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ESTIMATING THE DYNAMIC PROPERTIES OF WALL-FRAME STRUCTURES

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ABSTRACT

This study outlines a robust method to approximate the dynamic properties of wall-frame structures with a reasonable degree of reliability based on simple mechanics. The seismic response and drift demand of a structure are largely influenced by its first translational period and mode-shape. This study develops a method to estimate the storey stiffness of a wall, which can be combined with the storey stiffness of a frame to estimate the fundamental period and mode-shape for wall-frame structures. The fundamental period and mode-shape are calculated using this effective storey stiffness and Rayleigh's principle. A total of 301 wall-frame structures were sized to evaluate the reliability of the proposed method. Structures ranged from 2 to 25 storeys tall. The fundamental period and mode-shape estimated using the proposed method were compared with results from eigenvalue analysis of a detailed linear structural model. The proposed method leads to approximately 4 % error on average for estimating the fundamental period of regular structures. The proposed method leads to 5 % error on average for irregular structures with partial height walls, as well as variations in storey height, and lateral stiffness. For regular wall-frame structures, the proposed method led to average errors of 4% when estimating the roof mode-shape factor and 2% when estimating the maximum difference in mode-shape factor from one floor to the next, a proxy for storey drift. These errors were 5% and 15% for irregular structures with partial height walls. Results were also compared with estimates obtained from existing empirical equations to approximate the period of wall-frame structures, highlighting that empirical equations lead to greater error, between 15 and 70% depending on the equation. The method outlined in this paper enables users to estimate or corroborate the fundamental period, mode-shape, and lateral displacement for a dual wall-frame structure with a reasonable degree of reliability, suitable for preliminary design and linear analysis. Tools have been developed in MathCAD and python to automate the procedure for estimating the dynamic properties of wall-frame structures and are available here.

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INTRODUCTION

This study provides a reliable method to estimate the storey stiffness, fundamental period and mode-shape for wall-frame structures using first principles. The procedure can be used to estimate storey and roof displacement in preliminary design of the structure or to corroborate results from numerical models.

Building response to earthquake motion is sensitive to the dynamic properties of the structure such as the periods and mode shapes. These structural properties can be estimated with modern computing technology using eigenvalue analysis of detailed structural models. Nevertheless, there persists a need for simplified methods capable of reasonably approximating the response of structures to lateral loads. In the most basic scenario, it may not make sense to build an elaborate structural model while it is being proportioned. And even after proportions have been chosen, the engineer must have simple means to check software output. Additionally, complex structural models and nonlinear dynamic analyses are time-consuming and are difficult to apply to large building stocks. An old expression dating from early design methods for wind effects can be used to estimate the storey stiffness of an elastic frame structure [1]. For the idealised frames considered in the analyses by Schultz [2], estimates of storey stiffness were typically within 5% of the exact solution. Current methods to estimate the stiffness of RC wall structures often neglect the stiffness contribution of the gravity frame which can lead to an overestimation of the fundamental period, particularly for structures with slender walls or tall structures with large beams and columns [3,4].

A total of 301 linear wall-frame structures have been sized and the fundamental period and mode-shape of the structures were estimated. The subject structures range from 2 to 25 storeys and varied in: storey height, bay width, number of bays, as well as the dimensions of the beams, columns, and walls. Irregular structures with partial height walls and varying storey heights were also considered. The results of the proposed method are compared with results from structural analysis software as well as results from existing methods to estimate the period of wall-frame structures.

EXISTING METHODS TO ESTIMATE THE DYNAMIC PROPERTIES OF STRUCTURES WITH WALLS

There are several methods within literature to estimate the period of wall-frame structures. A common approach is to use empirical equations [5–9]. Typically, the periods of a range of structures are measured from ambient vibration tests, and a best-fit equation is derived to approximate the period of the structures. Often these

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empirical equations are simple and estimate period based on numerical coefficients and a few parameters of the building such as the structure height, length of shear walls, an assumed building mass, and the moment of inertia of the wall. A commonly used equation for concrete shear wall buildings was specified in the US building codes NEHRP-94 [10], SEAOC-96 [11], and UBC-97 [12] and adapted into the New Zealand code [13], converted to metric units. Equations 1-3 provided below can be used to estimate the period of RC wall structures.

$$T_1 = 1.25 \cdot k_t \cdot h_n^{0.75} \tag{1}$$

Where k_t is 0.05 for buildings with structural walls, and h_n is the height of the structure in metres. k_t can also be calculated based on the effective area of shear walls in the first storey in the building using Equations 2 and 3.

$$k_t = \frac{0.075}{\sqrt{A_c}} \tag{2}$$

Where A_c is calculated as

$$A_c = \sum A_i \left(0.2 + \left(\frac{l_{wi}}{h_n}\right)^2\right) \tag{3}$$

A_i is the cross-sectional area of shear wall i in the first storey of the building, in units of m^2, and l_{wi} is the length of wall i in the direction being considered. Empirical equations within design codes are typically used to provide an estimate of period based on historical averages, however they cannot often capture variations introduced during design, nor do they account for any contribution of stiffness from the frame. These equations are often recommended for preliminary design. Nevertheless, knowledge of the cross-sectional area and lengths of the walls in the structure (as required in Eq.3) are not typically known at the preliminary stages of design, and if they are known, then the proposed method can be used to obtain a more reliable and accurate estimate of the period and mode-shape.

Sozen [3] and Wallace et al. [14] have produced equations to estimate the fundamental period of wall structures based on an idealisation of a wall structure as a pure flexural cantilevered element, ignoring shear deformations and any lateral resistance provided by the frame (Figure 1b). The equation proposed by Sozen [3] is shown below where T_w is the period of the wall, E_c is the elastic modulus of the concrete, I_w is the moment of inertia of the wall, μ is the linear mass density up the height of the structure, and H is the height of the structure (Equation 4).

$$T_w = \frac{2\pi}{3.5}\sqrt{\frac{E_c I_w}{\mu H^4}} \tag{4}$$

The equation proposed by Sozen [3] can be applied to structures with dominant RC walls, however, the equation does not consider the contribution of the frame or any variation in the cross-section of the wall up the height. A procedural method using Rayleigh's principle based on the work by Sozen [3] is outlined in the book by Pujol et al. [4] and can be applied to walls with variations to the wall cross-section, storey heights, and different floor weights. Nevertheless, the method does not account for the contribution of storey stiffness from the frame, nor can it be applied to structures with partial height walls.

An alternative, and more rigorous approach to estimate the period of structures is by using the "couple beam method". For wall-frame structures, a linear continuum model is created where the

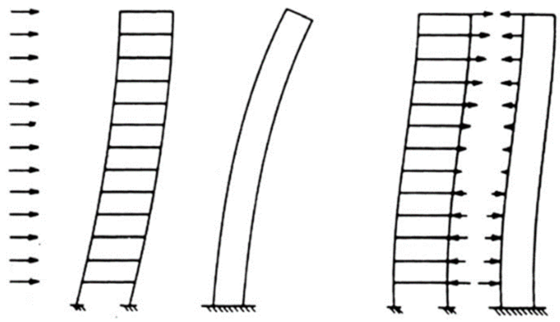


Figure 1: Deflected shape of (a) a frame structure, (b) a cantilevered wall, and (c) a wall-frame structure.

frame is assumed to be a pure shear cantilever, while the wall is modelled as a pure flexural cantilever. The structure is then modelled as two "coupled beams" as illustrated in Figure 1c.

