LATERAL FLEXIBILITY AND SEISMIC DEFLECTION ESTIMATES
How good is good enough?

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Frame flexibility and deflection estimates, needed for one or more of a number of purposes in design for seismic resistance, are most usually made from the bending moment diagram and the elastic properties of a column, by the moment area or a related method. (To state the obvious), the assessments thus obtained can only be accurate when the bending moment determinations are precise. It is perhaps less evident that small but tolerable bending moment errors may produce large intolerable deflection errors.

Popular approximations for bending moment determination are the cantilever method and the portal method, for both of which contraflexure is taken as occurring at midheight in all column spans. Consider a symmetrical single bay multi storey frame with fixed feet for which a deflection estimate is to be made from either of the approximate bending moment assessments, which are, in this case, equivalents. While, assuming arithmetical accuracy, the sum of terminal moments in any column span must be correct, there will, in general, be errors in individual moments. Fig. 1 shows a typical column span. From it can be derived an expression for the increment of lateral deflection at the top of the building due to shift of contraflexure from 0.5h to sh. The moment error everywhere in the column length, from the geometry of the figure, is M(2s-1). Thus, if the terminal curvature due to M is $\phi$, the error is a constant curvature of $\phi(2s-1)$. This produces an angle change $\delta h(2s-1)$ in the storey height. The product of this with the distance from the column mid-height to the top of the frame is the deflection increment.

For an n-storey building in which the column sections are progressively reduced from base to top in such a way that $\phi$ has the same value for every storey, and when all storey heights are the same, the top deflection, assuming contraflexure at midheight of every column, is $\Delta = \frac{n\delta h^2}{6}$.

Unless column stiffnesses are insignificant compared with girder stiffnesses, contraflexure will actually be above midheight in the lowest storey, reducing in successive storeys to be close to midheight for many of them and then to below midheight for one or two at the top. The increment of deflection due to a moment error in the lowest storey is $\delta(\Delta) = \frac{1}{2} \phi h^2(2s-1)(2n-1)$. 

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**Fig. 1.**
When \( n=10 \) and \( s=0.7 \), this increment from the lowest storey alone becomes \( 1.8 \varnothing h^2 \), already greater than \( 1.67 \varnothing h^2 \), the first estimate of the total top deflection. Increments from other lower storey columns add to it, and, especially when the column section is not reduced as the shear reduces up the building, the error builds up, often exceeding the first estimate of top deflection by many times its magnitude.

This example, not an extreme one, shows that moment determination must be precise if deflection estimates are to be made from it. Whether or not errors are serious depends upon the purpose of the estimate of deflection. If it is made to check an assumed division of shear between dissimilar frames coupled by floor diaphragms, error will mislead the designer badly. Estimation from an approximate BM diagram is thus indefensible, as it is when elastic deflections are needed to check whether the ductility and seismic separation provisions made are adequate. For load determination (indirectly from deflection through the period of the prime mode of vibration) the seriousness of error depends upon the problem at hand; but in general, deflections from approximate moments are not good enough.

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**PERSONAL**

It is with pleasure that we report the visit to New Zealand of Mr Tembe, M.Sc. (Lond.), M.I.C.E., M.I.Struct.E., M.I.E., Consulting Engineer of Bombay, India. After the recent severe earthquake in the Bombay area, Mr Tembe, who is responsible for the design of multi-storied government, semi-government and private buildings, wrote to the Society that he intended to visit New Zealand for a short period to study the design and construction of earthquake resistant buildings. Acting on the Society's advice, he attended the Seminar on Seismic Design at the University of Canterbury, Christchurch, from 13th to 16th May, and while in that city inspected multi-storied buildings in course of construction and discussed their design with the engineers concerned. Later, Mr Tembe visited Wellington where local members of the Society accompanied him on tours of inspection of both private and governmental buildings.