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SECTIONAL RESPONSE OF NON-RECTANGULAR CONCRETE WALLS WITH MINIMUM VERTICAL REINFORCEMENT

Tianhua Deng[1] and Richard S. Henry[2]

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ABSTRACT

Past research that investigated the behaviour of rectangular lightly reinforced concrete walls resulted in revisions to minimum vertical reinforcement provisions in concrete design standards in both New Zealand (NZS 3101:2006) and the United States (ACI 318-19). However, the minimum vertical reinforcement provisions developed for rectangular wall sections may not be suitable for non-rectangular walls due to the influence of flanges on the nominal flexural and cracking section capacities. A parametric study confirmed that non-rectangular wall sections with minimum vertical reinforcement in accordance with current NZS 3101 design provisions exhibit a lower margin between cracking and nominal flexural strength than comparable rectangular wall sections. The ratio of the sectional nominal flexural strength to cracking strength (M_n/M_{cr}) was less than 1.0 for non-rectangular sections with long flange lengths and low axial loads. The model results indicated that current vertical reinforcement requirements are insufficient to prevent a sudden and potentially unstable strength drop when cracking occurs in non-rectangular walls. A theoretical equation to calculate the required minimum vertical reinforcement was proposed for the typical I-shaped wall sections, including the impact of concrete strength and flange to web ratio. The proposed equation highlighted the need for an increase in the minimum vertical reinforcement limits for non-rectangular wall sections compared to the existing minimum vertical reinforcement requirement applicable to rectangular wall sections.

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INTRODUCTION

Reinforced concrete (RC) walls are one of the most prevalent structural systems implemented in New Zealand buildings. A survey of buildings constructed in Christchurch following the 2010/2011 Canterbury earthquakes showed that almost half used RC walls as part of the lateral load resisting system [1]. Rectangular walls are popular in low and mid-rise buildings due to the convenient geometry and precast construction methods, and core walls are common in taller buildings where strength/stiffness is required in multiple directions [2]. Representative wall cross-section can consist of various rectangular and non-rectangular configurations, including T-shaped, I-shaped, and core walls enclosed by multiple non-rectangular wall elements, as illustrated in Figure 1.

Following observations of the performance of reinforced concrete (RC) walls in the 2010/2011 Canterbury earthquakes, a series of studies were conducted to investigate the behaviour of the lightly reinforced rectangular RC walls [3-5]. The outcomes of this research resulted in revisions to the minimum vertical reinforcement provisions in both the New Zealand Concrete Structures Standard, NZS 3101:2006 [6], and the US Building Code requirements for Structural Concrete, ACI 318-19 [7]. A theoretical minimum vertical reinforcement ratio was also developed to achieve a consistent margin between cracking and nominal capacity for rectangular wall sections [8]. However, the behaviour of non-rectangular wall sections was not considered during this prior research, despite the potential difference in the response of rectangular and non-rectangular sections that might affect the minimum vertical reinforcement requirements. Compared with rectangular sections, the margin between the nominal strength and cracking strength for flanged walls has been shown to reduce significantly as the flange length increases [9]. In addition, the lightly reinforced non-rectangular core walls were found to be a contributing factor to the collapse of Pyne Gould Building during the 2011 Christchurch earthquake [10].

Figure 1: Representative cross-section used for structural walls.

[1] *Postdoctoral Researcher, Key Laboratory of Performance Evolution and Control for Engineering Structures of Ministry of Education, Tongji University, Shanghai, China: dth@tongji.edu.cn*

[2] *Associate Professor, Dept. of Civil and Environmental Engineering, University of Auckland, New Zealand: rs.henry@auckland.ac.nz (Member)*

The ratio between sectional nominal flexural strength and cracking strength is an important criterion for assessing the performance of lightly reinforced concrete wall sections. In this study, a set of analyses were conducted to evaluate the ratio between nominal flexural strength and cracking capacity for a range of non-rectangular wall sections designed with minimum vertical reinforcement in accordance with the current NZS 3101:2006 design provisions. A theoretical formula was proposed to calculate the required minimum vertical reinforcement for I-shaped sections to ensure a consistent margin between nominal flexural and cracking strength at a section level. The proposed formulas extended prior research that developed similar expressions for rectangular sections.

BACKGROUND

Ensuring that the nominal flexural strength exceeds the cracking strength ($M_n > M_{cr}$) has been identified as an essential criterion to ensure ductility in reinforced concrete sections [11]. The $M_n > M_{cr}$ criterion ensures that an irrecoverable loss of strength does not occur upon first cracking and is commonly used in the development of minimum longitudinal reinforcement limits of RC members. The criterion was previously adopted when assessing the section response of rectangular RC walls designed with minimum vertical reinforcement [8]. Rectangular RC walls with satisfying the minimum vertical reinforcement limits of $\sqrt{f_c'}/4f_y$ in NZS 3101:2006-A2 [12] exhibited a ratio of nominal to cracking strength ranged from 1.1 to 1.8, with most typically larger than 1.5. This margin of safety was deemed sufficient to prevent a non-ductile failure during lateral loading, and so this minimum distributed vertical reinforcement requirement was left essentially unchanged when Amendment 3 to NZS 3101:2006 was developed [6]. Despite these minimum vertical reinforcement requirements applied to all wall sections, no analysis or testing was conducted of non-rectangular sections at the time.

The $M_n > M_{cr}$ criterion can be used to assess the section response and prevent sudden failure after cracking, as demonstrated by lightly ref resulted in brittle failure with a bar fracture at less than 1.0 % drift ratio [4]. However, this criterion only assesses the section response and does not address the need for sufficient vertical reinforcement to generate well-distributed secondary cracking in plastic hinge regions. Test results for RC walls designed with minimum distributed vertical reinforcement in accordance with NZS 3101:2006-A2 indicated that walls with a M_n/M_{cr} ratio exceeding 1.0 were still susceptible to discrete irregular cracks with concentrated plasticity demands at wall base [5]. A secondary cracking index was introduced to calculate the required vertical reinforcement at the ends of the wall to generate distributed cracking, and was adopted for the plastic hinge region of RC walls in NZS 3101:2006-A3 [6].

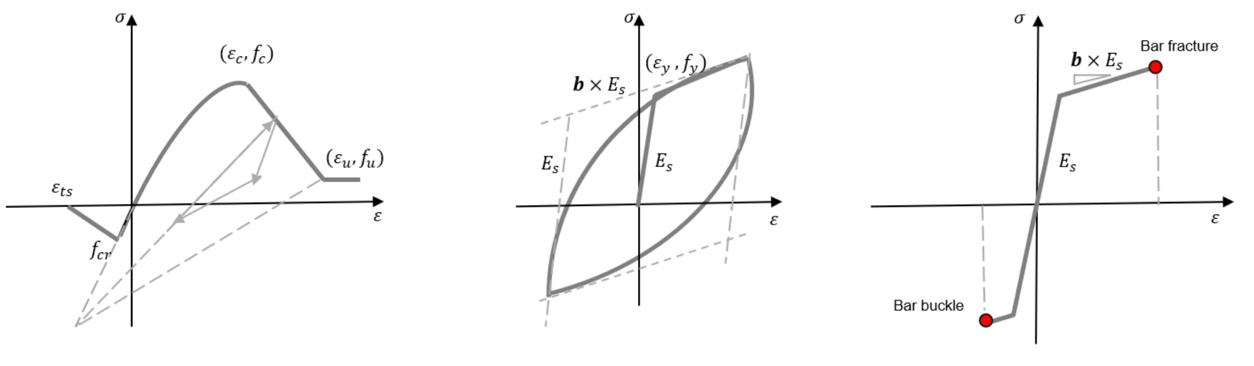
Prior investigations into minimum vertical reinforcement requirements for RC walls are based on rectangular wall sections, but were adopted consistently for all wall geometries in NZS 3101:2006-A3. Recent analysis has shown that the current vertical reinforcement requirement may be insufficient to provide a desirable seismic performance for non-rectangular RC walls as the increased flange length reduces the margin between the nominal strength and cracking strength [9]. It is essential to investigate the section response of non-rectangular walls to assess the vertical reinforcement required to satisfy the section moment criterion (M_n/M_{cr}) to ensure a ductile section response.

