THE SEISMIC RESISTANCE OF UNREINFORCED MASONRY CANTILEVER WALLS IN LOW SEISMICITY AREAS

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ABSTRACT

The time-history analysis method, the response spectrum method and the equal energy method are considered in this paper to determine the seismic resistance of unreinforced masonry (URM) cantilever walls. Fundamental problems with each of the methods in predicting the response of a simple cantilever wall model to the El Centro earthquake have been identified The response spectrum method underestimated the true seismic resistance of the wall by a factor of more than 2.0, whilst the equal energy method could overestimate the resistance of the wall by up to 3.5 times depending on the natural frequency adopted. Difficulties with establishing realistic dynamic properties of brickwalls are highlighted. A probabilistic procedure involving the analysis of a large number of representative ground motions and cantilever wall models is recommended for future investigations to derive reliable and simple expressions to predict the response of cantilever masonry walls.

1. INTRODUCTION

Domestic unreinforced masonry (URM) construction in most parts of Australia, like other countries with low seismicity, have not been designed for earthquake loadings. However, in the 1989 Newcastle earthquake, URM walls and particularly parapet walls, gable walls and facade walls suffered widespread damage demonstrating their potential hazard even in low seismicity regions [1,2].

URM walls in Australia are often unreinforced and do not have sufficient effective tie-backs to resist lateral load and consequently are considered to be brittle and vulnerable to out-of-plane earthquake excitation. Further, the mortar bond strength at the base of the walls is questionable due to uncertainty in the mortar strength, the presence of damp-proof courses and the effect of thermal stress cycles. For these reasons, the Australian Masonry Standard AS3700 [3] specifies zero tensile strength for free standing cantilever walls subjected to lateral load.

This paper examines various methods which can be applied to determine the out-of-plane seismic resistance of URM cantilever walls using a simple 1.0 m high wall model and the El Centro record for illustration. A physical model was first tested on the shaking table [4]. A non-linear time-history analysis was then carried out using a finite element model calibrated with the dynamic properties obtained from the experimental tests. As time-history analyses and dynamic tests are far too costly and time consuming for average design applications, simplified methods based on the use of the response spectrum and the equal energy principle have been considered and compared with the results obtained from time-history analyses.

The response of a structure to earthquake loadings depends significantly on the acceleration time-history record. Research is currently being undertaken to derive representative strong motion intraplate accelerograms representative of Australian conditions for seismic risk assessment [5]. In the interim, the Australian Earthquake Loading Standard AS1170.4-1993 uses a model response spectrum for stiff soil based on Californian interplate earthquake events and particularly the El Centro ground motion recorded in the north-south direction during the 1940 earthquake at Imperial Valley in Southern California. In this study, the El Centro record has been selected for analyses to demonstrate both the response spectrum approach and the equal energy approach.

2. SHAKING TABLE TESTS AND FINITE ELEMENT MODEL ANALYSES

A 1.0 metre high and 1.4 m long URM single-skin cantilever wall specimen was tested dynamically on a shaking table [4]. Based on the experimental results, a "stick" finite element model with masses lumped at discrete nodal positions was created to represent a cantilever wall with uniform properties and support conditions located away from any re-entrant corners. Non-linear time-history analyses were then carried out to determine the maximum rocking displacement of the wall subject to the El Centro ground motion scaled to various levels. The following sections describe the test specimen, the experimental set-up and the testing procedures and provide a summary of the test results.

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2.1 Description of the full-scale parapet wall specimen and experimental set-up

The masonry units supplied by Besser Masonry Victoria were traditional extruded clay building bricks of size 230 mm x 110 mm x 76 mm with a characteristic compressive strength of 50MPa. The masonry mortar was prepared with 1 part Type A Portland Cement, 1 part hydrated building lime and 6 parts masonry mortar sand with an average flexural tensile strength of approximately 0.2MPa determined from bond wrench tests [3].

Prior to laying the brick wall, the base course was positioned inside a stiffened steel channel section. The gap between the wall and the channel on both sides was then filled with compacted cement mortar. The brickwall was laid with a staggered bond configuration using a 10mm thick mortar bed finished flush by a qualified tradeperson [6].

A 2 m x 2 m three tonne payload computer-controlled shaking table was used to apply out-of-plane base excitation to the wall specimen which was secured against relative lateral movement and rotation at the base [4]. The set-up of the specimen and instrumentation on the shaking table are shown in Figure 1.

2.2 Brief description of free-vibration tests and results

The purpose of the free-vibration tests was to identify the frequency response characteristics and the damping characteristics of URM cantilever walls which then enabled calibrated computer models to be developed. In a typical test, a very short impulse was applied to the wall setting it off in free-vibration (refer Figure 2). The natural frequencies were obtained by measuring the time intervals between the peaks, and the damping ratio by measuring the rate of amplitude decay in the response envelope.

