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DEVELOPMENT OF PROVISIONS FOR SIMPLIFIED DESIGN OF ROCKING FOUNDATIONS

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ABSTRACT

A simplified design procedure has been developed for potential inclusion within NZS1170.5 to allow rocking shallow foundations for low- to mid-rise buildings without special study. Rocking foundations allow a nonlinear uplift and soil yielding mechanism to form at the soil-foundation interface, and can significantly reduce the required size of foundations (or avoid requiring deep foundations) and reduce seismic demands on a building. The procedure has been used to design a series of buildings and the performance of these buildings has been evaluated using a displacement-based assessment procedure that accounts for soil-foundation-structure interaction. Given that the equivalent static procedure in NZS1170.5 contains several conservative assumptions for multiple storey buildings, and the displacement-based approach is a first mode approximation, the majority of designs and assessments were based on a single degree-of-freedom (SDOF) system. Variations in the design and soil property assumptions were considered, as well as different thresholds for proposed limitations on the applicability of the simplified procedure. Seven performance measures were used to evaluate and demonstrate that the limitations in the proposal result in adequate behaviour for ultimate limit state and serviceability limit state actions.

A displacement correction, to conventional NZS1170.5 displacement procedures, is proposed as a concentrated rotation at the underside of the foundation. This correction accounts for foundation rotation prior to reaching the moment capacity.

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INTRODUCTION

Numerous experimental (e.g. [1-6]), and numerical studies (e.g. [7-10]) have demonstrated that rocking foundations can provide stable and predictable behaviour, that limits seismic actions in the superstructure. However, the design of rocking foundations under NZS1170.5:2004 [11] requires a special study from Clause 6.6:

> *Where energy dissipation is through rocking of structures or structural sub-assemblies (which include, but are not restricted to, walls, frames and foundations), the actions on the structures and parts being supported by the structures shall be determined by special study. Such studies are outside the scope of this Standard (see Clause 1.4).*

While the design actions are not fully prescribed in this clause, implicitly the foundation may only (without special study) be designed to resist the lesser of the overstrength actions or actions derived for a nominally ductile system (design ductility, μ=1.0 or 1.25). This clause prevents significant nonlinear behaviour (both uplift and soil yielding) at the foundation.

For many seismic systems, particularly shear walls, the prevention of significant uplift of a shallow foundation is not possible without using piles. This logical design step achieves compliance with NZS1170.5, however, it also maximises the seismic energy entering the structure and can be considerably more expensive than a well-designed rocking foundation. The previous New Zealand loading code, NZS4203:1992 [12], had more liberal provisions for rocking structures, Clause 4.11.1.2 stated:

> *Where dissipation of energy is through rocking of foundations, the structure shall be subject to a special study, provided that this need not apply if the structural ductility factor is equal to or less than 2.0.*

Under these provisions, foundations could be designed using actions considerably lower than the nominally ductile or overstrength loads, and many of the benefits of rocking foundations could be realised. Internationally, the 2005 Canadian National Building Code [13] added a rocking provision similar to NZS4203:1992. Similarly, the United States seismic assessment code, ASCE 41-17 [14], recognises that rocking foundations (quantified through the m-factor, similar to design ductility in 1170.5) can achieve the desired building performance.

The challenge with incorporating design ductility within foundation design for 1170.5 is that the published rocking foundation design procedures (e.g.[15,16,10]) are secant stiffness based (i.e. displacement-based), while 1170.5 is inherently initial stiffness based. To incorporate rocking foundations within a design procedure requires either the designer to be able to estimate displacements from soil-foundation-structure interaction (SFSI) or limit the design parameters to a subset, such that designs will implicitly avoid unacceptable behaviour. In this context displacements must consider both transient and residual horizontal, vertical and rotational displacements. The unacceptable behaviours considered in this study are outlined below in Table 1. The outcome of this study recommends that both approaches be used, depending on the elements being considered (i.e. the primary lateral force resisting system (LFRS) can avoid unacceptable behaviour implicitly through the proposed design limits, while secondary elements can be more sensitive to displacements and therefore recommended displacement estimates are provided to aid their design).

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Table 1: Measures of unsatisfactory behaviour for rocking foundations.

Behaviour	Evaluation measure
Global toppling	Compute the displacement factor of safety against global toppling: $$FOS_{topple} = \frac{\Delta_{topple}}{\Delta_{demand}}$$ Where the capacity is taken as the simplified estimate of the static toppling displacement: $$\Delta_{topple} = \frac{L_{ip}}{2}\left(1 - \frac{1}{FOS_v}\right)$$ The demand is taken as the displacement of the centre of mass.
Serviceability deflections exceed requirements	Compute the deflection at the serviceability limit state, SLS1, demand during assessment – also compare this to the estimated SLS1 deflection from design.
ULS deflections significantly greater than predicted	Compute the deflection at ULS demand during assessment and compare this to the estimated ULS deflection from design.
Forces significantly larger than expected	Compute base shear at 100% of design load during assessment and compare to design base shear.
Significant vertical uplift (reason: damaging to connections)	Compute the simplified vertical uplift: $$\delta_{up} = \left(\theta_f - \theta_{f,uplift}\right) L_{arm}$$ Where the lever arm was computed as: $$L_{arm} = L_{ip}\left(1 - \frac{1}{FOS_v}\right)$$
Significant permanent deformations	Estimate settlement and permanent tilt using two simplified methods, Deng et al. (2014) and Millen et al. (2020).

The intention of this study was to develop a simplified approach for low- to mid-rise buildings which allows the design of rocking foundations without special study, for limited classes of structures.

The development steps for producing the simplified approach were:

1. Development of an evaluation framework, including outlining potentially negative consequences of rocking foundations.
2. Develop a simplified design procedure compatible with 1170.5.
3. Develop an assessment procedure to evaluate whether designs satisfied the evaluation criteria.
4. Consideration of extensions and clarifications needed to achieve satisfactory design outcomes (e.g. estimation of peak displacements).
5. Consideration of limitations of applicability of the simplified procedure to allow designs to implicitly satisfy the evaluation criteria.

Ultimately, iteration between steps was required to achieve a design procedure that was simple while maximising its applicability. The paper is presented in the order of the development steps, however, steps 2 & 3 are combined into a single section. The paper should be read in conjunction with the proposed Technical Specification, TS 1170.5:2023, and corresponding commentary. While some details have been repeated within the paper, this paper focuses on the supporting evidence for the content of the TS, and does not contain the exact clauses.

EVALUATION FRAMEWORK

The evaluation framework was developed to outline performance measures that could indicate unacceptable behaviour. Noting that designing a rocking foundation encourages strong SFSI, and therefore displacements and forces can be significantly modified compared to fixed base assumptions. Table 1 outlines the performance measures used to track behaviour and how they were computed.

Proposed Design and Assessment Procedure

The design and assessment process was split into six steps, as outlined in Table 2.

Table 2: Design and assessment steps.

Steps	Approaches	Outputs
1) Select building and soil properties	• Case study building • Single property variation • Monte Carlo variation	• Inputs for building design/assessment
2) Convert to SDOF	• Linear displacement distribution	• Effective height • Effective mass • Effective stiffness
3) Design building and foundation	• SDOF equivalent static loads • MDOF equivalent static loads	• Design base shear • Foundation size • Design displacements (SLS1, ULS)
4) Consider soil uncertainty	• Median • Lower bound (50%) • Upper bound (200%)	• Soil shear modulus • Soil bearing strength
5) Determine rotational stiffness	• Elasticity theory [17] • Empirical [16]	• Foundation rotational stiffness
6) Assess system	• DBA (Displacement-based assessment)	• Assessed displacements (SLS1, ULS) • Base shear • Foundation rotation • Settlement • Foundation tilt

In the majority of the steps, alternative approaches were used, typically to address some uncertainty in behaviour or in the interpretation/application of the procedure.