MODEL DESCRIPTION

Moment-curvature analysis was conducted on a range of sections using a model scheme developed using a zero-length element in OpenSees. A fibre section was defined using the modified Kent-Park concrete model available as Concrete02 and Giuffré-Menegetto-Pinto steel hysteretic model [13] (extended by Filippou et al., [14]) defined as Steel02 to capture the non-linear material responses.

The compressive response of Concrete02 consists of three distinct regions, consisting of an ascending parabolic branch, a descending linear branch and a constant residual strength, as shown in Figure 2-a. The tensile response is defined by a bi-linear curve with zero residual strength after concrete cracking. The tensile strength was calculated in accordance with the *fib* Model code as $0.3(f_{ck})^{2/3}$, where f_{ck} is the characteristic compressive strength [15].

The Steel02 model consists of an initial slope E_s that is defined as the elastic modulus, and a strain hardening slope expressed as bE_s, as shown in Figure 2-b. The MinMax material command was coupled to define the tension and compression ultimate strain limit in the stress-strain response, as shown in Figure 2-c. Once the limits were triggered, the reinforcement stress immediately dropped to zero. Typical assumptions for nominal strength in design standards ignore reinforcement strain hardening contribution, so that an elastic perfectly plastic reinforcement definition was used in the model. The onset of reinforcement yielding was calculated from the specified yield strength and an elastic modulus of 200,000 MPa. The fracture strain was defined from expected values from typical test data, equal to 12% for Grade 500E reinforcing bars.



(a)Schematic Concrete02 stress-strain curve

(b)Schematic steel02 stress-strain curve

(b) Integrate Steel02 with MinMax Command

Figure 2: Constitutive material models.

ANALYSIS OF NZS 3101 DESIGNED NON-RECTANGULAR SECTIONS

Nominal Ductile Sections with Distributed Reinforcement

Compared with the rectangular sections, flanged wall sections with minimum reinforcement have been previously shown to amplify the cracking strength and reduce the ratio of nominal flexural strength to cracking strength (M_n/M_{cr}) [9]. A series of moment-curvature analyses were conducted to comprehensively investigate the sectional response for a range of different non-rectangular wall geometry designed in accordance with minimum vertical reinforcement requirements in NZS 3101:2006-A3. In accordance with common material properties for walls, concrete with a specified compressive strength of 40 MPa with a mean tensile strength of 3.5 MPa and Grade 500E reinforcement with a yield strength of 500 MPa were used in the models. A rectangular wall section with a length of 8.2 m and thickness of 0.3 m was designed as the baseline and compared to sections designed with flanges of 4.1 m length to generate either asymmetric T-shaped sections and a symmetric I-shaped section. The distributed vertical reinforcement arrangements for the rectangular, T-shaped and I-shaped sections are illustrated in ***Figure 3***. The reinforcement ratio in both the web and flange regions was 0.32% ($\sqrt{f_c'}/4f_y$) in accordance with the minimum distributed vertical reinforcement provisions in NZS 3101:2006-A3.

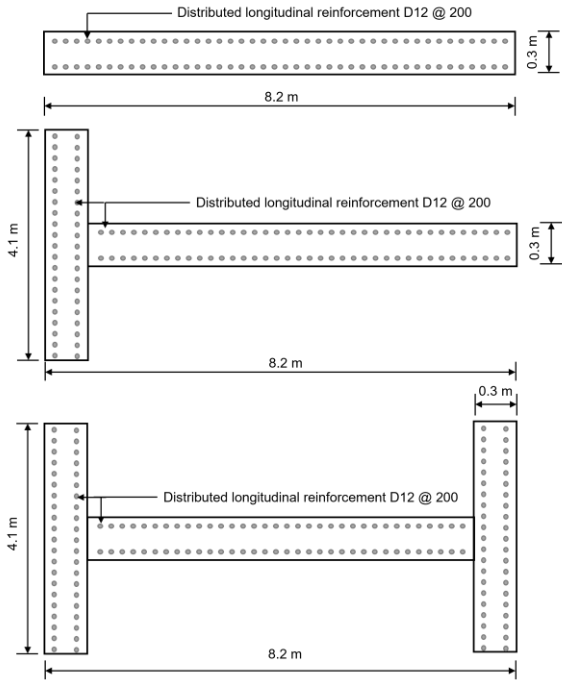
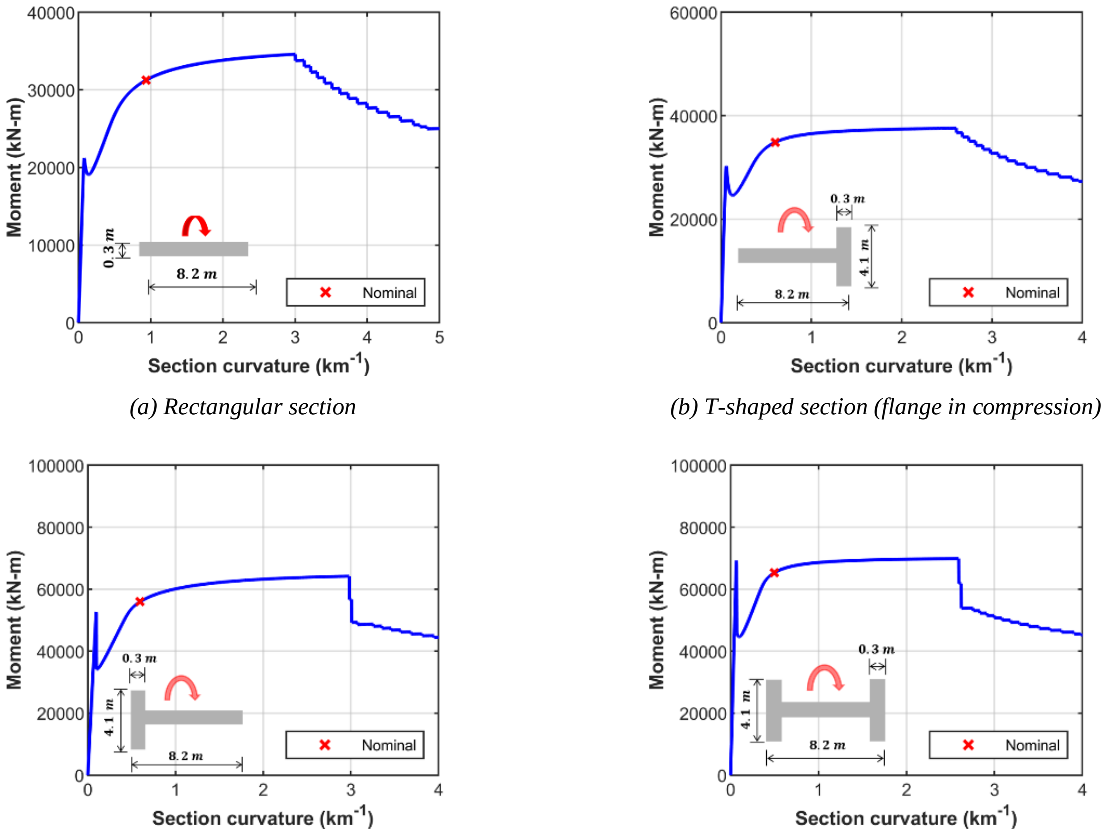