A large number of free-vibration tests were carried out to determine the response characteristics of the wall both before and after cracking. For the uncracked wall, the elastic natural frequency was found to be between 5-10 Hertz depending on the amplitude of vibration. For the cracked wall, the rocking frequency was around 1 Hertz when the displacement amplitude exceeded 20 mm (refer Figure 3). The damping ratio was around 3.0% for both the uncracked and cracked walls [4].

2.3 The stick finite-element model

A stick finite element model (refer Figure 4) was developed based on the dynamic characteristics obtained from the shaking

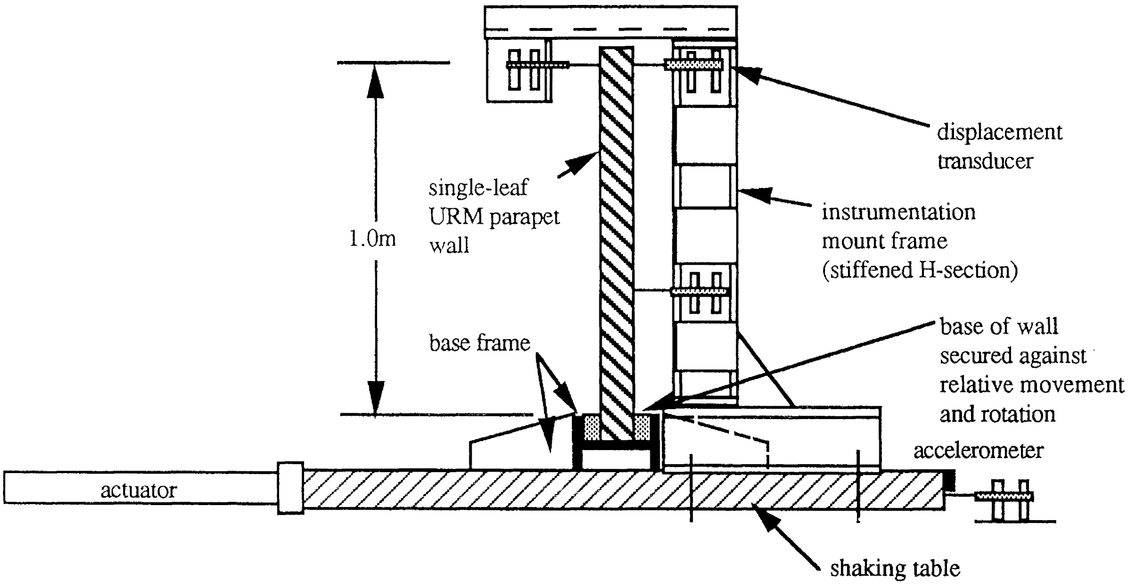

FIGURE 1 The set-up of the wall specimen and instrumentation on the shaking table

