INTRODUCTION

Seismic response history analysis (or so-called ‘time history analysis’) is frequently used as a design verification or assessment method for structures and infrastructure in New Zealand. Because of the complexity of earthquake-induced ground motions, an ensemble of motions with nominally similar ground motion intensities will produce a distribution of seismic demands to various components of the system considered. Such seismic demand distributions typically have a significant level of uncertainty (measured via the standard deviation), and as a result, the manner in which this distribution of demand is used in seismic performance assessment can have a significant effect on the overall decisions regarding the adequacy of the system's seismic resistance.

The purpose of this paper is to compare several alternative metrics by which seismic performance assessment results can be obtained from seismic response analyses. A brief overview of the uncertainty in seismic intensity and demand is first provided. Four different seismic performance assessment metrics are then directly compared and contrasted based on a central example. Discussion is focused on immediate practical work, but have not yet been implemented in codes of practice, or emerging methods which are common in research-based work, but have not yet been implemented in codes of practice. Discussion is focused on immediate practical implementation in a consulting environment, and therefore limited to seismic response metrics, rather than damage or loss metrics.

UNCERTAINTY IN SEISMIC INTENSITY AND DEMAND

Figure 1 illustrates the variation in ground motion intensity as a function of the annual rate of exceedance (the inverse of return period). Such a relationship can be obtained by site-specific seismic hazard analysis, or based on the general seismic hazard prescriptions in seismic design guidelines. In Figure 1, the ground motion intensity measure (IM) is represented via the spectral acceleration for a vibration period of $T = 1.25s$, which for site class C soils is numerically equal to the zone factor, $Z$, in the New Zealand loadings standard, NZS1170.5 (2004). Thus, the x-axis values in Figure 1 are simply equal to $Z$ multiplied by the return period factor, $R$ (NZS 1170.5, 2004); while the values on the y-axis are the inverse of the return period values specified for each return period factor. When plotted in log-log space it can be seen that the relationship is relatively linear, the implications of which are discussed subsequently in the paper.

As would be intuitively expected, the seismic demand imposed on a system is also a function of the ground motion intensity. Figure 2 provides a schematic illustration of such a relationship, which serves to elucidate several important features. Firstly, seismic demand generally increases with increasing intensity. Secondly, because of the fact that multiple ground motions with the same level of intensity produce different seismic responses, then there is a distribution of demand for a given level of ground motion intensity. Thirdly, a given seismic performance threshold can be exceeded at different levels of ground motion intensity, although the likelihood of this exceedance varies with ground motion intensity. The principal differences between the various seismic performance assessment metrics considered subsequently relate to the manner in which the demand distribution is considered, and whether single or multiple levels of ground motion intensity are considered for a specific seismic demand threshold.

ALTERNATIVE METRICS FOR SEISMIC PERFORMANCE ASSESSMENT

In this section, four different metrics for seismic performance assessment based on the results of seismic response history analyses are compared and contrasted.

"Maximum demand" – NZS1170.5:2004 approach

Prescriptions around the use of response history analysis for seismic assessment in New Zealand are covered in clauses 5.5 and 6.4 of NZS1170.5 (2004) (relating to ground motion selection and structural analysis, respectively). Two prescriptions of particular relevance to the discussion here are that "ground motion records for time history analysis shall consist of a family of not less than three records" (clause 5.5.1); and that "the most critical value of any response parameter … across the family of records shall be used to determine acceptability" (clause 6.4.7).

While the statement that at least three ground motions must be considered is similar to other codes (CEN, 2003; ASCE/SEI 7-05, 2006; FEMA-368, 2001), the use of the maximum response of all the considered motions (irrespective of the number of motions considered) is a clause which is not seen elsewhere. As the number of ground motions considered increases, the maximum response will also generally increase,
thus this provides a disincentive to analysts to consider more than three ground motions.