Figure 3: Rectangular and non-rectangular wall sections with distributed reinforcement.



(a) Rectangular section

(b) T-shaped section (flange in compression)

(c) T-shaped section (flange in tension)

(d) I-shaped section

Figure 4: Comparison of moment-curvature for rectangular and non-rectangular wall sections.

Prior research has highlighted that the M_n/M_{cr} ratio is sensitive to the axial load, and that lower axial loads are more critical to highlight the vulnerability of walls with the lightly vertical reinforcement [8]. As such a relatively low axial load representative of the lower bound load common for lightly reinforced multi-storey walls was used as a baseline. An example of the sectional model calculated moment-curvature responses were generated for the rectangular and I- and T-shaped sections (inc. flange in tension and flange in compression for T-shaped) with a representative axial load ratio of 5% are compared in Figure 4. The response for all sections showed a distinct dip in strength after cracking and a final failure due to reinforcement fracture. The rectangular section had cracking strength (21,000 kN.m) that was significantly lower than the nominal strength (31,000 kN.m). The T-shaped sections with flange in tension and compression resulted in cracking strengths (52,000 kN.m and 30,000 kN.m) that were slightly lower than the nominal strengths (55,000 kN.m and 35,000 kN.m). Lastly, the I-shaped section had a cracking strength (70,000 kN.m) that exceeded the nominal strength (65,000 kN.m) due to the section having the largest moment of inertia.

The ratio between nominal and cracking strength (M_n/M_{cr}) was further examined for the rectangular and non-rectangular sections in Figure 5. Compared with the rectangular section with a ratio of 1.47, the non-rectangular sections showed a significant reduction in the nominal and cracking strength ratios of 1.15, 1.05 and 0.95 for the T-shaped section with flange in compression, in tension and the I-shaped section. It is worth noting that a M_n/M_{cr} lower than 1.0 indicates that the minimum distributed vertical reinforcement required by NZS 3101:2006-A3 is insufficient to satisfy the intended performance criterion of nominal flexural strength exceeding the cracking strength for the non-rectangular wall sections.

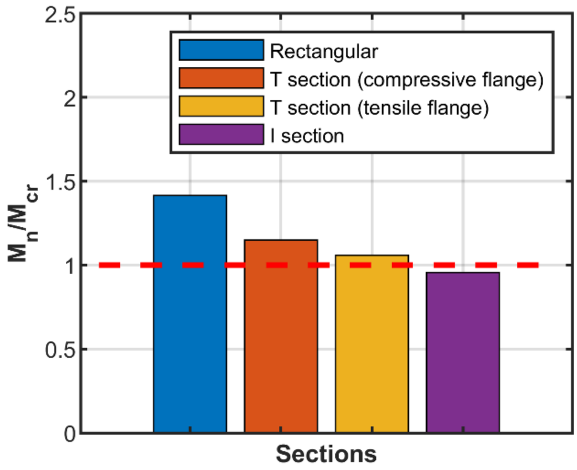
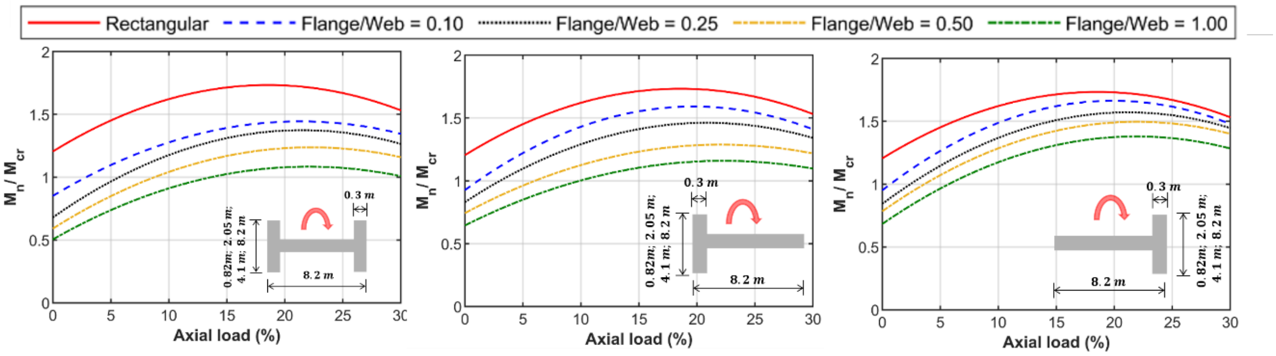


Figure 5: Comparison of the ratio of calculated nominal flexural strength to cracking strength between the rectangular and non-rectangular sections.



(a) I-shaped section

(b) T-shaped section (flange in tension)

(c) T-shaped section (flange in compression)

Figure 6: Calculated M_n/M_{cr} ratio for T- and I-shaped sections.

To investigate the impact of flange length, T-shaped sections (loaded in both directions) and the I-shaped section with a consistent web length of 8.2 m and flange lengths that varied from 0.3, 0.82, 2.05, 4.1, and 8.2 m were modelled. Again, the concrete compressive strength was 40 MPa with the mean tensile strength of 3.5 MPa, and reinforcement yield strength was 500 MPa, resulting in the vertical reinforcement ratio of 0.32% that was equal to the minimum required distributed reinforcement limit in NZS 3101:2006-A3. In addition, the range of axial load ratio was varied from 0 to 30%. The analysis results are shown in Figure 6, comparing the calculated ratio of nominal to cracking strength (M_n/M_{cr}) across the axial load ranges for the different wall sections. The trend of the section strength ratio showed the lowest values when zero axial load was applied, maximum values peaking at around 20% axial load (tension-controlled), followed by a descending trend in the strength ratios as the increased axial load ratio resulted in a compression-controlled response. For all section types, the ratio of nominal to cracking strength (M_n/M_{cr}) curves decreased as the flange length was increased. Notably, a significant reduction in the section strength ratio was observed between rectangular section (solid red line) and flange to web ratio of 0.1 (blue dash line), highlighting that even the addition of relative short flange (or enlarged boundary element) resulted in a critical change section response and ductility. The impact of flange length on section strength ratio was non-linear, with a flange to web length of 1:1 only slightly worse than a flange to web length of 0.5:1.