Each step is covered in detail in the following subsections, specific values and results for a baseline case study building are also provided where applicable. Full calculations of the case study building for the SDOF and MDOF design processes and assessments using Displacement-based assessment (DBA) with the theoretical stiffness using the median soil properties, are presented in Digital supplements A and B respectively. The Python codes for running all variations of the SDOF and MDOF design and assessment are presented in Digital supplements C and D respectively, as well as a summary of the case study outputs.

Step 1: Set building and soil properties

All designs were based on the same basic building layout shown in Figure 1. To provide reasonable coverage of the range of likely design scenarios, large variations in building properties were considered as listed in Table 3. The intention of these variations was to cover the following structural archetypes: industrial tilt slab, office building, residential multiple-storey units. Frame buildings were not considered due to the complex frame action that modifies the vertical loads in outer footings. Commercial buildings were not explicitly considered due to their often irregular foundation layout. The proposed design criteria address these excluded archetypes.

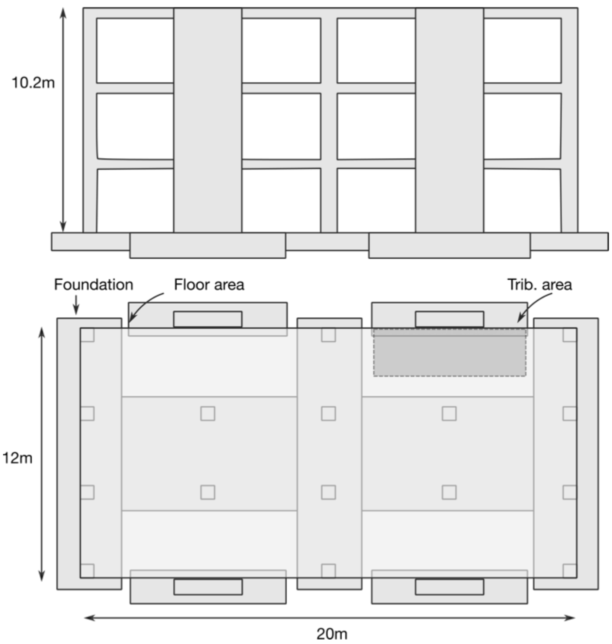

Figure 1: Case study building layout.

Ultimately the variation in input properties provided realistic ranges of vertical load and seismic mass to be resisted by a foundation element sitting on a range of soil properties. In the development of these loads the following simplifications were made:

- The internal frame was assumed to provide no seismic resistance in the direction of the walls
- There were deliberately no tie beams connecting to the rocking foundations, to simplify the calculations. However, tie beams should be used in design to connect foundation elements. Tie beam resistance can easily be incorporated into the foundation moment (See commentary on calculation of moment capacity). Tie beams would increase moment capacity and rotational stiffness (similar to the upper bound soil response, although potentially greater).

Table 3: Building and soil parameter ranges.

Parameter	Case study	Range
Z (Hazard factor from [11])	0.25	0.1-0.6
No. storeys	3	1-4
Interstorey height	3.4 m	3.2-3.6 m
Applied load per storey	8 kPa	5-10 kPa
Tributary area ratio (per 1 of 4 walls)	0.1	0.05-0.25
Site class [11]	C	B, C, D
G_{soil}	25 MPa	16.5-50 MPa
Soil shear modulus to bearing stress ratio (G/q_{ult})	35	20-70
Foundation height	$0.2{+}0.05h_n$	-

Additional inputs included the soil Poisson's ratio of 0.2, soil unit mass of 1650 kg/m^3, and foundation unit mass of 2.3 T/m^3. The adoption of a tributary area ratio requires some explanation, it is the ratio of the vertical load on one of four footings to the total seismic participating weight. The maximum value can therefore slightly exceed ¼ since the foundation weight, and the lower part of the structure, typically are not fully represented in the first mode participating seismic weight. The adoption of this ratio significantly simplified the evaluation, and could clearly identify the importance of this key parameter. The lower end of the range represented designs that contain a large gravity only or perpendicular frame system, where the vertical load on the rocking footing can be only a very small proportion of the total vertical load. The upper end of the range covers portal frames, and other industrial buildings, where foundations are typically only on the outer edge, and therefore the vertical load is completely supported by the rocking footings. A further advantage of adopting an tributary area ratio, is that conventional tributary area calculations are based strictly on area. However, if the gravity load system does not uplift but the LFRS does, some vertical load may redistribute to the foundations of the LFRS. Capturing this complexity would require understanding the vertical resistance of all elements connecting the gravity system to the rocking foundations.

In this study the soil properties were defined using an equivalent uniform small strain shear modulus (G_{soil}) and an ultimate bearing stress, q_{ult}. q_{ult} was determined from a ratio with G_{soil}. This deliberate simplification was because the two properties are coupled through both state (e.g. density) and material properties (e.g. angularity). Recent back-calculated ratios (G_{soil}/q_{ult}) from Hakhamaneshi et al. [18] for rectangular embedded and surface footings in centrifuge experiments were 25-80 and 40-100 respectively. The ranges adopted in this study differ since the small strain shear modulus is used here, whereas Hakhamaneshi et al. [18] used secant stiffness to 50% of moment capacity. Two further reasons justify this simplification, first, the ratio of G_{soil}/q_{ult} does not have a significant impact on results. Although higher G_{soil}/q_{ult} results in more conservative estimates of drift. Secondly, both the soil small strain shear modulus and peak bearing strength are very difficult to estimate in frictional material, due to their strong dependence on confining stress. Confining stress varies with depth but also varies significant under the footing depending on the applied load and footing shape. Finally, using a G_{soil}/q_{ult} ratio avoids the sometimes considerable iterations involved in establishing the peak friction angle and covers the majority of values for shallow foundations.

Of interest, a small study was conducted to investigate the soil parameters that produce the adopted G_{soil}/q_{ult} range. For sand it corresponded to relative densities of 50-80%, and clay with rigidity ratios (shear modulus divided by undrained strength) of 400-500, when computing bearing capacity according to Salgado [19], and shear modulus according to ASCE 41-17 [14].

Step 2: Convert to SDOF

For the purpose of isolating the implications of rocking foundations, each building was converted to a SDOF based on an assumed linear displacement profile. OpenSeesPy [20] eigen value analyses confirmed that this profile resulted in a period less than 5% shorter for the SDOF compared to the MDOF that had the same stiffness. The design and assessment was then performed directly on the SDOF. To evaluate this simplification the design and assessment was also repeated on the multi-storey building.

Discussion on Using SDOFs Versus MDOFs

The intention of this study was to identify the effects caused by the introduction of the proposed provisions, between the behaviour during design versus the likely behaviour (established through assessment). There are several simplifying assumptions within the NZS1170.5 design process that obscure the effects of rocking (e.g. assuming the total building weight participates in the first mode response for equivalent static design) and attempts were made to reduce these. Additionally, using an equivalent SDOF in both the design and assessment avoided a mismatch as the displacement based procedure adopted for assessment only considers the first mode response. The SDOF results were favoured since they excluded the complexities and biases of design for MDOFs, and because they isolated the rocking behaviour to enable the identification of biases in the proposed simplified rocking design.

A modal design approach was not considered, however, the modal design reduces the conservatism of using the full building weight as the seismic weight. The use of modal analysis for 1 or 2 storey buildings would result in the same design load and foundation size as SDOF design, since 90% or more mass is in the first mode.