The governing fourth order partial differential equation of the combined shear-flexural member can be expressed as:

$$\frac{d^4y}{dx^4} - \frac{GA}{EI} \cdot \frac{d^2y}{dx^2} = \frac{w(x)}{EI} \tag{5}$$

Where G is the shear modulus and A is cross-sectional area of the shear beam, E is the modulus of elasticity and I is the moment of inertia of the flexural beam, $w(x)$ is the distributed horizontal load, and $y(x)$ is the horizontal deflected shape of the shear-flexure beam. Several studies have proposed different approximations for GA for different types of structures such as frames [15], coupled walls [16–18] and braced frames [19]. Equation 5 can be solved to obtain the natural frequencies of the structure as well as the deformed shape. The model parameter α describes the relationship between EI and GA, the flexure, and shear cantilevers (Equation 6). As α increases towards infinity, the coupled beam exhibits pure shear deformation (Figure 1a), and as α approaches zero, the structure deforms in pure flexure (Figure 1b).

$$\alpha = \sqrt{\frac{GA}{EI}} \tag{6}$$

Full examples for calculating the fundamental period are provided by Heiderbrecht and Stafford-Smith [15] for a wall-frame structure, and in the study by Stafford-Smith and Crowe [19] for a frame structure. These continuum models consisting of coupled shear and flexural beams assume that the structure has constant properties up the entire height, and as such, cannot be applied to structures with large irregularities or partial height walls. Additionally, the approach by Heiderbrecht and Stafford-Smith [15] to estimate the shear stiffness of a frame does not consider changes in α along the height of the structure to account for variation in stiffness attributed to the boundary conditions at the lower storeys and the roof [20]. As a result, this method tends to lead to greater error for short structures, and examples provided within literature apply the method to structures with 20 or more storeys [15,19].

To the best of the authors' knowledge there is no method or equation proposed within literature to approximate the storey stiffness of a wall or wall-frame structure. The expression for storey stiffness proposed in this study is reliable, based on first principles, and can be applied to structures with vertical stiffness irregularities such as partial height walls, variation in element section sizes, and varying storey heights.

PROCEDURE FOR ESTIMATING WALL-FRAME STOREY STIFFNESS

The method proposed within this study to estimate the storey stiffness of a wall-frame structures involves estimating an effective storey stiffness of the wall elements in the structure, and combining this with the storey stiffness of the frame.

Estimating Frame Storey Stiffness

The storey stiffness of an elastic frame can be approximated using Equation 7 described in the paper by Schultz [2]. E_c is the elastic modulus of concrete and H is the storey height. Σk_c is the relative flexural stiffness of the columns in that storey, equal to the sum of the second moment of area, I_c, divided by the length of the element, H, for the number of columns. Σk_{ga} is the relative flexural stiffness of the girders above the storey, equal to I_b/L, where L is the length of the girder and I_b is the second moment of area of the girders. Similarly, Σk_{gb} is the relative flexural stiffness of the girders below the storey, equal to I_b/L, where L is the length of the girder and I_b is the second moment of area of the girders.

$$K_f = \frac{24E_c}{H^2} \cdot \frac{1}{\left(\frac{2}{\Sigma k_c} + \frac{1}{\Sigma k_{ga}} + \frac{1}{\Sigma k_{gb}}\right)} \tag{7}$$

Foundations are assumed rigid and therefore, the first storey stiffness is estimated using Equation 8

$$K_{f,1} = \frac{24E_c}{H^2} \cdot \frac{1}{\left(\frac{2}{\Sigma k_c} + \frac{1}{\Sigma k_{ga}} + 0\right)} \tag{8}$$

Once the storey stiffness is estimated, the period of the "shear building" illustrated in Figure 2, with lumped mass at each floor and translation springs representing the lateral stiffness of each floor, can be calculated using Rayleigh's principle.

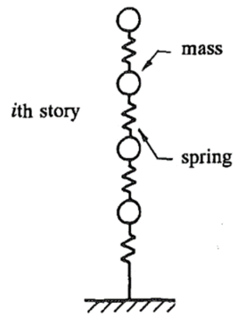


Figure 2: Lumped parameter model for a shear type structure (Schultz, 1992) [2].

Estimating Wall Storey Stiffness

In the analysis of wall structures, walls are typically represented as cantilevers bending in pure flexure [15]. Nevertheless, for the purpose of estimating initial period, this study demonstrates that wall elements can be represented as a shear type structure much like a frame (Figure 2). The storey stiffness of a structural wall can be estimated as the storey shear force divided by the storey deformation. The procedure to estimate storey stiffness is outlined below and illustrated in Figure 3.

A fictitious force distribution linear up the height is applied to the wall. The magnitude of the assumed forces is inconsequential. The shear demand in the wall is calculated as the sum of the forces above the floor being considered, where n is the number of storeys in the structure, and i is the storey being considered. Shear force in storey i is calculated using Equation 9 where $i = 1$ refers to the first storey above the foundation.

$$V_i = \sum_{j=i}^{n} F_j \tag{9}$$

Moments at each floor i are calculated as the sum of the product of shear forces above the floor and the height of each storey h_i where $i = 1$ refers to ground level (Equation 10). The curvature for each floor is then derived using Equation 11 as the moment divided by the product of modulus of elasticity, E_c, and second moment of area, I_w, for the wall.

$$M_i = \sum_{j=i}^{n} V_j \cdot h_j \tag{10}$$

$$\phi_i = \frac{M_i}{E_c \cdot I_w} \tag{11}$$

Change in slope is calculated as the product of curvature and storey height, with the exception that the change in slope for the ground level is equal to half the curvature multiplied by storey height to account for the assumed fixity of the base.

$$\Delta\theta_i = \phi_i \cdot h_i \tag{12}$$

$$\Delta\theta_1 = 0.5 \cdot \phi_1 \cdot h_1 \tag{13}$$

The slope of a storey is calculated as the cumulative sum of the changes in slope at each floor below where $i = 1$ refers to the first storey above the foundation. Storey deformation is calculated as the product of the slope, θ, and the storey height, h.

$$\theta_i = \sum_{j=1}^{i} \Delta\theta_j \tag{14}$$

$$\Delta_i = \theta_i \cdot h_i \tag{15}$$

The effective storey stiffness of the wall can then be expressed as the storey shear divided by the storey deformation.

$$K_{w,i} = \frac{V_i}{\Delta_i} \tag{16}$$

Results from this study suggest that the shape of the applied force distribution (whether it be uniform or linear) when calculating storey stiffness of the wall has little effect on the estimated period and mode shape.

Estimating the Storey Stiffness of a Wall-frame Structure

Once the storey stiffness of the wall and frame are obtained using the methods described in section 4.1 and 4.2, the storey stiffness of the wall-frame structure is calculated as;

$$K_{w-f,i} = K_{w,i} + K_{f,i} \tag{17}$$

The proposed method, approximating a wall-frame structure as a "shear building", makes several assumptions, a) structural elements are axially rigid, b) foundations are fixed, and c) beam and column dimensions are taken as centre-to-centre dimensions, ignoring any rigidity assumed for the beam-column joint.