For the rectangular section (solid red line), the margin of safety between cracking and nominal strength varied from 1.20 to 1.73 as the axial load increased, confirming that the minimum vertical reinforcement limits in NZS 3101:2006-A3 were sufficient to ensure a ductile section response for rectangular walls. For T-shaped sections with flange in tension (Figure 6-b), the ratio of nominal and cracking strengths ranged from 0.64 to 0.92 at 0% axial load ratio and 1.15 to 1.59 at 20% axial load ratio. For T-shaped sections with flange in compression (Figure 6-c), the ratios ranged from 0.68 to 0.95 at 0% axial load and 1.37 to 1.67 at 20% axial load ratio. However, the I-shaped section showed the worst performance when considering the ratio of nominal to cracking strength compared with the T-shaped sections. The section strength ratios ranged from 0.50 to 0.85 at 0% axial load to 1.08 to 1.43 at 20% axial load ratio (Figure 6-a). The geometry of the I-shaped section results in the largest second-moment inertia and thus cracking strength, and therefore, the lowest ratio or margin between nominal flexural strength and cracking strength among the non-rectangular sections. Hence, the I-shaped section was selected as the representative section that was used in the subsequent analysis.

A margin of safety between section cracking and nominal strength of less than 1.0 ($M_n/M_{cr} < 1.0$) indicates insufficient vertical reinforcement for wall sections and an irrecoverable strength drop after cracking. The T- and I- shaped sections with the large flange lengths exhibited a section strength ratio of less than 1.0 when the axial load ratio was below ~10%. It is worth noting that low axial loads are common for lightly reinforced walls, raising concerns about the ductility of such wall designs. The flanged walls designed in accordance with the current minimum distributed vertical reinforcement limits of $\sqrt{f_c'}/4f_y$ are likely to exhibit a non-ductile response with low drift capacity after cracking. These model results highlighted the vulnerability of non-rectangular RC walls and the need to reassess the minimum vertical reinforcement limits for non-rectangular wall sections to ensure a ductile response.

Influence of Material Properties

The minimum distributed vertical reinforcement limits in NZS 3101:2006-A3 are dependent on both the reinforcement and concrete strengths ($\rho_l > \sqrt{f_c'}/4f_y$). Other standards, such as ACI 318-19, rely on a fixed minimum vertical reinforcement ratio, where higher concrete and/or lower reinforcement strengths can significantly reduce the ductility and deformation capacity [16]. The influence of material strengths on the ratio between nominal to cracking strength was investigated for the I-shaped wall sections designed in accordance with the minimum vertical reinforcement provisions in NZS 3101:2006-A3.

The I-shaped section with a web length of 8.2 m, thickness of 0.3 m, and flange length of 4.1 m was analysed for axial load ratio ranging from 0 to 30% and G300E and G500E reinforcement grades in accordance with AS/NZS 4671:2019 [17]. The modelled concrete compressive strength was 40 MPa with a mean tensile strength of 3.5 MPa, resulting in distributed vertical reinforcement ratios of 0.52% and 0.32% for the G300E and G500E models to satisfy the minimum vertical reinforcement requirements in NZS 3101:2006-A3.

Figure 7 compares the ratio of nominal flexural strength to cracking strength (M_n/M_{cr}) for I-shaped sections designed with reinforcement G300E and G500E. Both reinforcement grades resulted in similar sectional responses, with the strength ratio starting at ~0.6 at 0% axial load ratio and peaking at ~1.25 at 20% axial load ratio. The analysis results indicate that the ratio between cracking to nominal strength was independent of reinforcement grade, with the minimum reinforcement provisions in NZS 3101:2006-A3 accounting for the reinforcement strength and adjusting the amount of reinforcement required to achieve consistent performance.

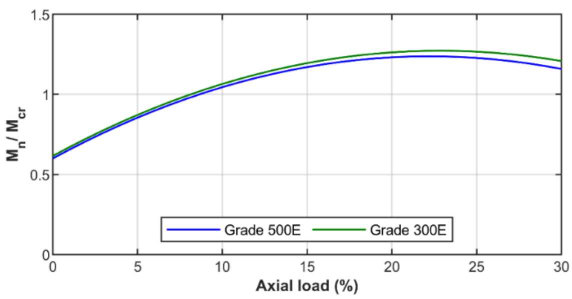


Figure 7: Calculated M_n/M_{cr} ratio for I-shaped sections with G300E and G500E reinforcement.

The I-shaped section with a web length of 8.2 m, thickness of 0.3 m, and flange length of 4.1 m was analysed for axial loads that ranged from 0% to 30% with concrete strengths varied from 30, 40 and 50 MPa, corresponding to mean tensile strengths were 2.9, 3.5 and 4.1 MPa. G500E reinforcement was used, resulting in required vertical reinforcement ratios of 0.28%, 0.32%, and 0.35%, satisfying the minimum vertical reinforcement provisions in NZS 3101:2006-A3.

Figure 8 compares the ratio of nominal flexural strength to cracking strength (M_n/M_{cr}) for I-shaped sections designed with the concrete compressive strengths of 30, 40 and 50 MPa. A close alignment of the analysis results is observed for the model, with the section strength ratio starting at ~0.6 at 0% axial load ratio and peaking at ~1.25 at 20% axial load. The model results showed the ratio between nominal to cracking strength of the section was not significantly influenced by the concrete compressive strength, with the minimum vertical reinforcement provisions in NZS 3101:2006-A3 already accounting for the concrete strength and adjusting the amount of reinforcement required to achieve consistent section performance. This would not be the case when applying equations for minimum reinforcement that do not account for concrete strength (e.g. ACI 318-19).

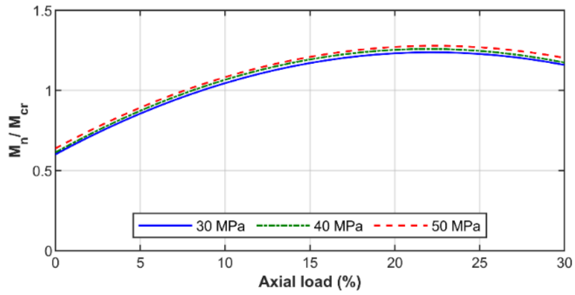


Figure 8: Calculated M_n/M_{cr} ratio for I-shaped sections with varied concrete strength.

Ductile Sections with Additional End Zone Reinforcement

Past research on the rectangular lightly reinforced concrete walls resulted in an increase in the reinforcement required in the end zones of walls to achieve sufficient ductility in plastic hinge regions [16]. In NZS 3101:2006-A3, a reinforcement ratio of at least $\sqrt{f_c'}/2f_y$ was adopted for both rectangular and non-rectangular walls, where the end zone length was defined as $0.15l_f$ or $0.15l_w$, as illustrated in Figure 9. The minimum distributed vertical reinforcement ratio of $\sqrt{f_c'}/4f_y$ was still required for the central web regions outside of the end zone.