Step 3: Design building and foundation

The design base shear was determined using Equation 1:

$$V_{base} = C_d(T_1) \cdot \mathrm{W_t} \tag{1}$$

Where W_t is the total seismic weight, and $C_d(T_1)$ is the horizontal design coefficient from 1170.5:2004. $C_d(T_1)$ was computed using a near fault factor of 1, return period factor of 1, and the period, T_1, was determined from the building height, h_n, using Equation 2 from 1170.5:2004 Commentary section C4.1.2.

$$T_1 = 1.25\ (0.05) \cdot h_n^{0.75} \tag{2}$$

The foundation was sized so that the moment capacity was equal to the design overturning moment. If the bearing capacity factor of safety for the seismic load case, FOSv, was less than 5, the design was removed in post-processing. This implicitly removed unrealistic designs that would not satisfy permanent load requirements, such as long term settlement. Alternatively, the foundation size could have been increased and smaller design ductility could be adopted. This alternative redesign was not considered since it would have introduced a bias in the results when back-calculating the displacement from the assessment procedure.

FOSv is used in this paper instead of a limit state design approach that considers strength reduction and load amplification factors. This deliberate choice allowed expected median properties to be used in both design and assessment.

The moment capacity was determined using the simplified moment capacity estimate based on the FOSv (Equation 3). Where N is the vertical load, including the weight of the foundation, and L_{ip} is the foundation in-plane length. The adoption of the simplified moment equation was considered appropriate compared to more refined estimates (e.g. Deng et al. [16]), since the uncertainty in bearing strength was directly considered in the assessment and this uncertainty results in a greater variation in moment capacity than the differences between equations.

$$\mathrm{M}_{f,ult} = N \cdot \frac{L_{ip}}{2}\left(1 - \frac{1}{FOS_v}\right) \tag{3}$$

The displacements were then directly calculated using 1170.5:2004 Clause 7.2.1.1a, where the inelastic displacement was taken as the elastic deflection multiplied by (μ-1). For the calculation of elastic displacements the structural stiffness was set to be consistent with the assumption of the vibration period (rather than an estimated cracked stiffness value).

Comparison between SDOF and MDOF Approaches

Table 4 compares the differences between the SDOF and MDOF approaches. Note that the SDOF approach is not an available approach under 1170.5 and was only used for isolating the influence of foundation rocking.

Table 4: Differences between SDOF and MDOF design approaches.

Approach	SDOF	MDOF
Seismic weight for base shear	SDOF effective mass multiplied by gravity	Building weight (excluding foundation weight)
Foundation overturning moment	Base shear multiplied by effective height plus foundation height	Equivalent storey forces multiplied by height from base of foundation
Displacements	Base shear multiplied by effective stiffness	Deflection at each storey computed using moment-area theorem due to equivalent storey forces.

Figure 2 compares the design and assessment behaviour when adopting the MDOF (i.e. multi-storey design) approach versus the SDOF approach. The SDOF approach results in lower design base shear due to only considering the first mode participation mass, whereas the multi-storey approach considers the total weight to contribute to the first mode (Equivalent static design). The foundation of the MDOF approach is considerably larger due to both the higher base shear and the distribution of lateral force being effectively higher than the SDOF height. The larger foundation results in slightly less foundation rotation in the assessment. Note that the assessment response is covered in step 6 below.

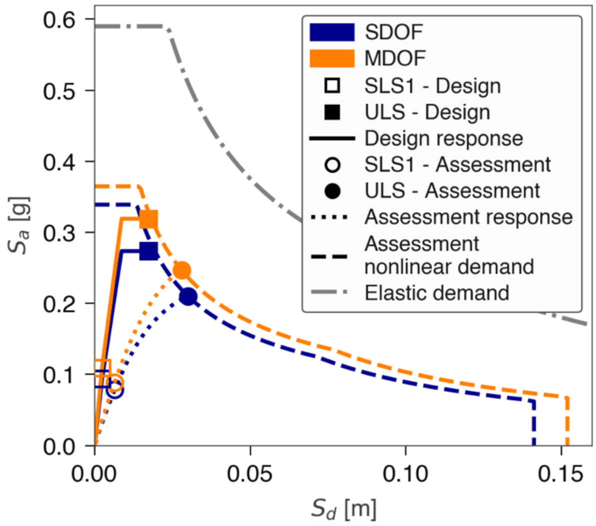

***Figure 2**: Comparison between the design and assessment of the case study building for both SDOF and MDOF*

Drift Correction for SDOF Design and Assessment

The superstructure drifts computed using the equivalent SDOF must be amplified by a factor of 1.4 for multi-storey buildings to obtain the maximum interstorey drift (as detailed in Figure 3). This is because the distribution of displacement over the building height is not linear, and larger drift occurs near the top of the structure. Figure 3 shows the ratio of maximum interstorey drift versus SDOF drift when using the storey forces distribution from NZS1170.5 and the same base shear. Note that this correction only applies to the elastic superstructure displacement.

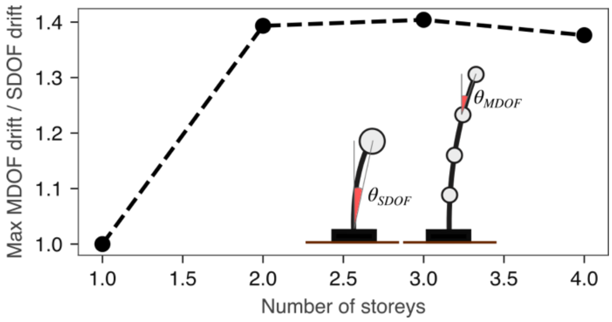

Figure 3: Ratio of interstorey drift versus SDOF drift.

Alternative Design Approach Directly Including SFSI Effects

An alternative design approach was considered where the period is updated to account for the increased flexibility from SFSI. The spectrum demands were calculated using the system period (considering SFSI), which was calculated based on soil and foundation properties. Iteration was required as the foundation needed to be sized to estimate the period, which subsequently influenced the design loads and the required foundation size. The foundation rotational stiffness was taken as half the initial stiffness to account for nonlinearity.

Comparing the two design procedures, the increased period resulted in lower equivalent loads and therefore smaller foundations. Typically, displacements were larger and in some cases required significant iterations when slightly close to the cut off for P-delta provisions. Remarkably, foundations were in some cases less than half the length of a design based on the fixed base period (See Figure 4).

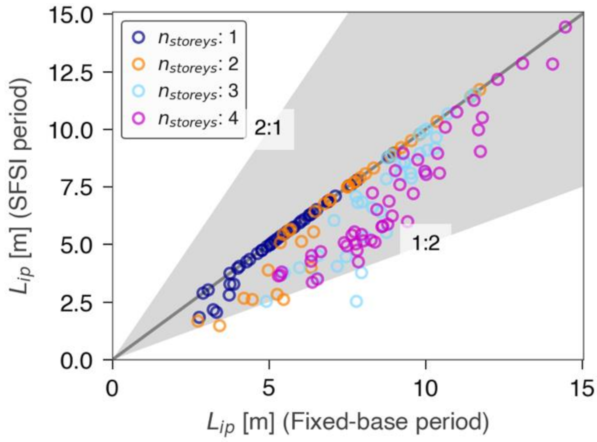

Figure 4: Comparison of foundation sizes when using a period updated to account for SFSI effects.