The author is aware that this NZS1170.5:2004 prescription is often avoided by NZ practitioners in favour of the using an international code approach, based on the mean response from analyses using the considered ground motions, as long as at least 7 ground motions are used (which is discussed further in the next section). The author supports the avoidance of these NZS1170.5:2004 prescriptions. The reason for this is that the maximum of 7 records (which is a typical number used in NZ practice), for typical seismic response problems, represents the 92nd percentile of the distribution of demand for a given level of ground motion intensity. For example, if the seismic demand distribution for maximum inter-storey drift had a mean of 1.5% and a lognormal standard deviation (approximately equal to the coefficient of variation) of 0.4, then the maximum of 7 responses would be, on average, 2.5% drift. In this regard, it must be borne in mind that the ground motion intensity level considered is not an absolute maximum, but represents an intensity level with a certain rate of exceedance. For example, the use of the ground motion intensity with a 1/500 annual exceedance rate combined with the maximum response from seven ground motions is likely to give a level of response which has an exceedance rate much less than 1/500 years (in fact, on the order of 1/1300 years, as shown in a subsequent example), and is therefore arguably overly conservative.

Figure 1: Illustration of the variation in ground motion intensity with annual rate of exceedance for low and high seismicity areas in New Zealand (Auckland and Wellington, respectively) based on NZS1170.5:2004, as well as a power-function approximation to this relationship.

Figure 2: Schematic illustration of the variation in seismic demand with ground motion intensity, and the uncertainty in seismic demand for a specific level of ground motion intensity as represented by a demand distribution. The three illustrated levels of seismic intensity correspond to the 1/50, 1/500, and 1/2500 annual exceedance rates for Wellington shown in Figure 1.
“Maximum of 3, average of 7” – An international code approach

Rather than requiring the use of the maximum seismic response obtained from response history analyses, as in NZS1170.5:2004, the majority of international codes of practice (CEN, 2003; ASCE/SEI 7-05, 2006; FEMA-368, 2001) make use of a "maximum of 3, average of 7" clause. Such clauses essentially state (in code language) that a minimum of 3 ground motions must be considered; if less than 7 ground motions are considered then the maximum response must be used, and if seven or more ground motions are considered then the average response can be used. While such clauses allow for any number of ground motions to be considered, almost always either 3 or 7 ground motions are considered. This is because the maximum response will increase, on average, as the number of motions increases from 3 to 6 (and therefore one would be always tempted to only consider 3 motions); and because the average response should be relatively stable for 7 or more motions, and therefore practical considerations would result in the minimum (i.e. 7) number of motions being considered.

Average demand accounting for uncertainty – Bradley (2011) approach

Bradley (2011) noted several limitations of the use of the ‘maximum of 3, average of 7’ approach, which include:

1. It addresses variability in seismic response in a deterministic manner and as a result it is not clear, for example, what is the likelihood that the maximum of three seismic response analyses is smaller than the “true” mean seismic response (Bradley, 2011);  
2. It provides no explicit incentive for conducting larger (i.e., greater than 7) numbers of seismic response analyses, something which is now routinely possible considering the time to conduct analysis versus the time to develop and validate a seismic response model;  
3. It does not directly provide any motivation for reducing uncertainty in seismic response analyses by careful ground motion selection (Baker and Cornell, 2006; Bradley, 2010; Bradley, 2012; Hancock, et al., 2008);  
4. It does not account for the fact that some measures of seismic response are significantly more sensitive to the input ground motion than others (i.e., have a higher variability).

On the basis of the above limitations, Bradley (2011) developed a simple probability-based method for determination of the design seismic demand using the distribution of the sample mean of the seismic response history analysis results. While further specifics are omitted, Bradley (2011) proposed that the design seismic demand could be computed from:

\[ EDP_{design} = EDP + R_{EDP,0.84} \]  

where \( EDP_{design} \) is the design value of the engineering demand parameter (EDP); \( EDP \) is the sample mean of EDP obtained from the performed response history analyses; and \( R_{EDP,0.84} \) is a ‘design factor’ to account for the number of ground motions considered (\( N_{gm} \)), and the standard deviation in the distribution of EDP (\( \sigma_{EDP} \)) and is given by the equation:

\[ R_{EDP,0.84} = \exp \left( \frac{\sigma_{inEDP} \left( \frac{N_{gm}}{\sqrt{N_{gm}}} \right)^{1/10}}{0.84} \right) \]  