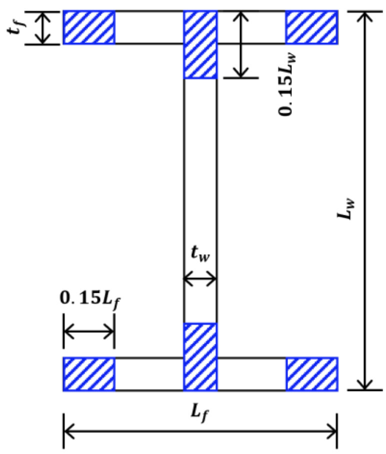


Figure 9: End zone definitions for non-rectangular I-shaped section.

The ratio between nominal to cracking capacity was also examined for non-rectangular wall sections designed to the minimum vertical reinforcement provisions for ductile plastic hinge regions in accordance with NZS 3101:2006-A3. The I-shaped section was designed with a web length of 8.2 m, thickness of 0.3 m, and varied flange lengths of 0.3, 0.82, 2.05, 4.1, and 8.2 m and axial load ratios from 0 to 30%. The concrete compressive strength was 40 MPa with a mean tensile strength of 3.5 MPa, and G500E reinforcement was used in the model.

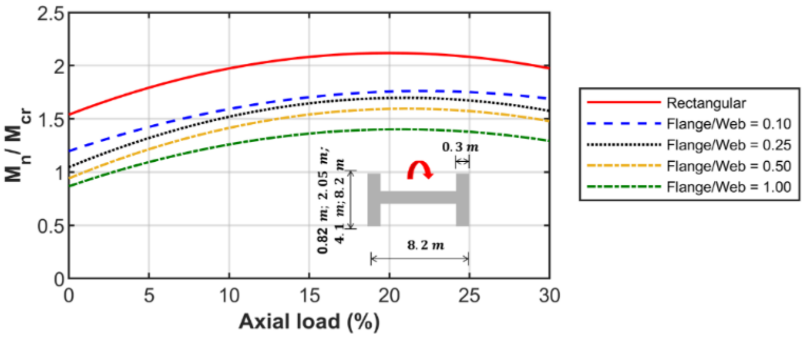


Figure 10: Margin of safety M_n/M_{cr} for ductile designed non-rectangular section in accordance with NZS 3101:2006-A3.

Figure 10 compares the calculated ratio between nominal flexural strength to cracking capacity (M_n/M_{cr}) for the I-shaped sections designed in accordance with the minimum vertical reinforcement provisions for ductile plastic hinge regions in NZS 3101:2006-A3. As with the previous analysis results, the section strength ratio had the lowest values at 0% axial load and peaked at 20% axial load, followed by a descending trend in the strength ratios as the increased axial load ratio resulted in compression controlled responses. For the rectangular wall section (solid red line), the margin of safety between nominal flexural strength to cracking capacity varied from 1.52 to 2.20 as axial load increased, which was higher than the corresponding ratios for the rectangular wall section with distributed minimum reinforcement (Figure 6). Although the sectional response does not explicitly relate to the formation of secondary cracks over the plastic hinge length, a M_n/M_{cr} ratio of ~2 can be considered as a substitute for the secondary cracking index based on the results of the rectangular wall section with additional end zone reinforcement [8].

For the non-rectangular wall sections with end zone reinforcement, the ratio of nominal to cracking strength decreased as the flange length was increased (see Figure 10). For the I-shaped sections, the strength ratios ranged from 0.92 to 1.28 at 0% axial load ratio to 1.45 to 1.78 at 20% axial load ratio. Of particular interest, the I-shaped sections with flange/web lengths of 0.5 (yellow dash line) and 1.0 (green dash line) still exhibited a margin of safety below 1.0 when the axial load ratio was low (<0.3% axial load ratio). The envelopes of the strength ratios indicate that even with the additional end zone reinforcement required for ductile plastic hinge regions, the amount of vertical reinforcement in the non-rectangular sections was still insufficient to ensure sufficient section ductility.

PROPOSED MINIMUM VERTICAL REINFORCEMENT LIMITS

The analysis results presented highlight that the minimum vertical reinforcement requirements in current concrete standards are insufficient to provide a desirable margin of safety for the nominal ductile non-rectangular wall sections with low axial loads. A theoretical equivalent minimum vertical reinforcement limit for representative I-shaped sections was developed to address the deficiencies for the nominal ductile designed non-rectangular sections.

As discussed in a previous study of rectangular walls [8], the M_n/M_{cr} criterion can be used to evaluate the ductility of the section response of the walls and is a useful metric to determine the required minimum distributed vertical reinforcement. As shown in Eq. (1, the nominal flexural strength (M_n) should exceed the cracking strength (M_{cr}) with a safety factor Ω is to ensure a satisfactory margin is achieved to account for variability in material strength and sectional response. While the nominal flexural strength (M_n) can be calculated with reasonable accuracy, the estimated cracking strength (M_{cr}) is highly depended on the assumed concrete tensile strength. As discussed by Henry [3] the cracking strength should be calculated using a mean estimate of tensile strength and the safety factor (Ω) used to ensure a sufficient margin to accommodate the range of possible tensile strengths.

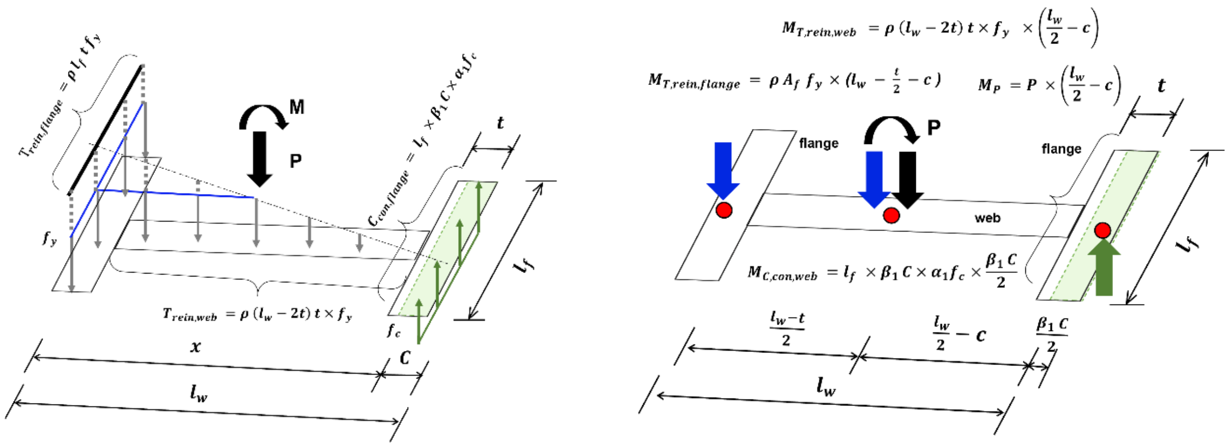
$$M_n \geq \Omega M_{cr} \qquad (1)$$

RC walls are usually subjected to a combination of bending and axial actions. The wall section will crack when the tensile stress in the extreme tension fibre reaches the concrete maximum flexural tensile strength. The calculated cracking moment of the non-rectangular wall section can be expressed by Eq. (2 assuming plane sections remain plane and linear-elastic response up until cracking, where M_{cr} is the cracked moment capacity, f_t is taken as the mean value of concrete tensile strength in *fib* Model code [15]. It is worth noting that using a higher value of f_t to calculate M_{cr}, e.g. an upper characteristic value, would effectively reduce the resulting safety factor if the calculated M_n remained constant. P is the axial load, A is the gross area of the wall section, I is the second moment of inertia for the wall sections, x is the distance from the extreme tension fibre to the neutral axis, which can be simplified as the centroid location as the neutral axis is approximately equal to the centroid before cracking.