Step 4: Consider soil uncertainty

Soil uncertainty was considered through an adjustment to both G_{soil} and q_{ult}. G_{soil} is used to obtain the foundation rotational stiffness using the elastic theory approach. q_{ult} was used to obtain both the FOSv, and the foundation overturning capacity. Uncertainty in these values was considered by adopting lower and upper bound scenarios where these values were adjusted to be 50% and 200% of the original estimates, consistent with the approach in ASCE 41-17 Section 8.4.2.3.1 [14].

Step 5: Determine rotational stiffness

The initial rotational stiffness can be extremely difficult to estimate. Two alternatives were considered.

Approach 1: Elastic Theory

The conventional approach based on elastic theory typically assumes that both the soil shear modulus and Poisson's ratio, ν, are uniform under and around the foundation. The initial rotational stiffness, $K_{M,i}$, is computed using Equation 4 from Gazetas [17] (or a similar equation from Pais and Kausel [21]), where I_{yy}, is the area moment of inertia about the short axis, L_{ip} is the in-plane length, L_{oop} is the out-of-plane length, k_{dyn} is an adjustment factor based on the frequency of vibration and n_{emb} is an adjustment factor to account for the embedment of the foundation and soil sidewall contact (taken as 1 in this paper).

$$K_{M,i} = \frac{GB^3}{1-\nu} I_{yy}^{0.75} \cdot 3 \cdot \left(\frac{L_{ip}}{L_{oop}}\right)^{0.15} \cdot k_{dyn} \cdot n_{emb} \quad (4)$$

The value of G used in the stiffness calculations was reduced from the small strain stiffness using Equation 5 to account for static shear stress (Gazetas et al. [22]).

$$G = G_{soil}\left(1 - \frac{0.8}{FOS_v}\right) \quad (5)$$

Approach 2: Empirical

The empirical approach first determines the rotational stiffness at half of the moment capacity using the correlation from Deng et al. [16] in Equation 6. Equation 6 avoids the difficulties of obtaining an accurate value for G, and is based on a reasonable experimental data set. To then obtain the initial stiffness, $K_{M,i}$, the relationship between $K_{M,50}$ and $K_{M,i}$ is computed from the moment-rotation equation from Millen et al. [10], which provides a factor of 1.7, as adopted in Equation 7.

$$K_{M,50} = 300 \cdot M_{f,ult} \tag{6}$$

$$K_{M,i} = 1.7K_{M,50} = 500 \cdot M_{f,ult} \tag{7}$$

Step 6: Assess system

The expected performance of the building was determined using a simplified version of the DBA procedure from Sullivan [23]. The simplification is due to the assumption that the superstructure remains elastic. The foundation rotation is computed using the moment-rotation equation from Millen et al. [10] (Equation 8). Where θ_f is the foundation rotation, M_f is the foundation moment demand, $M_{f,ult}$ is the foundation moment capacity, and the nonlinearity factor, $f_p = 0.5$. No foundation sliding deformation was considered. The steps of the procedure are outlined in Figure 5. For SDOF-based design, the assessment used the same SDOF properties as in the design. For the MDOF-based design, the equivalent SDOF differed slightly from the one computed in design, mainly due to the assumption of the displaced shape. P-Delta effects were considered in the procedure.

$$\frac{\theta_f}{M_{f,ult}/K_{M,i}} = \frac{\left(\ln\left(\frac{M_{f,ult}}{M_f}\right) + f_p\right)}{\frac{M_{ult}}{M_f}\ln\left(\frac{M_{f,ult}}{M_f}\right)} \tag{8}$$

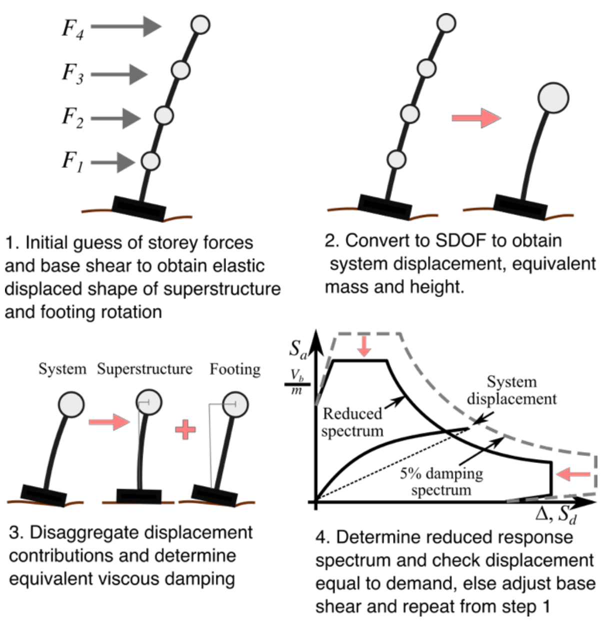

Figure 5: Key steps in the displacement-based assessment procedure.

Sliding was only evaluated in post-processing, where the resistance was computed assuming a soil friction angle of ϕ =36° and the interface friction coefficient was taken as $tan(0.67 \cdot \phi)$. Active and passive pressure was assumed on the out-of-plane sidewalls and an at rest earth-pressure of $1 - \sin(\phi)$ was assumed for the in-plane sidewalls. The foundation depth was taken as the lesser of 0.6 times the foundation height and 1/5 of the out-of-plane foundation length.

Case Study Assessment Results

Figure 6 shows the assessment results from the case study building SDOF design using elastic theory for the initial stiffness and considering the median, lower, and upper bound soil values. The results show that the assessment performance does not reach the full base shear expected in design, even for the upper bound soil properties. The assessment behaviour also shows both a considerably softer initial response and early nonlinearity, compared to the initial design behaviour. The design displacement with the additional pre-rocking base rotation allowance (explained in Section: Pre-rocking Foundation Rotation Allowance) shows a slight overprediction of displacements for both the serviceability, SLS1, and ultimate limit state, ULS, actions.

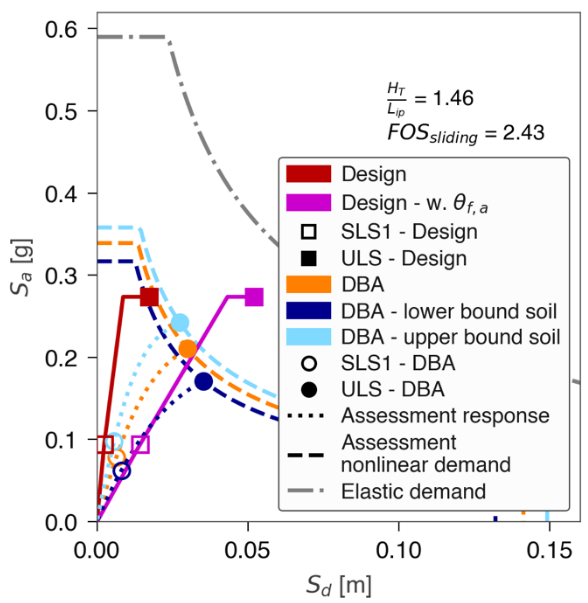

***Figure 6**: Case study design and assessment results, SDOF design using theoretical estimation of rotational stiffness.*

KEY RESULTS

SDOF Design Results

Figure 7 shows the prediction versus assessment SDOF results for the parametric study using the theoretical estimate of initial rotational stiffness with the median, lower and upper bound values from assessment shown as a line. Only results that satisfy the proposed limitation criteria of: $h_{sys} \leq 15m$ and $\frac{h_{sys}}{L_{ip}} \leq 3$ are presented, as well as the limit of FOSv>5, for an adopted μ of 2. The empirical stiffness results produced similar comparisons and are not shown here but are included in the digital supplement F.