For example, considering 7 ground motions for seismic response analyses, with a seismic response parameter which has a distribution with lognormal standard deviation of \( \sigma_{inEDP} = 0.4 \), would result in a design factor of \( R_{EDP,0.84} = 1.15 \) (i.e. the design demand would be 1.15 times the sample mean demand from the 7 analyses performed). This is notably less than taking the maximum of 7 ground motions (which is, on average, 1.7 times the mean of a lognormal distribution with \( \sigma_{inEDP} = 0.4 \)), but acknowledges that using the sample mean value alone does not account for the importance of the number of ground motions considered and the uncertainty in the seismic response distribution itself. It also therefore provides motivation to perform additional analyses (i.e. increasing \( N_{gm} \)), or reducing the uncertainty in the demand distribution through careful ground motion selection (i.e. reducing \( \sigma_{EDP} \)).

Seismic demand hazard approach

While the third approach above, which has a probabilistic basis, may be considered as an improvement over the first two approaches which provide arbitrary deterministic prescriptions; all three approaches suffer from a fundamental limitation. In all three cases, the likelihood of exceeding the specific demand threshold is only considered for a specific level of ground motion intensity. In order to explain this idea in more detail, consider a ‘typical’ multi-objective seismic performance criteria used in current codes of practice shown in Table 1.

<table>
<thead>
<tr>
<th>Likelihood</th>
<th>Ground motion exceedance rate</th>
<th>Performance objective</th>
<th>Inter-storey drift limit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequent</td>
<td>1/500 years</td>
<td>Continued operation</td>
<td>0.3%</td>
</tr>
<tr>
<td>Rare</td>
<td>1/500 years</td>
<td>Life safety</td>
<td>1.5%</td>
</tr>
<tr>
<td>Very rare</td>
<td>1/2500 years</td>
<td>Collapse prevention</td>
<td>3.0%</td>
</tr>
</tbody>
</table>

Each of the three performance objectives in Table 1 is associated with a single level of ground motion exceedance rate. At each level of ground motion intensity, the likelihood that the response exceeds the corresponding performance criteria is considered, however it is also possible that the response exceeds the performance criteria due to other levels of ground motion intensity, as illustrated in Figure 2 for a seismic demand of 0.015 (1.5%). As a result, these so-called ‘intensity-based assessments’ (which typically use the mean response at the design intensity level as discussed in the ‘International code approach’) tend to underestimate the actual likelihood that the design response level occurs (Bradley, 2013a), sometimes significantly.

In order to account for the above deficiencies, it is possible to directly integrate the seismic hazard and seismic response analyses together to obtain a ‘seismic demand hazard’ (Bradley, 2013a; Bradley, 2013b). This can be done in a number of ways, and here only the simplest approach is illustrated for the purpose of conventional seismic assessment, which makes use of the so-called SAC/FEMA approach (Cornell, et al., 2002), widely used in the US.

**Step 1**: The seismic hazard curve is assumed to be of the form \( h_T = k_0 f \cdot M^{-k} \), where \( k_0 \) and \( k \) are empirical constants. This equation is fit to the seismic hazard curve over the exceedance rates of interest. Here, the values for the 1/500 and 1/2500 year exceedance rates are adopted to determine the constants, \( k_0 \) and \( k \). Figure 1 illustrates the appropriateness of this assumption for the NZS1170.5:2004 representation of the seismic hazard. The empirical constant, \( k \), in the parametric expression represents the slope of the hazard curve which has a value of \( k = 2.74 \) (because the risk factor, \( R \), in NZS1170.5:2004 is independent of location). By
manipulation, it can be shown that $k_0 = Z^b / 500$, where $Z$ is the zone factor in NZS1170.5:2004.