$$M_{cr} = \left(f_t + \frac{P}{A}\right)\frac{I}{x} \qquad (2)$$

The nominal strength is implemented to calculate the design capacity for the section based on nominal material properties and standardised assumptions. NZS 3101:2006-A3 states that nominal flexural strength should be calculated assuming a strain of 0.003 at the extreme compression fibre and reinforcement at yield stress. For the purpose of the theoretical equations developed here, the following assumptions were applied in the calculation of nominal flexural strength of the non-rectangular wall section:

- The strain profiles are linear (Navier-Bernoulli "Plane sections remains plane" hypothesis).
- Neglect concrete tensile strength at cracked sections when calculating nominal flexural strength.
- Axial force is applied at the section centroid considering the symmetrical geometry.
- All the vertical reinforcement yields in either tension or compression at nominal flexural strength.
- The compressive region concentrates within the flange with no extension into the web region.
- Reinforcement stresses in the compression flange are neglected.
- The entire flange length is effective in tension and compression, ignoring tension lag effects.



(a) Illustration of force equilibrium *(b) Illustration of moment equilibrium*

Figure 11: Calculation of nominal strength for the non-rectangular I-shaped section.

To solve the unknown variables for the neutral axis depth and the required reinforcement ratio, both force and moment equilibrium equations were established for the I-shaped section, based on the parameters illustrated in Figure 11. Force equilibrium of the I-shaped section is expressed in Eq. (3 and Eq. (4, where $T_{rein,flange}$ and $T_{rein,web}$ are the tensile reinforcement forces in the flange and web regions, $C_{con,flange}$ is concrete compressive force in the compressive flange, P is the axial load, ρ is the equivalent vertical reinforcement ratio, l_f and l_w is the flange and web lengths, t is the flange and web thickness, f_y is reinforcement yield strength, f_c is concrete compressive strength, α_1 and β_1 is the rectangular stress block parameters as defined in NZS 3101:2006, c is the neutral axis length.

$$T_{rein,flange} + T_{rein,web} + P = C_{con,flange} \tag{3}$$

$$\rho l_f t f_y + \rho(l_w - 2t)tf_y + P = l_f \times \beta_1 c \times \alpha_1 f_c \tag{4}$$

Taking bending moments about the neutral axis c location, the nominal flexural strength can be expressed as shown in Eq. (5 based on the actions illustrated in Figure 11. By substituting Eq. (2) and Eq. (5 into Eq. (1, the formula in Eq. (6 can be obtained. Rearranging Eq. (6, the minimum vertical reinforcement ratio required to satisfy the section moment criterion can be expressed as shown in Eq. (7. It is worth noting that Eq. (7 should only be applied when the assumptions used to derive this expression are considered valid for the section being analysed.

$$M_n = T_{rein,flange} \times \left(l_w - \frac{t}{2} - c\right) + T_{rein,web} \times \left(\frac{l_w}{2} - c\right) + P \times \left(\frac{l_w}{2} - c\right) + C_{con,flange} \times \frac{\beta_1 c}{2} \tag{5}$$

$$\rho l_f t f_y \times \left(l_w - \frac{t}{2} - c\right) + \rho(l_w - 2t)tf_y \times \left(\frac{l_w}{2} - c\right) + P \times \left(\frac{l_w}{2} - c\right) + l_f \times \beta_1 c \times \alpha_1 f_c \times \frac{\beta_1 c}{2} \geq \Omega \times \left(f_t + \frac{P}{A}\right)\frac{2I}{l_w} \tag{6}$$

$$\rho \geq \frac{\Omega \times \left(f_t + \frac{P}{A}\right)\frac{2I}{l_w} - P \times \left(\frac{l_w}{2} - c\right) - l_f \times \beta_1 c \times \alpha_1 f_c \times \frac{\beta_1 c}{2}}{\left[l_f t f_y \times \left(l_w - \frac{t}{2} - c\right) + (l_w - 2t)tf_y \times \left(\frac{l_w}{2} - c\right)\right]} \tag{7}$$

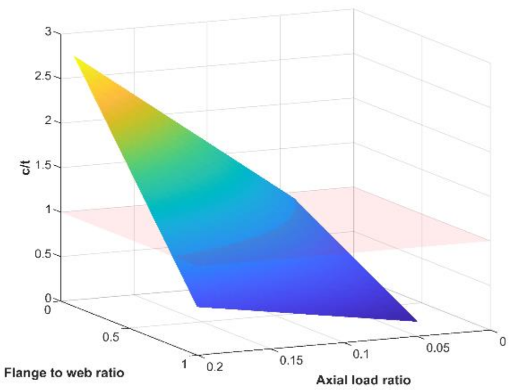
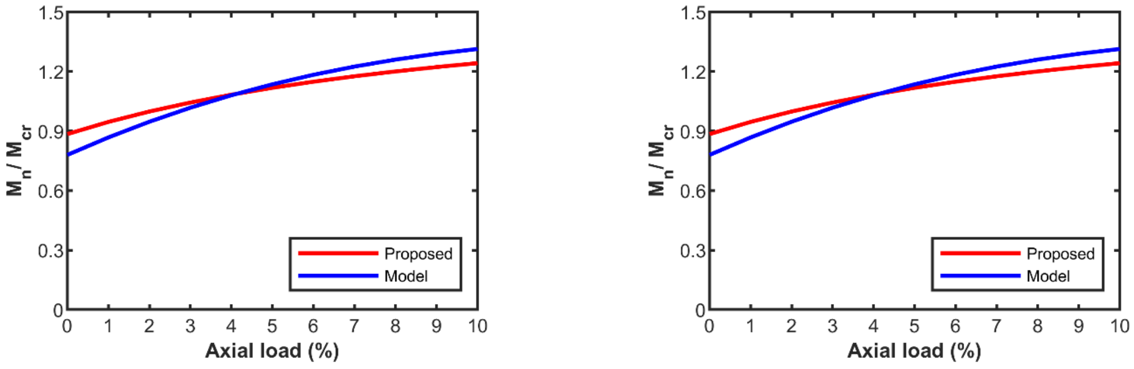


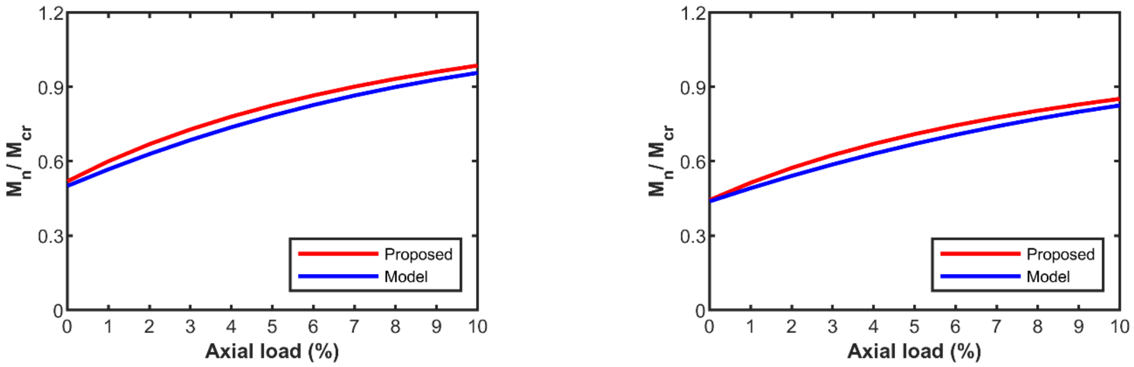
Figure 12: Ratio of neutral axis depth to wall thickness across a range of axial load and flange to web ratios.