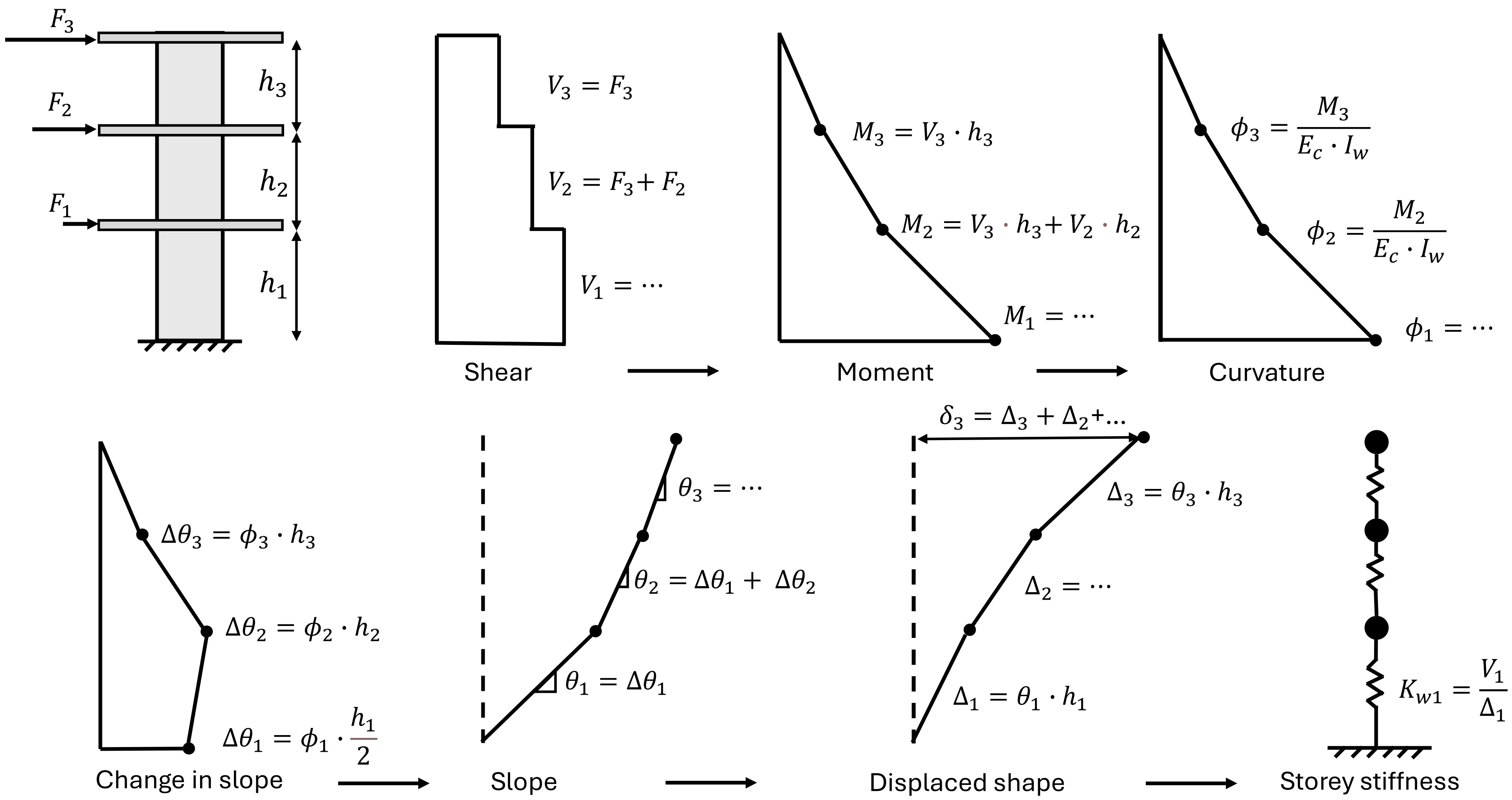


Figure 3: Illustration of procedure for calculating effective storey stiffness of a wall and modelling the structure as a "shear building".

CALCULATING DYNAMIC PROPERTIES OF WALL-FRAME STRUCTURES

To estimate the fundamental period and mode-shape of the structure, arbitrary lateral forces are applied at each floor (as illustrated in Figure 4) and the storey deformations and floor displacements are calculated based on the applied shear force and the estimated storey stiffness (Equations 18-20). The applied force distribution is linear up the height of the structure.

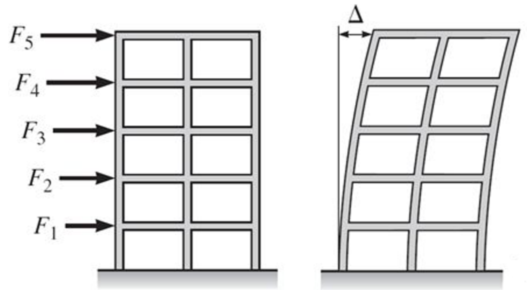


Figure 4: Applied lateral force distribution up the height of the structure and calculated displacements at each floor.

Shear force at storey *i* is calculated using Equation 18

$$V_i = \sum_{j=i}^{n} F_j \tag{18}$$

Storey deformation at storey *i* is

$$\Delta_i = \frac{V_i}{K_{w-f,i}} \tag{19}$$

Floor displacement at floor *i* is

$$\delta_i = \sum_{j=1}^{i} \Delta_j \tag{20}$$

Based on Rayleigh's principle, the fundamental frequency of an oscillating system is proportional the ratio of maximum potential energy to the maximum kinetic energy. This concept is used to compute the fundamental period of the structure (Equation 21). m is the seismic mass at each floor, δ is the floor displacement, relative to the ground, and F is the applied force at each floor. The numerator in equation 21 is related to the relative kinetic energy of the system and the denominator is related to potential energy.

$$T = 2\pi\sqrt{\frac{\sum m_i \cdot \delta_i^2}{\sum F_i \cdot \delta_i}} \tag{21}$$

The periods of 110 regular wall-frame structures were estimated using the described method and are compared against the period of the equivalent structures modelled using OpenSees [21]. The mode-shape is assumed equal to the displaced shape of the structure, calculated using Equation 20. The results are presented in the following sections.

STATISTICAL PROPERTIES AND MODELING OF RC WALL-FRAME STRUCTURES

A total of 110 regular wall-frame structures were sized. Table 1 provides information regarding the typical building dimensions and the range of dimensions that were varied. The typical storey height ranged from 2.7 to 3.6 m. All columns had square cross-sections and their depth varied between 400 and 1200 mm. Beams had a width of 200–400 mm and a cross-section depth of 300—800 mm. Bay widths varied between 5 and 8 m with a median span of 6.0 m. Walls varied in length between 2.0 and 12.0 m and wall density, calculated as the total wall area divided by the floor area, ranged from 0.2 to 1.7% in one direction. Column density, calculated as the total column area divided by the floor area, ranged from 0.2 to 3.3%. The elastic modulus of the concrete was assumed to be 28 GPa, and the seismic mass of the structure was 8 kPa.

Table 1: Range of wall-frame structure dimensions.

	Minimum	Maximum
No. of storeys	2	25
Storey height (m)	2.7	3.6
Column cross-section (mm)	400	1200
Beam width (mm)	200	400
Beam depth (mm)	300	900
Beam span (mm)	5000	8000
Wall length (mm)	2000	12000
Column density (%)	0.21	3.31
Wall density (%)	0.22	1.71

Structural models of the wall-frame buildings were developed in OpenSees. Beams, and columns were modelled as linear elastic beam-column elements. Walls were first modelled as shell elements using SFI-MVLEM (shear-flexure interaction - multiple vertical line element model) and then modelled as elastic beam-column elements to understand how explicitly incorporating shear stiffness would influence the estimated period. Figure 5 plots the results comparing the differences in period obtained using the two different models. The average difference in period obtained between the different models was 0.02 seconds for models with wall aspect ratios between 1 and 10. The results suggest that incorporating shear stiffness led to a small increase in period of approximately 5-10% for wall structures with aspect ratios less than 2. Comparisons made in following sections were in terms of structural models neglecting shear deformation (walls modelled as elastic beam-column elements).