The key outcomes include:

1. The largest SLS1 interstorey drift (note that the SDOF drift has been corrected to an equivalent interstorey drift for multi-storey buildings) was approximately 0.3%, a level of drift that might start to cause slight damage to conventionally fixed partitions, and would generally be accommodated by modern glazing systems.
2. The median ULS interstorey drift was in the order of 1.6%, which is below the 2.5% limit of 1170.5. Even in the event of a larger earthquake, instability leading to collapse is not expected although there could be significant damage to any secondary structure. However, it is not expected that this would be any greater level of damage than would be

expected with a conventional moment resisting frame structure on fixed foundations with similar design drift.

3. The assessed base shear is virtually always less than the design base shear, suggesting that designing the superstructure for the μ=2 actions will be suitable, noting also that there were no material overstrength considerations and no consideration of overstrength associated with gravity. However, there may be variation in the vertical load due to load redistribution when the foundation uplifts. Furthermore, foundations were sized so that the median moment capacity exactly matched the design moment demand, however, in cases where foundations are required to be larger (due to settlement, bearing or size of above ground elements), the moment capacity and assessed base shear would increase. In this scenario the designer could adopt a lower design ductility, so that the design demand is equal to capacity, to obtain the correct base shear.
4. The drift at both SLS1 and ULS was poorly predicted by the conventional 1170.5:2004 equations, so an adjustment to displacement estimates is proposed in Section: Pre-rocking Foundation Rotation Allowance.

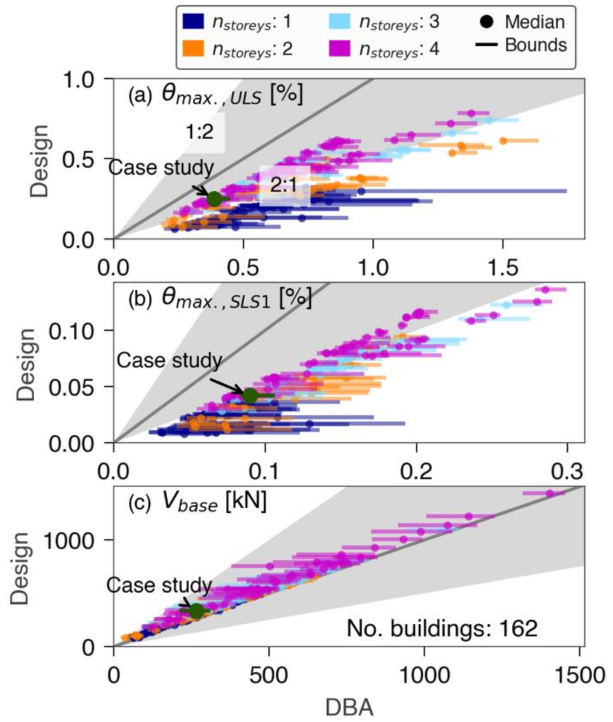

Figure 7: Comparison between design and assessment behaviour from SDOF design using elastic theory for initial stiffness (a) Equivalent interstorey drift at ULS; (b) Equivalent interstorey drift at SLS1 (c) Base shear at ULS.

Figure 8 show the other performance measures used to determine whether the behaviour was satisfactory. Due to space limitations, only results from the empirical rotational stiffness approach are shown, which produced worse outcomes than the theoretical rotational stiffness option. Note the large number of cases where the walls would slide ($FOS_{slide} < 1$), however, adequate tie beams to the remainder of the foundation would prevent sliding or result in rigid body sliding of the whole foundation system.

The key outcomes were that SFSI-induced settlement was generally expected to be less than 50 mm and residual tilt of the foundation to be less than 0.5%. Uplift of the centre of the footing would be in the order of 50 mm, and there is a considerable factor of safety against toppling, $FOS_{toppling} > 25$.

More detailed results from the parametric study for using elastic theory for initial foundation stiffness are presented in Digital Supplement E.

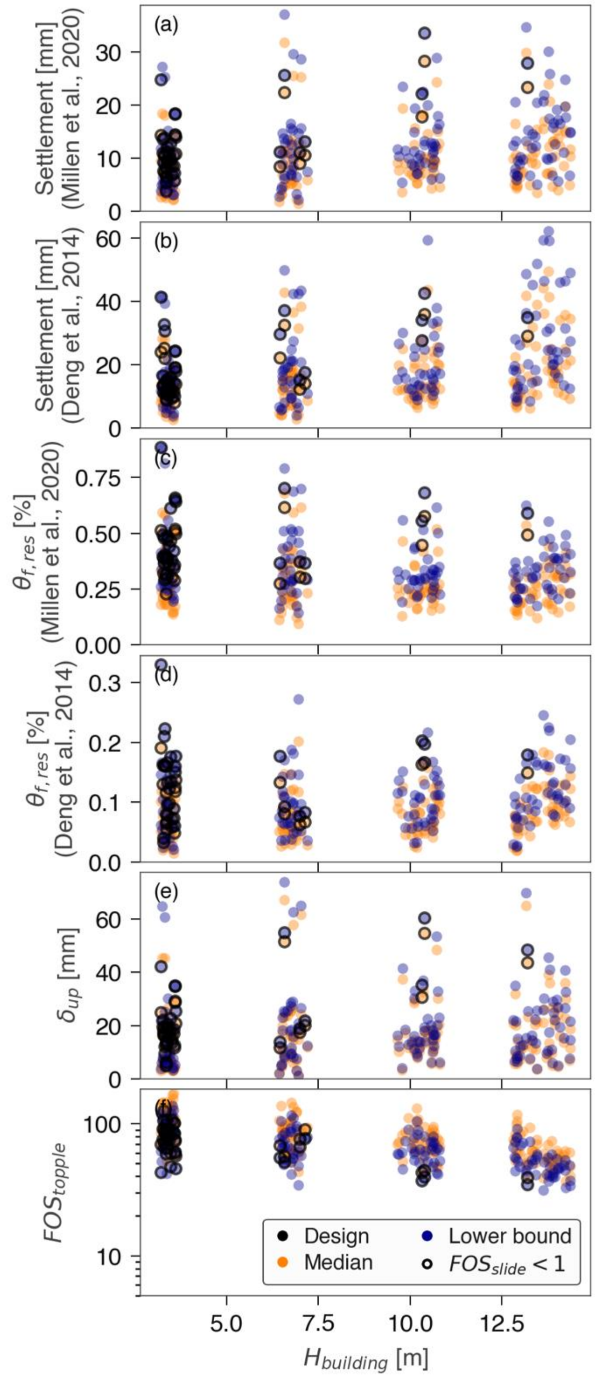

Figure 8: Performance of SDOF parametric design using empirical estimate for initial stiffness, (a & c) settlement and tilt estimated using Millen et al. [10]), (b & d) settlement and tilt estimated using Deng et al. [14], (e) estimated peak uplift of centre of foundation (f) Estimation of displacement factor of safety against global toppling, all shown versus building height.

SDOF Design with Period Adjustment for SFSI Effects

Figures 9 and 10 summarise the behaviour for the designs where the first mode period is adjusted to account for SFSI effects. Only 125 out of the 200 designs are shown as the remainder were excluded due to not satisfying the proposed criteria, mainly FOSv>5. The increase in the number of low FOSv cases is due to the smaller foundation that can be used when accounting for SFSI effects. The designs typically result in larger deformations than the original designs that adopted the fixed base period. Note that no displacement adjustment is needed, since the base rotation was directly included in the displacement estimate.

Note that the SLS1 interstorey drift is still generally less than 0.3%. Furthermore the displacement estimates are reasonably consistent with those obtained from DBA, such that a competent designer could increase the foundation size to reduce displacements if need be.