Eq. (7 is complex and not easy to implement in design, so a simplified expression was developed. As previously discussed, the lowest strength margin occurred in I-shaped section with a long flange and low axial loads where the neutral axis was confined within the flange width. The most significant parameters that affect the ratio of the neutral axis to the thickness (c/t) was the flange to web ratio and the axial load ratio. A parametric study was conducted to show the relation for c/t, flange to web ratio and the axial load ratio, as shown in Figure 12. The portion below the red plane represented the I-shaped sections with a lower neutral axis than the flange thickness. The model results indicated that the long flange length and low axial load ratio resulted in a short neutral axis.



(a) Flange/web = 0.5 *(b) Flange/web = 1.0*

Figure 13: Comparison of the ductile I-shaped section with modelled and proposed method.



(a) Flange/web = 0.5 *(b) Flange/web = 1.0*

Figure 14: Comparison of the nominally ductile I-shaped section with modelled and proposed method.

The fitting equation in Eq. (8 was provided to estimate the neutral axis length, where n is the axial ratio. It was worth noting that the $\frac{c}{t}$ should be less than 1.0 to satisfy the neutral axis assumption. The neutral axis c derived from the fitting equation was substituted into Eq. (7 to simplify the calculation for the proposed minimum equivalent vertical reinforcement ratio.

$$\frac{c}{t} = 0.43 + 13.1n - 0.39\frac{l_f}{l_w} - 10.6n\frac{l_f}{l_w} \leq 1.0 \quad (8)$$

Figure 13 and Figure 14 compare the M_n/M_{cr} criterion calculated using the derived expressions Eq.2 and Eq.5 with the results of a detailed section analysis using a fibre element model for the ductile and nominal ductile I-shaped sections and axial load ratios ranging from 0% to 10%. The geometry of the I-shaped section was identical to the previously modelled section, with a web length of 8.2 m and flange lengths of 4.1 and 8.2 m. The concrete strength was 40 MPa and reinforcement yielding was 500 MPa, resulting in the equivalent reinforcement ratio of 0.42% and 0.32% for the ductile designed and nominal ductile designed I-shaped sections. In general, the results of the proposed theory closely matched the results from the details sectional analysis model across the range of variables considered, with errors typically less than 10%. The close alignment to the analysis results indicated that the proposed theoretical method can accurately calculate the M_n/M_{cr} criterion and that the assumptions used are generally appropriate for such calculations.

EVALUATION OF THE PROPOSED REINFORCEMENT LIMIT

The analysis in the previous sections demonstrated that the flange to web length ratio and concrete strength were the critical factors that significantly influenced the margin between nominal and cracking strength for non-rectangular I-shaped sections with low axial loads. These critical variables and the influence on the required minimum vertical reinforcement were evaluated using the theoretical expression presented in Eq. (7. For the purpose of this evaluation, the safety factor (Ω) for the margin between nominal flexural strength and cracking strength was set to 1.6, which was consistent with the average range of M_n/M_{cr} for the minimum reinforcement applied to rectangular beams [3] and has been used previously for a similar study on rectangular walls [8]. Note that the safety factor is included as a variable in Eq. (7 which allows engineers to apply their own judgement when selecting an appropriate value. The proposed minimum vertical reinforcement ratio for the I-shaped sections was compared with the requirements for the nominal ductile walls in accordance with NZS 3101:2006-A3, ACI 318-19, and the theoretical reinforcement requirement for the rectangular sections proposed by Lu and Henry [8].

Concrete Strength

The I-shaped section designed with a web length of 8.2 m, thickness of 0.3 m, and flange length of 4.1 m was used to investigate the influence of concrete strength on the minimum vertical reinforcement amount with a 5% of axial load ratio. The assumed reinforcement yield strength was 500 MPa, and the concrete compressive strength ranged from 30 MPa to 60 MPa. The tensile strength was calculated in accordance with the *fib* Model code as $0.3(f_{ck})^{2/3}$ [15].

The proposed vertical reinforcement for the I-shaped sections in Eq. (7 was compared with the minimum vertical reinforcement requirements in accordance with NZS 3101:2006-A3 [6], ACI 318-19 [7] and Lu and Henry [8] in Figure 15. As the concrete compressive strength increased from 30 MPa to 60 MPa, the required minimum distributed vertical reinforcement in accordance with Eq. (7 increased from 0.65% to 1.01%. These reinforcement ratios were considerably higher than the fixed minimum 0.25% required by ACI 318-19, the 0.27% to 0.38% required by current NZS 3101:2006-A3 provisions, and the theoretical estimates for rectangular sections proposed by Lu and Henry [8] that ranged from 0.22% to 0.31%. Although both the NZS 3101:2006-A3 [6] and Lu and Henry [8] requirements considered the concrete strength, these requirements were still insufficient to ensure the required margin of safety for non-rectangular sections.

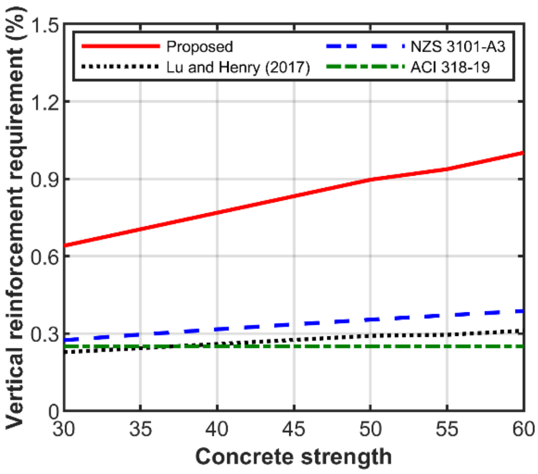


Figure 15: Minimum vertical reinforcement for I-shaped sections with different concrete strengths.

The theoretical minimum vertical reinforcement ratio proposed in Eq. (7 indicated that the higher concrete strength combined with the flange length for the I-shaped sections significantly increased the cracking strength, and as such a higher reinforcement ratio was required to improve the nominal flexural strength by the same proportion. The comparison results revealed that the minimum vertical reinforcement required in ACI 318-19 [6], NZS 3101:2006-A3 [7] and Lu and Henry [8] are insufficient to provide a desirable safety factor for non-rectangular sections with higher concrete strength.

Flange to Web Ratio

I-shaped sections designed with a web length of 8.2 m, thickness of 0.3 m, and varied flange lengths of 0.82, 2.05, 4.1 and 8.2 m were to investigate the impact of the flange to web ratios on the minimum vertical reinforcement with a 5% axial load ratio. The concrete compressive strength was 40 MPa with a mean tensile strength of 3.5 MPa, and the assumed reinforcement yielding strength was 500 MPa.