The beams and columns in the models are based on centreline dimensions, and the use of rigid offsets was ignored. To provide meaningful comparisons of estimated period using the empirical equations from the NZ standards, the second moment of area for cracked sections was used for beams, columns, and walls. It was assumed that $I_{cr} = I_g/3$ for all sections. 2% Rayleigh's damping was applied to the first and third-mode period of the structure using the committed tangent stiffness matrix. The columns and walls were fixed at the base of the structure and soil-structure interactions were ignored. The fundamental period and mode-shape were calculated after gravity loads were applied to the structure, incorporating P–Delta effects. Modelling the walls as elastic beam-column elements in OpenSees assumes that there is no shear deformation and that the shear stiffness is infinite.

ESTIMATES OF FUNDAMENTAL PERIOD OF WALL-FRAME STRUCTURES USING THE PROPOSED METHOD

Using the method proposed in the previous sections, the periods and mode-shapes of 110 regular wall-frame structures with properties listed in Table 1 were estimated. The estimated period was compared against the period calculated using eigenvalue analysis. The results from structural analysis and the proposed method are compared in Figure 6.

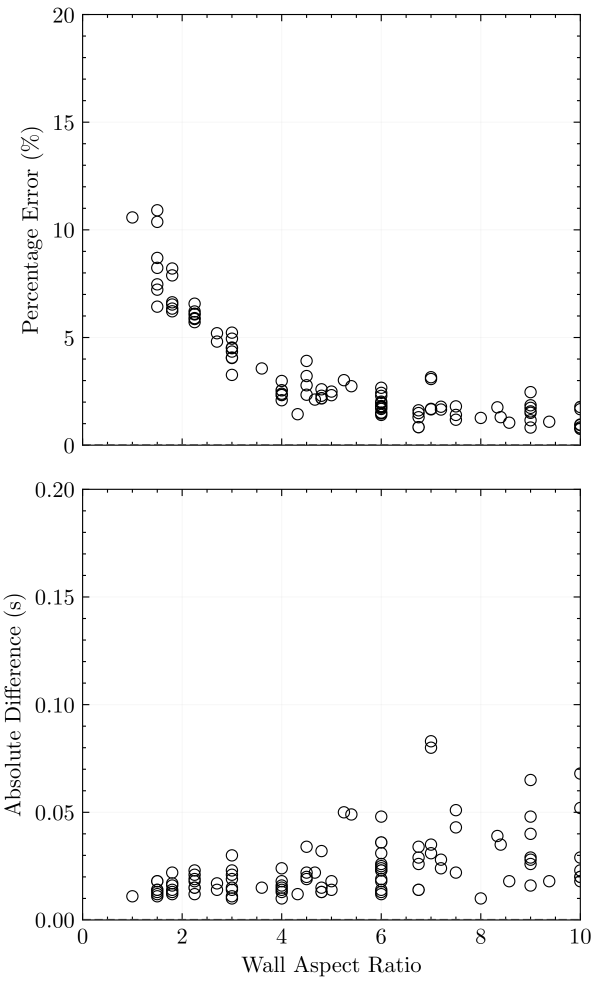


Figure 5: Percentage difference and absolute difference between period obtained using SFI-MVLEM and elastic beam-column elements representing RC walls.

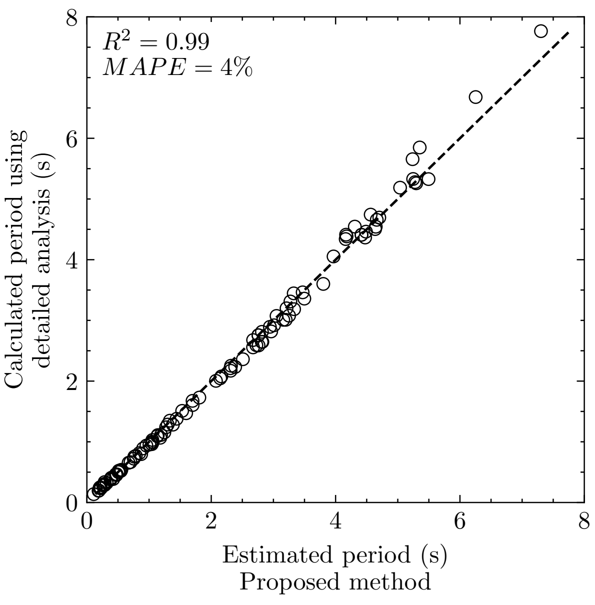


Figure 6: Estimated fundamental period using proposed method vs calculated period from eigenvalue analysis of linear structural model.

The scatter is close to the 1-to-1 line and the mean absolute percentage error (MAPE), calculated using Equation 22, is approximately 4% which is inline with the method highlighted by Schultz [2] for frame structures. The MAPE measures error in terms of percentage and is calculated as the average of the absolute value of the error for each sample, $y_i - \hat{y}_i$, divided by the calculated period from OpenSees, y_i. n is the number of samples, in this case, the 110 wall-frame structures. $\hat{y}_i$ is the estimated fundamental period using the proposed method.

$$\text{MAPE} = \frac{100\%}{n} \sum_{i=1}^{n} \left| \frac{y_i - \hat{y}_i}{y_i} \right| \tag{22}$$

R^2 is calculated using equation 23 where: y_i are the observed values of period from OpenSees, $\hat{y}_i$ are the estimated values of period, $\bar{y}$ is the mean of the observed values, and n is the number of data points. The results illustrated in Figure 6 show a clear correlation with an R^2 value of 0.99.

$$R^2 = 1 - \frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{\sum_{i=1}^{n}(y_i - \bar{y})^2} \tag{23}$$

For the broad range of buildings considered in this study, many had slender walls with large aspect ratios where the period and mode-shape was often controlled by the stiffness frame. Figure 7 is plotted to assess the accuracy of the proposed method for structures that are either wall dominant or frame dominant. Structures were considered wall dominant if the walls contribution to storey stiffness on the first storey was greater than 75% ($K_{w,1} > 0.75 \cdot K_{w-f,1}$). The method proposed in this study is shown to be reliable for either sub-group of structures with MAPE of 3% for wall dominant structures, and 5% for frame dominant structures. In general, the period of frame dominant structures tended to be overestimated when using the proposed method.

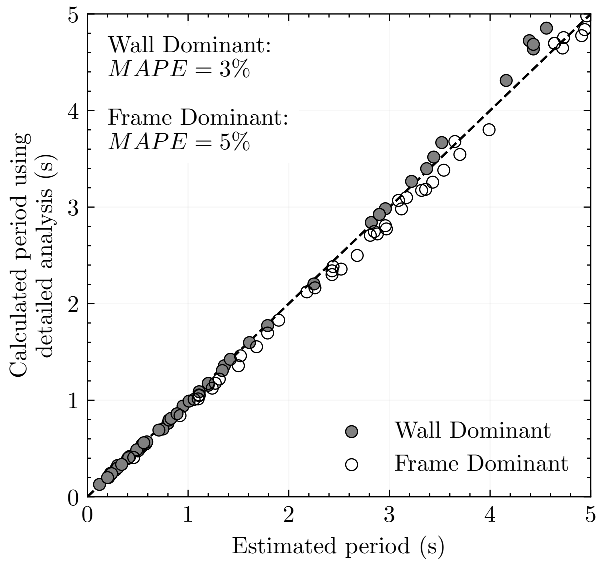


Figure 7: Estimate of initial period using proposed method for structures that are wall dominant vs frame dominant.

Estimating Fundamental Period for Irregular Structures

In this section, the proposed method was used to estimate the fundamental period of structures with different vertical stiffness irregularities. Schultz [2] previously demonstrated that the method for estimating the storey stiffness of a frame (Eq. 7) is robust, and accurate for structures with large variations in the stiffness of adjacent floors. In this study, emphasis was placed on irregularities that would influence wall stiffness. The following irregularities were considered, a) changes to the storey height of adjacent storeys, and b) structures with partial height walls. The method proposed in this study is not suited for structures with significant plan irregularities which lead to torsional modes that dominate the response of the structure.

