence

\[ V_i = 2.21 \text{ MPa} \]

Using \( N_u = 549 \text{kN}, N_u/A_q = 2.75 \text{ MPa,} \)

whence from Figure 6
\[ v_c = 0.38 \text{ MPa}, \text{ in the end region.} \]

Therefore
\[ \rho_h > (2.21 - 0.38)/275 = 0.0067 \]

Use R10-100 stirrups (0.0079) in the end-region.

Beyond the end-region,
\[ v_c = 0.76 \text{ MPa} \]

whence
\[ \rho_h = (2.21 - 0.76)/275 = 0.00527 \]

Use R10-150 stirrups (0.00523) beyond the end-region.

Comparisons

It is of interest, but not a design requirement for the strength method, to determine the overstrength flexural capacity and the corresponding neutral axis depths. For this condition, only axial loads due to \( D \) or \( D + L \) need be considered in accordance with NZS 4203 - Amendment A2. These values (i.e. 610 kN and 910 kN) are plotted thus on the interaction diagram. The critical load is \( D + L \)
\[ = 910 \text{ kN when } M = 580 \text{ kNm at } c = 335 \text{ mm.} \]

Thus \( \phi = 580 \times 1.25/375 = 1.93 \)

approximately. The corresponding critical neutral axis depth is \( c_c = 0.10 \times 1.93 \times 1.6 \times 1000 = 309 \text{ mm, confirming that confinement would be required.} \)

However, if the capacity design procedure had been applied, significant reductions could have been gained because a smaller \( S \) would have been used, and \( \phi \) for flexure = 0.9 would apply. In addition shear demand would have been reduced and \( v_c \) could have been taken equal to \( v_c = 0.68 \text{ MPa minimum, for } N_u = 610 \text{ kN, in the hinge zone.} \)

For the present example \( c/b_w = 1.7, \)

and \( c/l_w = 0.34 \) so that lateral instability (9) is not likely to be a problem.

Example No. 3: Design of T-Shaped Shear Wall:

Figure 9(a) shows the cross section of the example wall, for which the aspect ratio \( h/w \) equals 3.0. Preliminary flexural design indicates that the quantity of vertical reinforcement shown to be required. Design for confinement and for shear in the end region will be carried out using the strength design method, for the load direction producing tension in the flange. Note that analysis has used the geometrical centroid for the reference axis and has derived the following critical design actions

\[ M_u = 2.0 \text{ MNm} \]
\[ P_{u,\text{min}} = 800 \text{ kN} \]
\[ P_{u,\text{max}} = 1.4 \text{ MN} \]
\[ V_u = V_e = 330 \text{ kN} \]

For confinement these are related to the mid-depth axis by elementary transformation producing

\[ M_u = 1.1 \text{ MNm} \]
\[ P_{u,\text{max}} = 1.4 \text{ MN} \]

Figure 9(b) is a portion of the ideal (\( \psi=1 \)) interaction diagram. The bold line refers to the centroidal axis, and the thin line to the mid-depth axis. The section is seen to be adequate for flexure and axial load for the relevant loading combinations referred to the centroidal axis, shown thus *.

Confinement

From 'Confinement in the End-Region' and Eq(C-3)

\[ \gamma = \frac{1.1 \times 10^2 + 0.3 \times 1.4 \times 10^6 \times 3000}{0.12 \times 20 \times 250 \times 3000^2 \times 0.8} = 0.55, \text{ with } \psi = 0.8, (P_u/f'cA_g = 0.05) \]

No confinement is required.

The ratio of vertical reinforcement at the end of the web

\[ \rho_v = 6 \times 314/250 \times 250 = 0.03 \]

Since this is greater than \( 3/f_c \), in accordance with 'Spacing of Confinement Reinforcement' tying as for columns is required. In the end-region, spacing should not exceed 200 mm. (Away from the end region 16 longitudinal bar diameters, 48 tie bar diameters, or the wall thickness, whichever is least, would suffice).

The closed R6 ties shown, spaced at 175 mm to suit the horizontal shear reinforcement spacing, will be satisfactory.

Shear

From 'Design Shear Force'

\[ V_i \geq 2 \times 330/0.85 = 776 \text{ kN} \]

whence

\[ V_i = 776 \times 10^3/250 \times 0.8 \times 3000 = 1.29 \text{ MPa} \]
\[ v_c = 0.38 \text{ MPa in the end-region} \]
\[ \rho_h = (1.29 - 0.38)/275 = 0.0033 \]

Use D10-175, EF (0.0036)