The proposed minimum vertical reinforcement ratio for the I-shaped sections in Eq. (7 was compared with the requirements in accordance with NZS 3101:2006-A3 [6], ACI 318-19 [7] and Lu and Henry [8] in Figure 16. As the flange to web ratio increased from 0.1 to 1.0, the calculated minimum distributed vertical reinforcement in accordance with Eq. (7 increased from 0.35% to 0.91%. The comparative minimum vertical reinforcement contents required by NZS 3101:2006-A3 [6], ACI 318-19 [7] and Lu and Henry [8] ignore the flange effect that required a fixed reinforcement ratio of 0.25%, 0.31% and 0.26%, representatively, significantly lower than the ratio required by the proposed theoretical approach. The model results indicated that the minimum vertical reinforcement requirements in current design standards and theory based on rectangular wall sections are insufficient to ensure adequate vertical reinforcement is provided in non-rectangular wall sections to meet the section strength criterion for a ductile response.

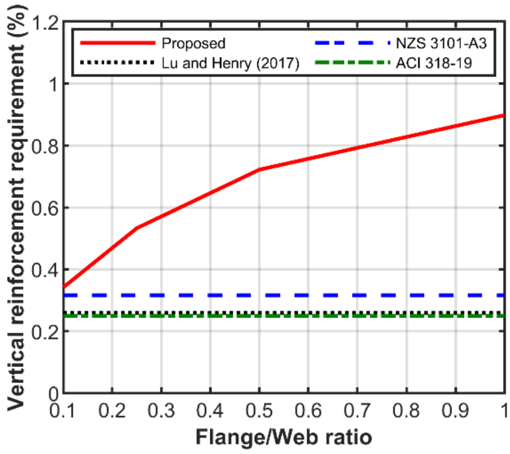


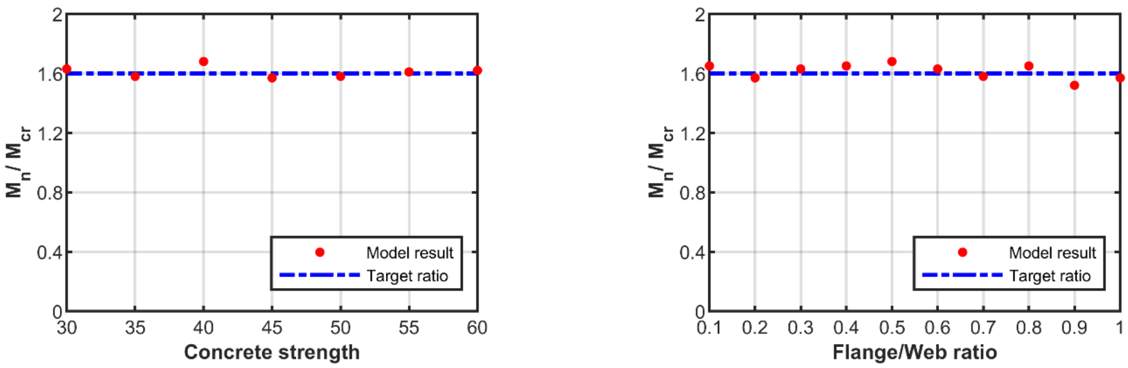
Figure 16: Minimum vertical reinforcement for I-shaped sections with different flange to web ratios.

Comparison of Theoretical and Model Results

The vertical reinforcement ratios obtained using Eq. (7 for the analyses presented in Figure 15 and Figure 16 were used to re-design the distributed vertical reinforcement for the I-shaped sections with different concrete strength and flange lengths. The re-designed wall sections were then analysed and the calculated M_n/M_{cr} ratios are compared in Figure 17 across the range of concrete strength and flange to web ratios. The calculated M_n/M_{cr} values for the sections designed using the proposed theoretical approach were close to 1.6 safety factor that was applied in Eq.7 for all the design cases with errors less than 5%. These results confirmed that the theoretical method proposed and expressed in Eq. (7 can accurately estimate the minimum vertical reinforcement ratio to satisfy the section moment criterion. Note that the safety factor is included as a variable in Eq.7 which allows engineers to apply their own judgment when selecting an appropriate value.

CONCLUSIONS

The minimum vertical reinforcement requirements in NZS 3101:2006-A3 were derived based on assessments of the seismic performance of the rectangular walls. However, non-rectangular flanged sections amplify the cracking strength, influencing the section response and ratio between cracking strength and nominal flexural strength. This study proposed an alternative minimum vertical reinforcement expression for non-rectangular sections to satisfy the section moment criterion for a ductile response. Sectional moment-curvature responses were compared for different wall geometries to determine the impact on the ratio between nominal to cracking capacity designed with the light reinforcement contents. A parametric study was conducted to investigate the ratio of nominal flexural to cracking strength for the nominal ductile and ductile designed I-shaped sections in accordance with the minimum vertical reinforcement provisions in NZS 3101:2006-A3. New theory and expressions were proposed to calculate the required minimum vertical reinforcement for the I-shaped sections to ensure a consistent margin between cracking and nominal capacity. The main conclusions drawn from this investigation include:



(a) Concrete strength *(b) Flange to web ratio*

Figure 17: Calculated M_n/M_{cr} ratio for I-shaped wall sections designed according to the proposed theoretical approach.

- Compared with rectangular wall sections, flanged wall sections exhibited a significant reduction in the margin between the nominal and cracking strength. Due to the largest second-moment of inertia among non-rectangular sections, the I-shaped section had the lowest M_n/M_{cr} ratios representing the most vulnerable section geometry.
- Non-rectangular sections designed in accordance with current provisions in NZS 3101:2006-A3 for both the minimum distributed reinforcement as well as additional end zone reinforcement in plastic hinge regions were found to result in M_n/M_{cr} ratios below 1.0 when long flanges and low axial loads were applied. These results highlight the vulnerability of currently designed non-rectangular walls to non-ductile section response.
- The section responses of I-shaped sections with minimum vertical reinforcement in accordance with current provisions in NZS 3101:2006-A3 confirm that the M_n/M_{cr} ratios were independent of both the reinforcement and concrete strength as both are included as variables in the required minimum reinforcement ratio.
- The comparison of minimum vertical reinforcement limits for NZS 3101:2006-A3, ACI 318-19 and Lu and Henry [8] showed that current vertical reinforcement requirements that exclude the consideration of flange length were insufficient to ensure a desirable section response for non-rectangular wall sections, especially for walls with, higher concrete strengths and longer flange lengths.
- The proposed reinforcement limits for the I-shaped sections were shown to achieve a consistent margin of safety between section cracking and nominal strength which can be set by the engineer. To achieve this consistent margin, the theoretical expression developed results in a significant increase in the minimum vertical reinforcement limits for non-rectangular wall sections when compared to existing requirements that are only suitable for rectangular wall sections.
- The proposed expression should only be relied on in design after confirming that the assumptions used to derive this expression are valid for the section being analysed. Alternatively, the nominal and cracking strengths of the section can be calculated directly and the margin of safety between section cracking and nominal strength checked considering the range of expected variation in each value.

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