Wall-frame Structures with Partial Height Walls

The results presented in Figure 6 relate to wall-frame structures that have a) a constant storey height, b) constant cross-sections of walls, columns, and beams, and c) walls that span from the foundations to the roof. Often, mid-rise, or high-rise RC wall structures have walls that do not span the full height of the structure. An example of this is the 10-storey RC Wall-frame structure tested at E-Defense, in Japan in 2015 (Figure 8) where the wall spans the first 7 storeys, and the structural systems consist of a moment frame in the floors above.

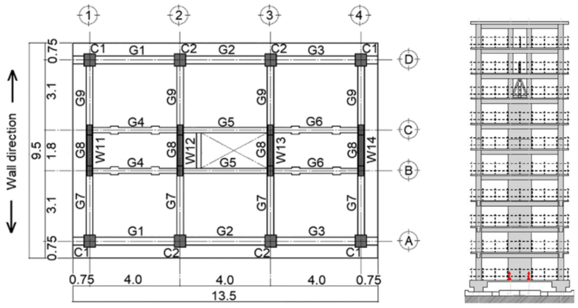
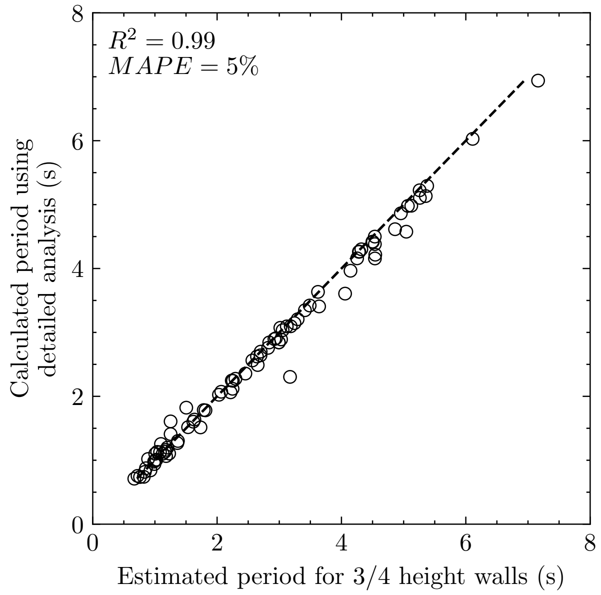


Figure 8: a) Typical floor plan for 2nd to 7th floor and b) elevation of 10-storey RC wall structure tested at E-Defense [22]

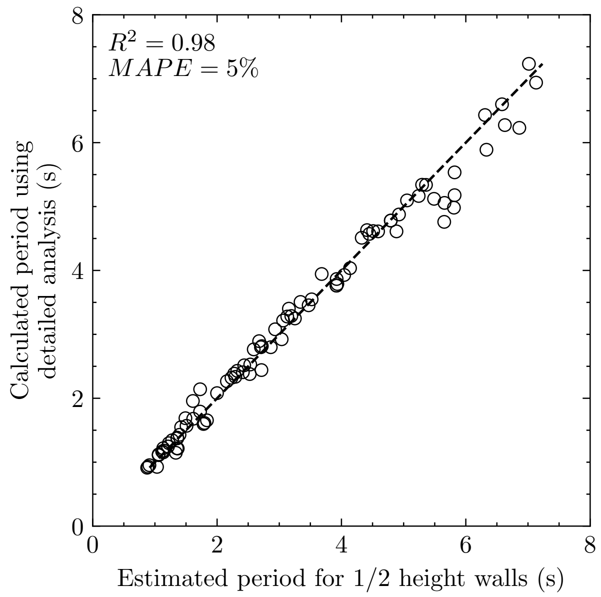
To test the reliability of the proposed method at estimating the dynamic properties of wall-frame structures with partial height walls, the structures sized in the earlier section have been re-designed to include walls that span half the height of the structure, and walls that span three quarters of the structure height. Structures with fewer than five storeys were removed from the dataset, as it is unlikely that low-rise structures would include partial height walls. A total of 176 structures were sized, with 88 having walls that spanned half the height of the structure, and 88 with walls that spanned from the foundation to three quarters of the structure height. The proposed method for calculating storey stiffness does not change for a structure with partial height walls. The storey stiffness of the wall is only calculated for storeys in the structure with a wall. The periods of structures with partial height walls were estimated using the proposed method and the results are compared with the period calculated using detailed structural analysis (Figure 9). For the structures with partial height walls, the scatter is again close to the 1-to-1 line and the mean absolute percentage error (MAPE) is approximately 5%. The results suggest that the proposed method to estimate fundamental period is robust and can be applied to structures with irregularities in storey stiffness.

Wall-frame Structures with Variations in Storey Heights

A total of 15 structures were sized with storey heights varying up the structure. Five different 5, 10, and 15-storey structures were considered. Table 2 provides a summary of the different section dimensions and ranges of storey heights. The fundamental period was estimated using the proposed method and results



(a)



(b)

Figure 9: Estimated period of wall-frame structures with a) walls that are 3/4 the height of the structure, and b) walls that are half the height of the structure.

Table 2: Range of irregular wall-frame structure dimensions with varying storey heights.

	Minimum	Maximum
Storey height (m)	2.7	6.0
Column cross-section (mm)	400	800
Beam width (mm)	200	400
Beam depth (mm)	300	800
Beam span (mm)	5000	9000
Wall length (mm)	3000	10000
Column density (%)	0.38	1.3
Wall density (%)	0.25	1.5

were compared with the period calculated using a detailed structural model (Figure 10). The error calculated for structures with variations in storey height up the structure is 3% on average, suggesting that the proposed method can be reliably applied to estimate the period of structures with irregularities in storey height.