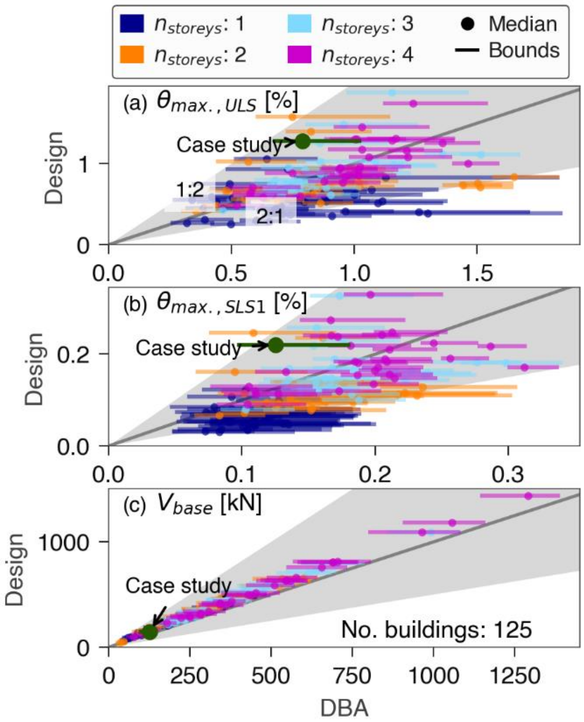

Figure 9: Comparison between design and assessment behaviour from SDOF design with SFSI period adjustment using empirical estimate for initial stiffness (a) Equivalent interstorey drift at UL;S (b) Equivalent interstorey drift at SLS1 (c) Base shear at ULS.

Change to Nonlinear Load Reduction Factor

In the proposed TS 1170.5:2023, the nonlinear load reduction factor, k_{μ}, is taken equal to μ for all periods, compared to NZS1170.5:2004, where it was lower for $T_1 < 0.7\ s$. The influence of this change on the simplified rocking foundation design was investigated. It resulted in significantly smaller foundations for one and two storey buildings. Some buildings showed no significant difference in behaviour. However, several buildings with very short periods and small foundations had larger than estimated displacements. In these cases the soil-foundation-structure system was outside the applicability of the DBA procedure. The assessments were out of scope since the large normalised foundation rotation meant that the foundation rotation displacement reduction factor, DRF, used in DBA reached its limit of 0.41 (based on the data range in the original study (Millen et al. [24]), combined with having very short periods well below the shortest period considered in the original study of 0.4 s. Given that the spectral displacement demand in this range is small, the performance was still considered suitable to not require additional intervention within the design for short period buildings.

Extensions to Rocking Foundation Procedure

While the criteria of: $h_{sys} \leq 15m$ and $\frac{h_{sys}}{L_{ip}} \leq 3$ implicitly avoided many of the unacceptable performance outcomes outlined in Table 1, the design estimation of displacement was notably less than the displacement from the assessment procedure. To allow more accurate displacement estimates, a correction is proposed where an additional rotation is applied to the base of the foundation and projected up to estimate storey displacements.

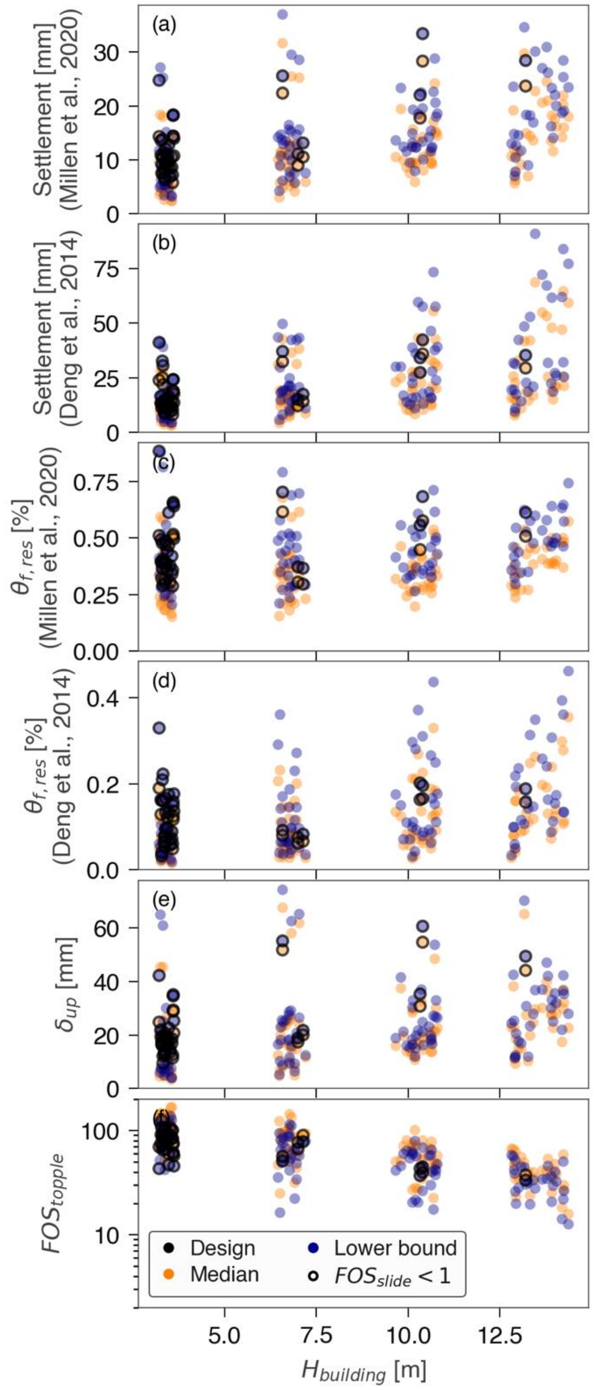

Figure 10: Performance of SDOF parametric design with SFSI period adjustment using empirical estimate for initial stiffness.

Pre-Rocking Foundation Rotation Allowance

In the adopted design, the elastic displacement contribution from foundation rotation is ignored (i.e. the period is computed assuming a fixed base). The displacement contribution from the foundation is assumed to be zero until the moment capacity is reached, and then deformation happens in a concentrated manner at the foundation base. The initial design rotation shown in Figure 11 is the assumed rotation that would be adopted from a direct application of NZS1170.5:2004, and gives a poor estimate, particularly for low levels of moment demand. A bi-linear correction (Corrected design rotation) is proposed to achieve a reasonable fit for SLS1 and ULS demands by applying a 'Pre-rocking Foundation Rotation Allowance', $\theta_{f,a}$, that linearly increases up to the moment capacity. $\theta_{f,a}$ was set equal to a normalised rotation of 2. Assuming the empirical relationship between stiffness and moment (Equation 7), then $\theta_{f,a}$ at ULS is 1/250 rad. Figure 12 shows the comparisons of prediction versus assessment SDOF results for the parametric study similar to Figure 7, but with the Pre-rocking Foundation Rotation Allowance included. This shows a much better correlation between design and assessment.