**Step 2:** The median seismic demand, $EDP_{50}$, is assumed to vary with the ground motion intensity as given by $EDP_{50} = a l M^b$, where $a$ and $b$ are empirical constants; and the uncertainty in the seismic demand assumed as a lognormal distribution with standard deviation, $\sigma_{INEDP}$.. Seismic response analysis results for two levels of ground motion intensity (e.g. the 1/500 and 1/2500 year exceedance rates) are needed to determine these two constants, $a$ and $b$, directly. However, it is generally reasonable to assume that $b = 1$ (Cornell, et al., 2002) and therefore the constant $a$ is simply computed as the median demand divided by the ground motion intensity level.

**Step 3:** Based on the above two assumptions, the seismic demand hazard, which directly provides the likelihood of exceeding the seismic demand value, can be computed as:

$$\lambda_{EDP}(edp) = k_0 \left( \frac{edp}{a} \right)^{-k/b} \exp \left[ \frac{1}{b^2} \sigma_{INEDP}^2 \right]$$

(3)

where all parameters have been previously defined. The first part of the above equation, $k_0 \left( \frac{edp}{a} \right)^{-k/b}$, simply represents the rate of exceedance of the median level of seismic demand for the considered ground motion intensity level, while the second expression, $\exp[.]$, essentially represents an ‘amplification factor’, which accounts for the uncertainty in the seismic response, $\sigma_{INEDP}$, and the nature of the seismic hazard. One of the benefits of the use of Equation (3) is that it directly provides the rate of exceedance of a seismic level of seismic demand over a continuum of values.

**Comparison of the considered metrics**

In order to compare the results of the previous four metrics which have been discussed, use is made of the seismic hazard results for the example Wellington site in Figure 1 (i.e. $k_0 = 1.6 * 10^{-4}$; $k = 2.74$) as well as assuming that seismic response analyses performed at ground motion levels corresponding to the 1/50, 1/500, and 1/2500 year exceedance rates provide the seismic demand distributions shown in Figure 2 (i.e. $a = 0.0231$; $b = 1$, $\sigma_{INEDP} = 0.4$), which for the 1/500 year exceedance rate corresponds to a median drift of 0.10, and a mean drift of 0.11.

Figure 3 provides a comparison between the results that would be obtained from these various four methods for the 1/50, 1/500, and 1/2500 year exceedance rates, all based on the use of 7 ground motions (which is most commonly adopted in practice). Monte carlo simulation was used to generate statistically stable estimates based on the relationships previously mentioned. The demand hazard can be considered as the ‘exact’ solution in that it explicitly considers the uncertainty in the seismic demand due to ground motion variability and also the full seismic hazard curve. As previously mentioned, the maximum of 7 ground motions is generally significantly conservative (i.e. ~140% of the seismic demand hazard EDP value for the same exceedance rate), while the use of the mean response of 7 motions is generally unconservative (i.e. ~85% of the seismic demand hazard EDP value for the same exceedance rate). The use of the Bradley (2011) approach produces design EDP values which are in between the “max of 7” and “mean of 7” approaches, and are similar to the seismic demand hazard for the 1/500 and 1/2500 year exceedance rates, but more unconservative for the 1/50 year exceedance rate.

It is important to note that while results for three different exceedance rates are shown here for illustration, all four methods can be used to conduct a performance assessment at a single exceedance rate. Furthermore, all four approaches require the same level of input information (i.e. seismic response history analysis results). While the seismic demand hazard computation also requires information to define the seismic hazard curve, as was noted above, this can be easily obtained from NZS1170.5:2004 (i.e. $k = 2.74$; $k_0 = Z^b / 500$) if site-specific information is not available.

![Figure 3: Comparison of four different approaches for determining the seismic demand for seismic performance assessment at the 1/50, 1/500, and 1/2500 year exceedance rates.](image)
CONCLUSIONS

This paper has examined four different seismic performance metrics which relate to the determination of design seismic demands from seismic response history analyses. It was illustrated that the use of the "maximum demand" metric in the NZ loadings standard, and the "mean demand" in international codes of practice are notably conservative and unconservative, respectively. Both of the other two examined metrics provide a significant improvement, and given that both require the same information from an analyst’s perspective, are recommended as replacements.

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REFERENCES