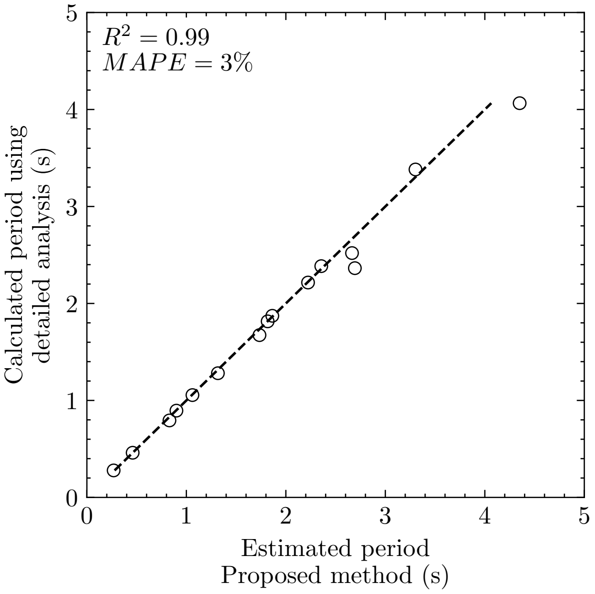


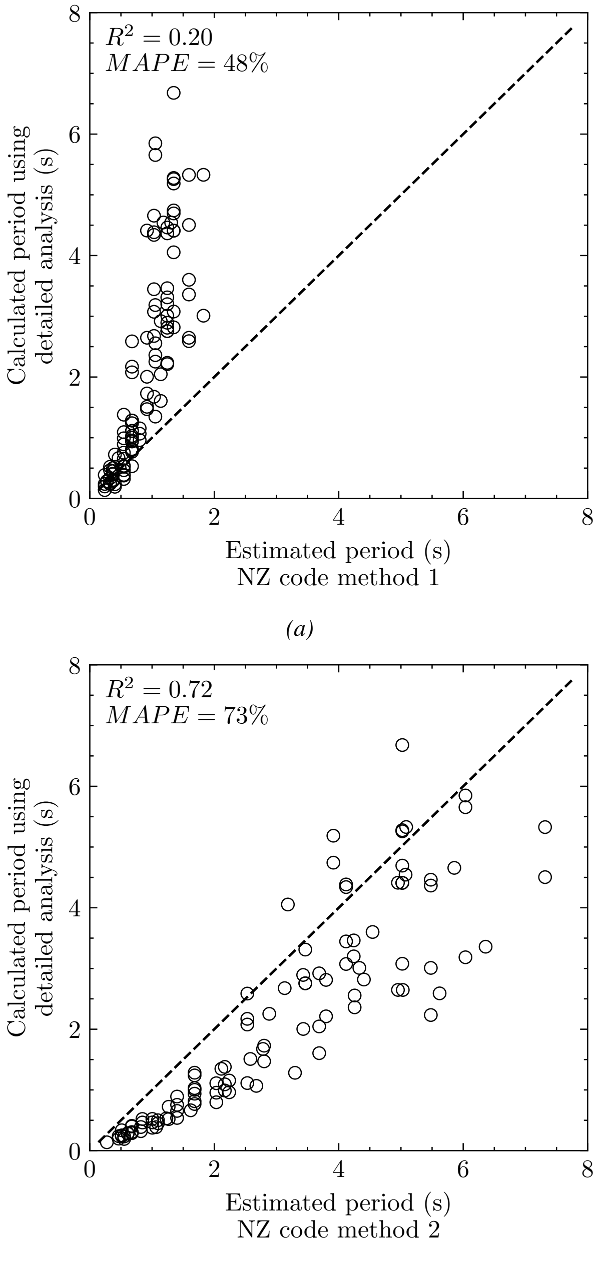
Figure 10: Estimated period of 15 wall-frame structures with height irregularities vs period computed using eigenvalue analysis of a linear structural model.

ESTIMATING FUNDAMENTAL PERIOD USING CURRENT STATE-OF-PRACTICE METHODS

In this section, the fundamental period of the 110 regular wall-frame structures is estimated using methods proposed within design codes and literature to compare with the method proposed in this study. Several of the state-of-practice methods, including equation 4 and the empirical equations within the New Zealand standards (Eq. 1–3) do not account for structures with partial height walls, and are not considered in the following sections.

Empirical Equations from the NZ Code

There are numerous methods proposed in the literature to approximate the period of a structure with walls. The simplest of these are empirical equations. In the New Zealand building code [13] the period of RC wall structures can be initially estimated using Equations 1-3 described in the literature review. The periods of the 110 wall-frame structures were estimated using the approximate methods within the NZ code and the results are illustrated in Figure 11. Figure 11a refers to Equation 1 with $k_t = 0.05$. Figure 11b refers to Equation 1 with k_t calculated using Equations 2 and 3. It can be seen from Figure 11a that there is considerable difference between the period estimate using Equation 1 and the period calculated from detailed structural analysis. The results suggest that there is little value in using Equation 1 with $k_t = 0.05$, even as an initial estimate of period. Moreover, these methods are not recommended for assessing existing buildings and "should only be used to estimate a fundamental period for the purposes of preliminary design". While the NZ code states that "generally these methods predict fundamental periods that are lower than the corresponding values calculated by analytical methods", the results from Figure 11b demonstrate that this is not always the case.



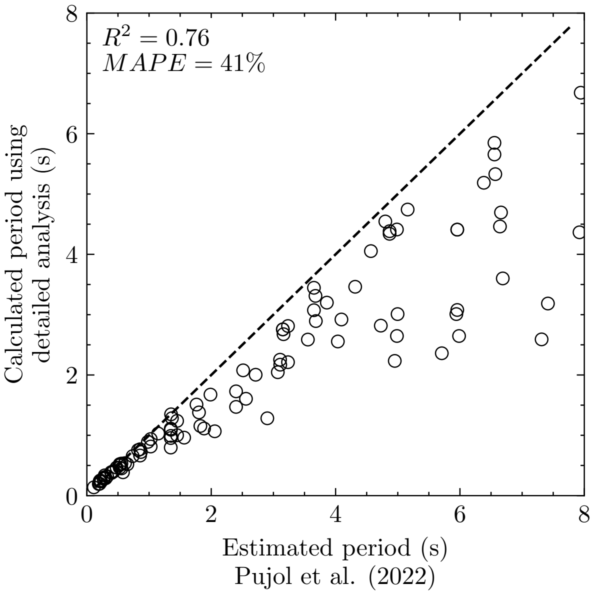
(b)

Figure 11: Empirical methods in NZS1170.5 to estimate the period of structures with walls vs calculated period from eigenvalue analysis of a linear structural model.

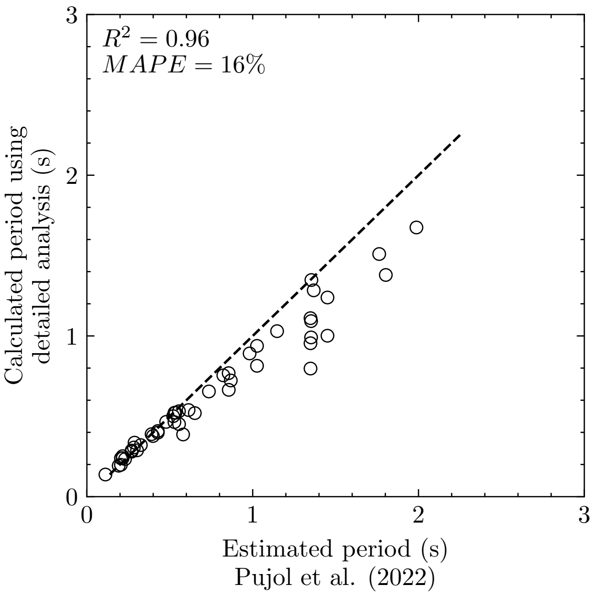
The results from Figure 11b depict differences between the period estimated using Equations 2 and 3, and the period calculated from detailed structural analysis. The alternative method using Equations 2 and 3 leads to an estimated period that is typically longer than the calculated period with the majority of points falling below the 1-to-1 line. While both of these methods are intended to be used for preliminary design, knowledge of the cross-sectional area and lengths of the walls in the structure are not typically known at the preliminary stages, and if they are, the method proposed within this study could be used to obtain better results instead.

Estimating the Period of a Cantilevered Wall

Another common approach to estimate the period of a building with structural walls is to conservatively ignore the contribution of the frame and consider the lateral resistance provided by the wall only. To estimate the period of a structure in this manner, Equation 4 can be used. The periods were estimated for the 110 wall-frame structures and the findings are shown in Figure 12. As expected, ignoring the contribution of the frame leads to an overestimate of the period of the structure. For tall structures, or structures with slender walls, the storey stiffness can be dominated by the frame.



(a)



(b)

Figure 12: Estimated fundamental period for a wall structure, ignoring the contribution of the frame, vs calculated period from eigenvalue analysis of a linear structural model. (b) Structure with fewer than 6 storeys and wall aspect ratios greater than 5.

Nevertheless, in short buildings with fewer than 6 storeys and wall aspect ratio of less than 5, where structural walls dominate the response, the method provides a reasonable approximation of period (Figure 12) with 16% error on average. The method proposed in this study leads to 4% error for the same sub-group of buildings.