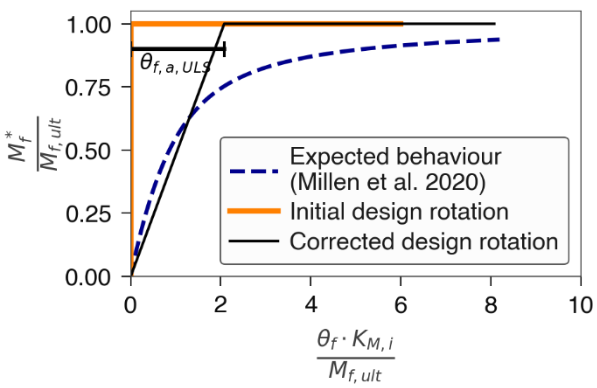

***Figure 11**: Normalised foundation rotation behaviour and proposed base rotation correction.*

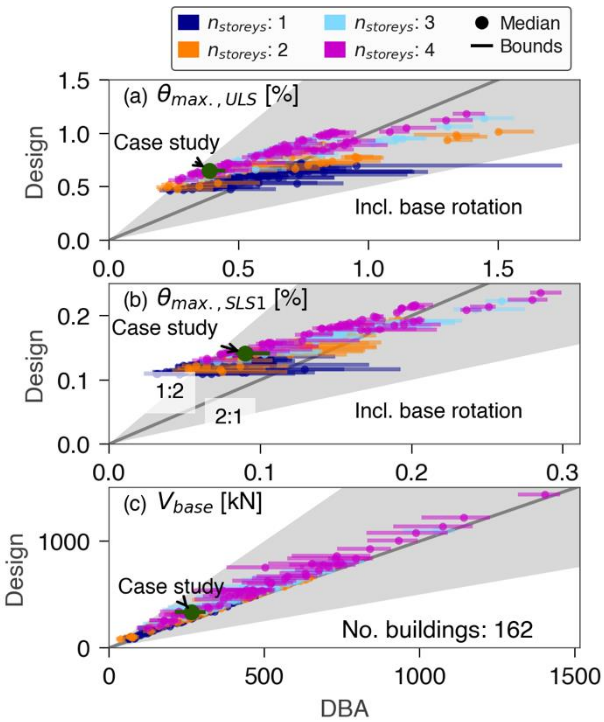

***Figure 12**: Comparison between design and assessment behaviour from SDOF design using elastic theory for initial stiffness with base rotation adjustment (a) Equivalent interstorey drift at ULS; (b) Equivalent interstorey drift at SLS1 (c) Base shear at ULS.*

Sliding of Foundations

Foundation sliding in high seismic environments with elastic or near elastic structural response can be expected unless shallow foundations are deeply embedded or a deep foundation is used. Sliding can act as a load limiting mechanism, and provided the foundation system slides as a single unit, it can significantly reduce demands in the structure. Allowance for sliding of the whole foundation is provided within the proposed TS when the rocking limitations are met, such that the structure and foundation are simple and suitably robust. However, sliding under SLS1 has been explicitly not allowed. This is in contrast to VM4, Verification Method 4 [25], which requires sliding to be supressed in all scenarios, with a sliding strength reduction factor of 0.8-0.9.

Design Actions for Foundations

During rocking the foundation is supported only on a small portion of the footing. The foundation element should be checked that it can sustain the moment and shear demand at this point of loading. The worst case for this is typically the upper bound strength of the soil. The upper limit should be developed in consultation with a geotechnical engineer as covered in the commentary.

Estimation of Moment Capacity

To avoid ambiguity about the moment capacity, a descriptive definition was provided in the commentary.

Conflict with VM4 Eccentricity Requirement

Assuming that the simplified method provides a full compliance path, designers would not be expected to comply with both VM1 and VM4. VM4 states that:

> *"3.1.4 Foundations subject to moment loading shall not be proportioned such that the point of application of the reaction force on the underside of the foundation is closer to the edge than [Lip]/6, for a rectangular foundation, or [radius]/2, for a circular foundation."*

This criteria was not considered, and explicitly allowed to be exceeded as stated in the commentary. Comparing design options, the rocking foundation design resulted in a 17-50% reduction in size – assuming $\mu = 2$ for both design options and no overstrength factor for the VM4 loads. Noting that for a bearing factor of safety, FOS_v, of 3, the sizes are identical.

Estimation of Lateral and Vertical Displacements

In some cases the estimation of lateral and vertical displacements will be important in assessing displacement compatibility in the structure. In particular the vertical displacement between two adjacent rocking foundations, where one edge goes down while the edge of the adjacent foundation goes up, potentially putting large demands on connecting elements. Simplified equations have been proposed and explained in the commentary.

Redistribution of Loads

Additional practical consideration was given to load redistribution in the case where the available individual foundation moment capacities differ from the complementary moment demand from their connecting LFRS elements across the building footprint. Redistribution is acceptable, provided that the overall system resistance to torsion is not reduced. Details are provided in the commentary.

Considered Design Limitations

There is always a trade-off between simplicity and applicability. To provide a design procedure that did not require any additional design steps (at least for the LFRS) a number of limitations were imposed. Several different limitations were considered, some were adopted, while others were not, as outlined in the following sections.

Adopted Design Limitations

Height Limit

Height of the LFRS is a convenient geometric limitation to apply to minimise the effects of higher modes on the influence of SFSI – and vice versa, which has not explicitly been considered in the simplified design procedure. Single parameter variations showed that increasing the height increases the predicted design drift but drifts estimated using displacement-based assessment (DBA) considering the effects of SFSI remain nearly constant (noting only the first mode is considered in DBA).

In considering a height limit, there are several benchmarks used elsewhere in the New Zealand Building Code:

1. NZS 1170.5:2004 Clause 6.1.3.1, One criteria for the equivalent static method is h_n < 10 m
2. NZS1170.5:2004 Clause 6.5.2, Specific P-delta considerations are not required if $h_n \leq 15$ m and $T_1 \leq 0.6$ s
3. NZS 3604:2011 [26] limits height from the lowest ground level to the highest point of the roof to 10 m and a maximum of three storeys.

A height limit of $h_{sys} \leq 15$ m was adopted, with allowance for a lightweight roof structure above this.

Height to Length Aspect Ratio Limit

The aspect ratio (h_{sys}/L_{ip}) of a lateral force resisting element is a conventional measure used in SFSI studies and is strongly linked to the contribution of foundation rotation to overall displacement. The evaluation framework showed that h_{sys}/L_{ip} was always below 3, unless the FOSv=5 limit was removed, the height limit was increased, or the ductility limit was increased. At large h_{sys}/L_{ip} ratios, the factor of safety against toppling is lower – however, it was always above 30. Noting that the study only considered soil sites (where the FOSv limited the aspect ratio), an aspect ratio limit of 3 was adopted and is considered more applicable on rock sites, or sites where ground improvement has occurred to increase bearing resistance.

Design Ductility Limit

Performance at larger design ductility values is generally very similar to that observed for μ=2, with slightly lower base shear and larger displacements (see comparison of behaviour of case study building in Figure 13). Displacements and drifts were larger for μ=3 compared to μ=2, however, the displacements were still well predicted when applying the pre-rocking foundation rotation.

Figure 14 shows the difference between μ =2 and μ =3 for the parametric design results. Note that the significant increase in assessment ULS displacement for some 1 storey buildings is due to the DRF being limited to 0.41 (explanation provided in the section: Change to nonlinear load reduction factor).

Ultimately, a design ductility of 2 was adopted, partly due to some results being out of scope of the assessment procedure, and partly since higher foundation rotations result in greater relative contribution of higher modes (which were not considered in the study).

All Foundations Unrestrained

To prevent forces concentrating on some foundation elements, all foundations that support the LFRS are required to be unrestrained from rocking, as detailed in the commentary.