Estimating the period of wall-frame structures (Heidebrecht and Stafford Smith, 1973)

Heidebrecht and Stafford–Smith [15] proposed a method to approximate the deflected shape and natural frequency of tall wall-frame structures using relatively simple hand calculations. The procedure involves approximating the frame and the wall as a combination of a shear and a flexural vertical cantilever, deforming in the manner illustrated in Figure 1c. The response of the combined shear-flexural continuum member is expressed in Equation 6. The equation can be solved to obtain natural frequencies of the structure. Full examples are provided by Heidebrecht and Stafford–Smith [15] for a wall-frame structure, and in the study by Stafford–Smith and Crowe [19] for a frame structure. The fundamental period of the wall-frame buildings was estimated using the procedure outlined by [15] and results are illustrated in Figure 9.

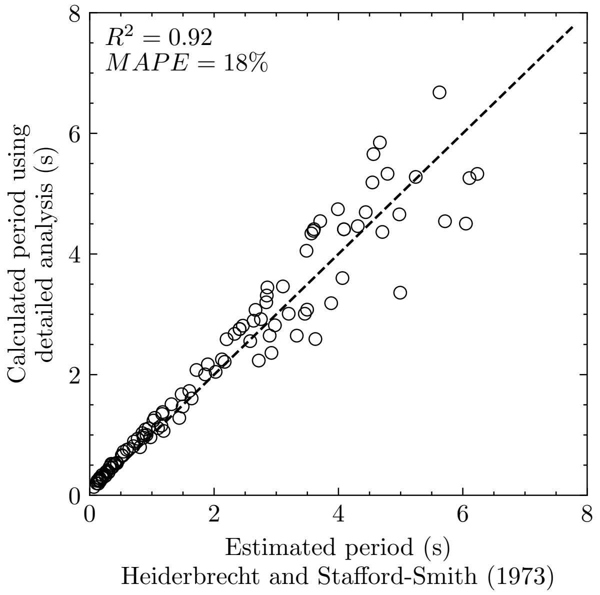


Figure 13: Estimate of initial period using method proposed by Heidebrecht and Stafford–Smith [15] for Wall-frame structures, vs period calculated from eigenvalue analysis of a linear structural model.

Compared with other alternative methods to estimate the fundamental period of wall-frame structures, the procedure developed by Heiderbrecht and Stafford-Smith [15] leads to less error and on average, the estimated period is within approximately 20% of the period calculated from structural analysis software. The procedure developed by Heiderbrecht and Stafford-Smith [15] can be applied to non-uniform structures, however, doing so requires a greater number of steps including solving simultaneous equations and trial and error. The procedure proposed in this study to estimate the initial period, illustrated in Figure 6 leads to better results, and is reliable to within approximately 4% of the period calculated from OpenSees. The proposed method can be easily applied to irregular and non-uniform structures unlike the method by Heiderbrecht and Stafford-Smith [15].

ESTIMATING FUNDAMENTAL MODE-SHAPE AND PARTICIPATION FACTOR

To obtain reliable estimates of linear drift and displacement up the height of a structure, one needs a reliable estimate of the fundamental period, and the deflected shape, or mode-shape, of the structure.

For the method proposed in this study, idealizing a wall-frame structure as a 1-D MDOF (Figure 3), the mode-shape and participation factor were calculated and compared against values obtained from linear structural models. The mode-shape of each wall-frame structure was estimated based on the displaced shape of the structure obtained from equation 20. The participation factor is calculated using Equation 24 where ϕ_{1i} is the mode-shape factor for the 1st mode at floor i, m_i is the mass of floor i, and n is the number of floors in the structure.

$$\Gamma_1 = \frac{\sum_{i=1}^{n}(m_i \cdot \phi_{1i})}{\sum_{i=1}^{n}(m_i \cdot \phi_{1i}^2)} \tag{24}$$

Figure 14 provides a comparison of the product of mode-shape and participation factor ($\Gamma_1 \cdot \phi_{1,i}$) of a RC wall-frame structure obtained using the method proposed in this study vs $\Gamma_1 \cdot \phi_{1,i}$ from eigenvector analysis of an equivalent structural model. $\Gamma_1 \cdot \phi_{1,i}$ is a factor used to project single-degree-of-freedom (SDOF) displacements to a MDOF system. The values of $\Gamma_1 \cdot \phi_{1,i}$ illustrated in Figure 14 show that the proposed method to estimate the deflected shape of the structure closely resembles that of a detailed structural model.

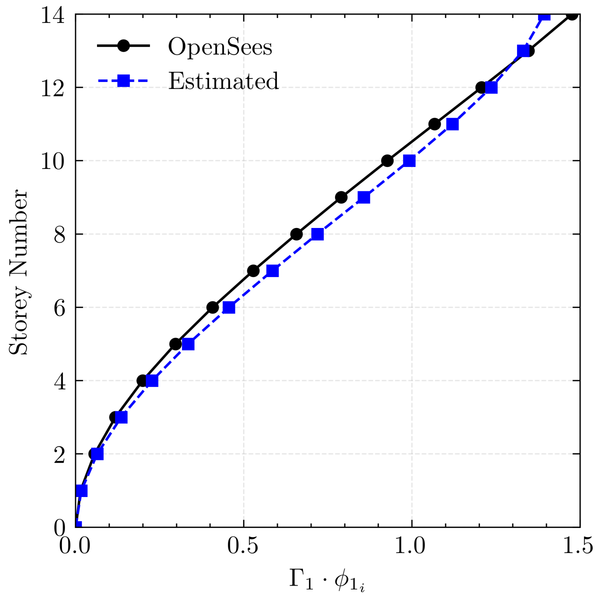


Figure 14: Illustration of $\Gamma_1 \cdot \phi_{1,i}$ for a RC wall-frame structure obtained from proposed method and eigenvalue analysis.

To provide a comparison for all of the RC wall-frame structures, two metrics were used to quantify the reliability of the proposed method. First, the projection factor at the roof ($\Gamma_1 \cdot \phi_{1,R}$) is estimated using the proposed method and compared against the results from the linear structural model. This provides a measure of the accuracy of the proposed method at estimating roof displacement or drift. The results are illustrated in Figure 15 showing the ratio of the estimated projection factor $\Gamma_1 \cdot \phi_{1,R}$, to the value obtained from the linear structural model. The data are divided into two groups. Regular RC wall-frame structures with full height walls are represented by the black circles in Figure 15, while wall-frame structures with partial height walls are depicted as red squares in the scatter plot. Both groups of data show a strong correlation between the ratio of estimated $\Gamma_1 \cdot \phi_{1,R}$ to the value obtained from analysis of the structural models, with a mean of 0.95 and a coefficient of variation –calculated as the standard deviation normalised by the mean– of 0.03. The error, in terms of the mean absolute percentage error (MAPE), is approximately 5%. The results from Figure 15 suggest that the method proposed in this study leads to reliable estimates of roof deflection or drift.

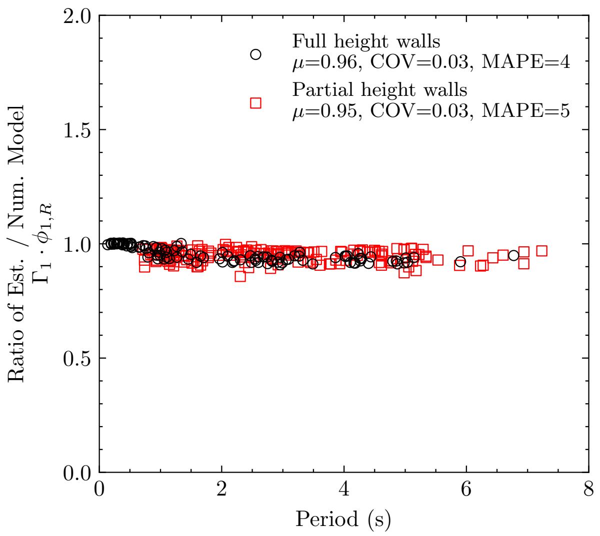


Figure 15: Ratio of estimated $\Gamma_1 \cdot \phi_{1_R}$ using the proposed method vs value obtained from eigenvalue analysis of numerical models of RC wall-frame structures. Black circles represent structures with full height walls, and red squares represent structures with half-height of three-quarter height walls.

The second metric used to compare the deflected shape obtained from the proposed method and the linear structural models looks at the the maximum storey displacement, or difference in projection factor from one floor to the next, $\Gamma_1 \cdot |\phi_{1,i} - \phi_{1,i-1}|$. The ratio provided in Eq. 25 is used to quantify the reliability of the proposed method at estimating storey displacement or drift. The results are illustrated in Figure 16. For regular structures with full-height walls, the proposed method results in a mean of 1.0, COV = 0.02, and an MAPE of 2%. The results suggest that the proposed method can reliably estimate the deflected shape of a regular RC wall-frame structure.

$$Ratio = \frac{Estimated - max[\Gamma_1 \cdot |\phi_{1,i} - \phi_{1,i-1}|]}{Num.Model - max[\Gamma_1 \cdot |\phi_{1,i} - \phi_{1,i-1}|]} \tag{25}$$

For an irregular wall-frame structure, with partial height walls, the proposed method leads to slightly more scatter (Fig. 16). The proposed method tends to over-estimate the difference in mode-shape factor from one floor to the next, with a mean of 1.13 and a COV of 0.13. The MAPE is 15%, higher than for regular wall-frame structures. The larger scatter illustrated in Figure 16 is attributed to the sharp change in slope of the mode-shape as a result of the partial height wall. Figure 17 shows a comparison of $\Gamma_1 \cdot \phi_{1,i}$ estimated for a RC wall-frame structure with a half-height wall along side $\Gamma_1 \cdot \phi_{1,i}$ obtained from eigenvalue analysis of an equivalent structural model. The estimated deflected shape leads to slightly larger storey displacement on the 6th and 7th floor, likely due to underestimating the storey stiffness of the floors directly above the wall. Nevertheless, the method tends to lead to conservative estimates of the deflected shape of wall-frame structures with partial height walls (μ = 1.13, COV = 0.13).

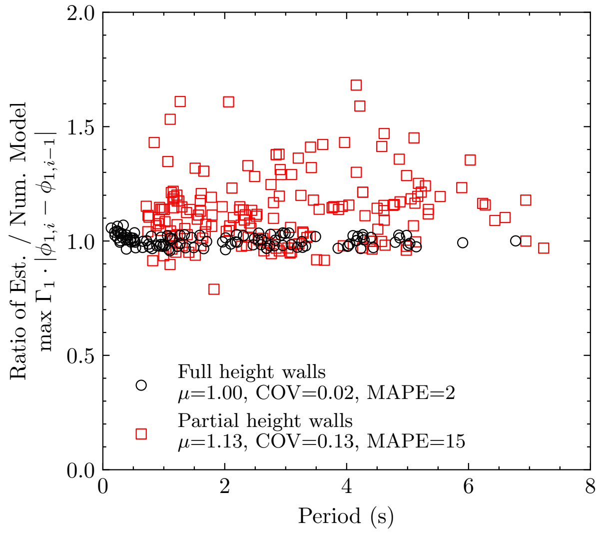


Figure 16: Ratio of maximum estimated $\Gamma_1 \cdot |\phi_{1,i} - \phi_{1,i-1}|$ using the proposed method vs value obtained from eigenvalue analysis of numerical models of RC wall-frame structures.

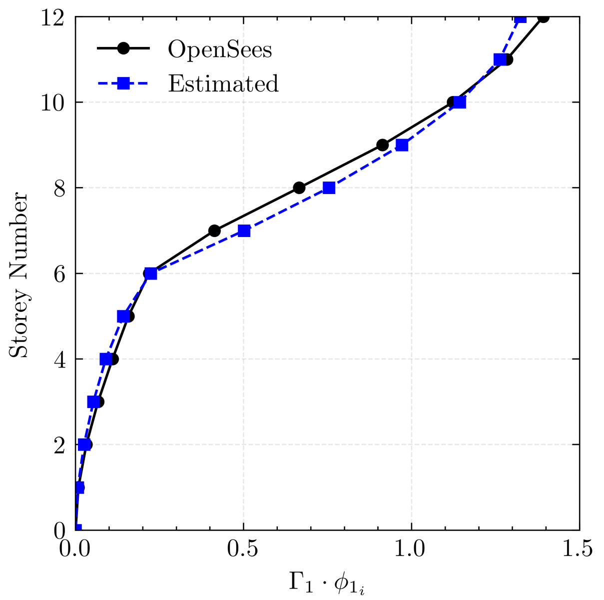


Figure 17: Illustration of fundamental mode-shape of a RC wall-frame structure with a partial height wall obtained from proposed method and eigenvalue analysis.

CONCLUSIONS

The dynamic properties of a structure can be derived using modern computing technology and structural analysis software. Nevertheless, a need still exists for simplified methods to check computer outputs and to facilitate preliminary design. A procedure idealizing a wall-frame structure as a 1-D MDOF shear spring model has been proposed to estimate the dynamic properties and lateral displacements of wall-frame structures. The fundamental period was estimated for 301 wall-frame structures and compared with the period calculated using detailed structural models. The structures considered ranged from 2 to 25 storeys tall, column area densities varied between 0.2 and 3.3% and wall area densities ranged from 0.2 to 1.7%. Structural irregularities including partial height walls and variations in storey heights were also considered. Periods obtained using the proposed method had average errors of approximately 5% relative to results obtained from detailed linear structural analysis. Comparisons of the deflected shape demonstrated that the proposed method led to average errors of approximately 4% when estimating the projection factor at the roof $\Gamma_1 \cdot \phi_{1,R}$, and 2% when estimating the maximum difference in projection factor from one floor to the next ($\Gamma_1 \cdot |\phi_{1,i} - \phi_{1,i-1}|$) for regular wall-frame structures. When assessing irregular structures –with partial height walls–, these errors were 5 and 15% respectively.

Estimates of the fundamental period using the proposed method were also compared with estimates of the fundamental period obtained from existing procedures and empirical methods for wall-frame structures. The results highlight that empirical equations lead to unreliable estimates of period with average errors between 15 and 70 % depending on the method or equation used. The method outlined in this study enables users to estimate the fundamental period, mode-shape, and displacement for a dual wall-frame structure within tolerances suitable for preliminary linear design of the structure. The proposed method only considers the 1st translational mode and as such, it is less suited to structures with significant vertical stiffness irregularities such as discontinuous walls in tall structures which can lead to higher mode effects. The method is not applicable to structures with large plan irregularities which lead to dominant torsional modes. In both cases, it is recommended that additional analysis is performed using computer software which accounts for higher mode effects. While this study provides a mechanics-based method for estimating the dynamic properties of numerical models of structures, future work could explore the merits of machine learning for estimating the period of existing structures using measurements of period from the field and basic structural properties.

The following link provides a design example for a 10-storey wall-frame structures using the procedure described in this study. https://github.com/LiamPledger/Estimating-the-dynamic-properties-of-Wall-frame-structures. Tools have been developed in MathCAD and python to automate the process outlined in the study for estimated the dynamic properties of wall-frame structures. The link also includes additional information related to the structures designed and modelled in this study including the dimensions of individual structures, and all of the python scripts / OpenSees models developed as part of this study.

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