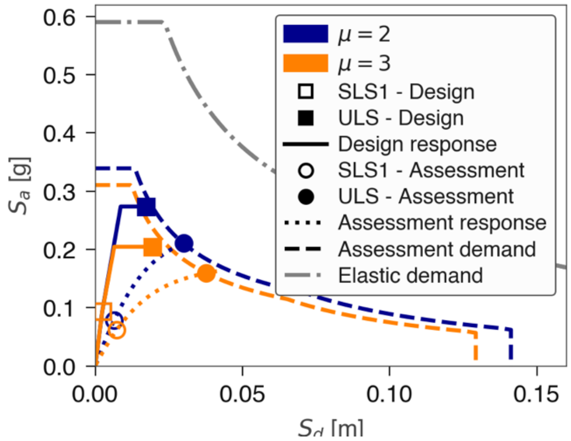

Figure 13: Design and assessment values when adopting mu=3 vs mu=2.

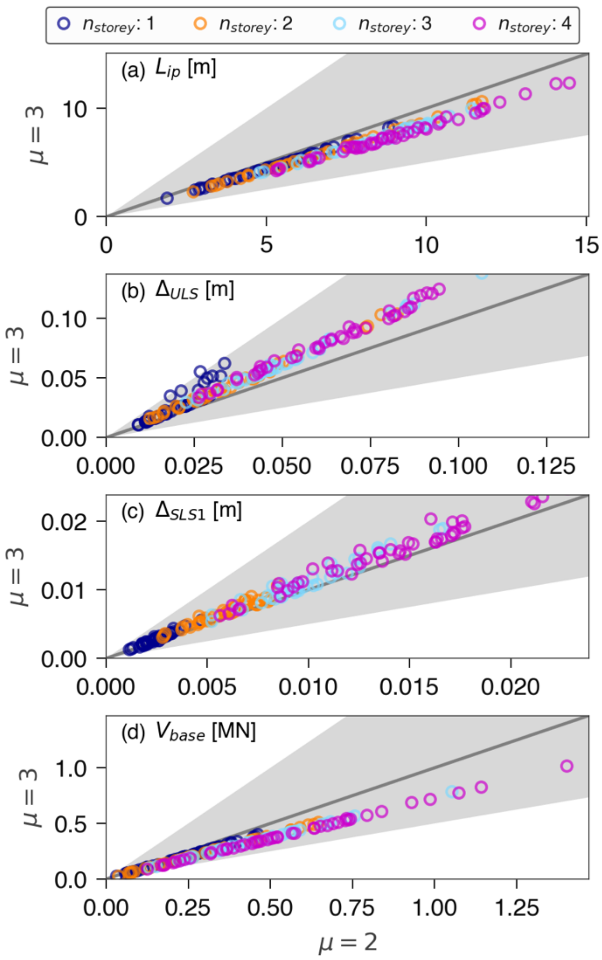

Figure 14: Comparison in performance between design ductility 2 and 3.

Foundation Base Elevation

A requirement was included for the difference in elevation of the base of foundation elements to be less than 1 storey, as detailed in the commentary.

Foundation symmetry

Non-symmetric foundations (e.g. L-shaped and C-shaped) result in considerable load-to-resistance eccentricities and out-of-plane movement. While restraints can be included to counteract act these undesirable behaviours, unrestrained non-symmetric foundations were excluded from the simplified design procedure as outlined in the commentary.

Design Limitations that were not Adopted

Critical Contact Area Ratio

A common parameter in the estimation of permanent settlement is the critical contact area ratio [16]. The critical contact area ratio is the total area of the foundation divided by the critical contact area, similar to the FOSv. When foundations are heavily loaded (i.e. low critical contact area ratio) the seismic action in the soil, as well as the inertia load of the structure result in significant permanent settlements and tilting under rocking. A limitation on the critical contact area ratio would implicitly limit the development of permanent deformations to an acceptable level. However, this requirement was considered to add considerable complexity to the design process as the critical contact area ratio would not be known at the start of the design. Furthermore, common strength reduction factors used in NZ, as well as permanent settlement requirements under non-seismic conditions, would typically result in foundations with sufficient critical contact area ratio (greater than 5-8), and therefore this requirement was considered unnecessary.

Regular Structures

A requirement that would not allow irregular structures to be designed using the proposed simplified design was considered but not included. The arguments against this provision include: That small structures with rocking foundations have been observed generally to have performed well in earthquakes, regardless of regularity. That the structures that this will frequently be applied to are often irregular, so it would be limiting the application too much. This is reasonably consistent with other provisions (e.g. P-delta or ratcheting), where small, simple structures are generally excluded from the more detailed provisions without regularity being a requirement.

Bi-directional Loading

Since the development of a large moment demand perpendicular to the desired rocking direction reduces the moment capacity, a limitation on the extent of bi-directional loading was considered. This was not adopted, since for a square footing with FOSv=5, forming rocking mechanisms simultaneously in both directions results in a reduction to 70% of the one-way capacity. A similar level of reduction (75%) would occur if the soil bearing strength was 50% of the expected strength for the same foundation. Furthermore, the majority of investigated buildings in the validation study only developed 90% of the moment capacity, and simultaneous peak response in both directions is unlikely. To address the concerns of capacity reduction a statement was included in the commentary.

CONCLUSIONS

A simplified design procedure was developed and evaluated for rocking shallow foundations for a subset of low- to -mid-rise buildings. The procedure is compatible with NZS 1170.5 provisions, with some extensions to achieve satisfactory and practical designs, and some limitations to implicitly control design outcomes. The design procedure was evaluated using a displacement-based assessment procedure considering the effects of soil-foundation-structure interaction. Throughout the evaluation procedure different inputs, assumptions and methods were investigated to cover the range of realistic values, different interpretations of the proposed provisions, and uncertainty in behaviour. The key outcomes included:

1. The maximum SLS1 interstorey drift was in the order of 0.3% and median ULS interstorey drift was in the order of 1.6%.
2. The assessed base shear was virtually always less than the design base shear.
3. A displacement correction was proposed to obtain more accurate estimates when the demands and displacements are obtained from a fixed based structure. The correction applies a foundation rotation to account for rotation that would happen prior to developing a rocking mechanism.
4. Limitations on building height, height to foundation in-plane length, and design ductility, were proposed to implicitly achieve satisfactory design displacements.

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APPENDIX

Table A1: List of symbols.

Symbol	Description
Δ_{demand}	Displacement demand on SDOF
Δ_{topple}	Displacement of SDOF at which it would topple over under static conditions
$FOS_{sliding}$	Factor of safety against sliding for a foundation element
FOS_{topple}	Factor of safety against toppling for a foundation element
FOSv	the bearing capacity factor of safety of a foundation for the seismic load case
G	Equivalent uniform small rotation soil shear modulus under a foundation
G_{soil}	Equivalent uniform small strain soil shear modulus under a foundation
h_n	Height from the top of the foundation to top of the structure or heavy roof
h_{sys}	Height from the underside of the foundation to top of the structure or heavy roof
I_{yy}	The area moment of inertia about the short axis of a foundation element
$K_{M,50}$	Foundation secant rotational stiffness at moment of 50% of the moment capacity
$K_{M,i}$	Foundation initial rotational stiffness at low levels of moment capacity, typically less than 10% of the moment capacity
L_{ip}	In-plane length of foundation element
L_{oop}	Out-of-plane length of foundation element
M_f	Foundation moment demand
$M_{f,ult}$	Foundation moment capacity
μ	Design ductility
N	The vertical load on a foundation element
$n_{storeys}$	Number of storeys
q_{ult}	Bearing stress capacity of soil under foundation
ν	Soil Poisson's ratio
S_a	Spectral acceleration
S_d	Spectral displacement
θ	Interstorey drift
θ_f	Foundation peak rotation
W_t	Total seismic weight of structure