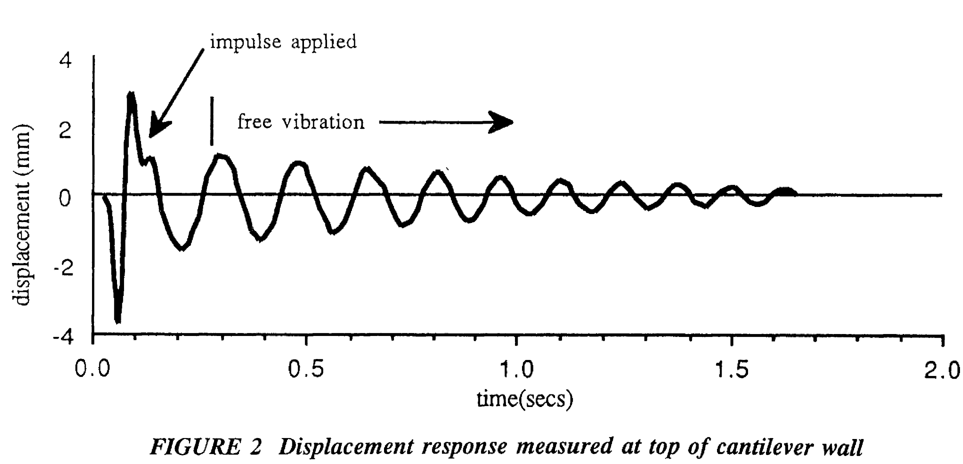

FIGURE 2 Displacement response measured at top of cantilever wall

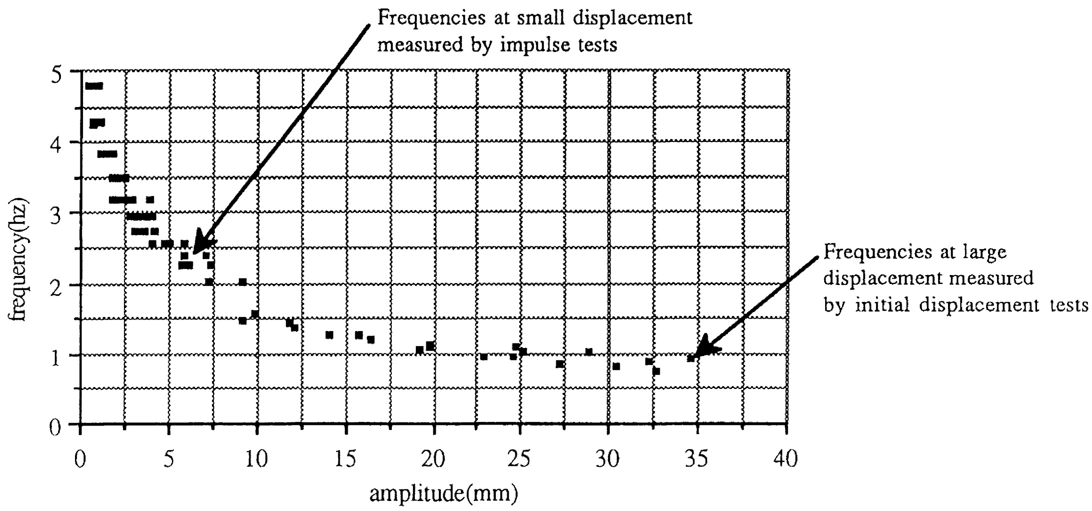

FIGURE 3 Rocking frequencies of cracked parapet wall

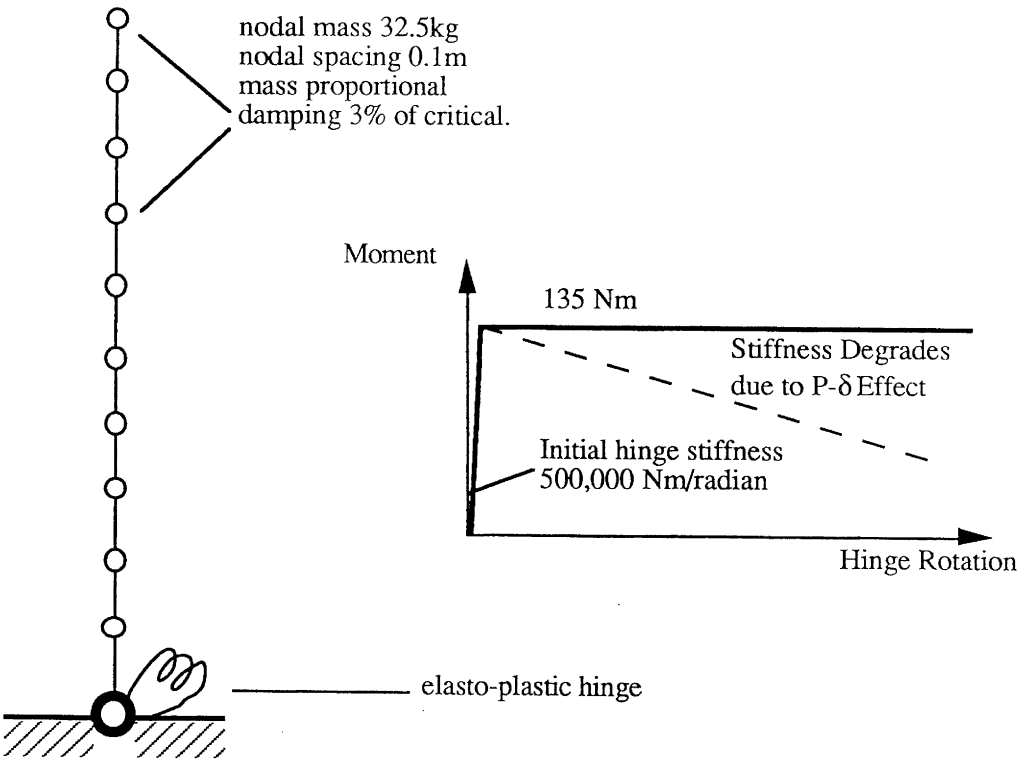

FIGURE 4 The "Stick" finite element model

table tests. The major features of the model can be summarised as follows:

a) The nodal mass of 32.5 kg was obtained by dividing the total mass of the wall by the number of nodes.

b) Damping values obtained from dynamic tests were consistently within the range of 2.6% - 3.4% [4]. The viscous damping matrix in the finite element model was assumed to be mass proportional and selected to represent 3% damping.

c) An elastic modulus value of 410 MPa was obtained from dynamic tests and was used in the model. This value is significantly smaller than the 1000 MPa elastic modulus obtained from static compression tests [4].

d) An elasto-perfectly plastic hinge was added to the base of the stick model to allow for rocking.

e) The unconditionally stable average acceleration method with a constant time-step of 0.0025 seconds was used in the time-history analysis.

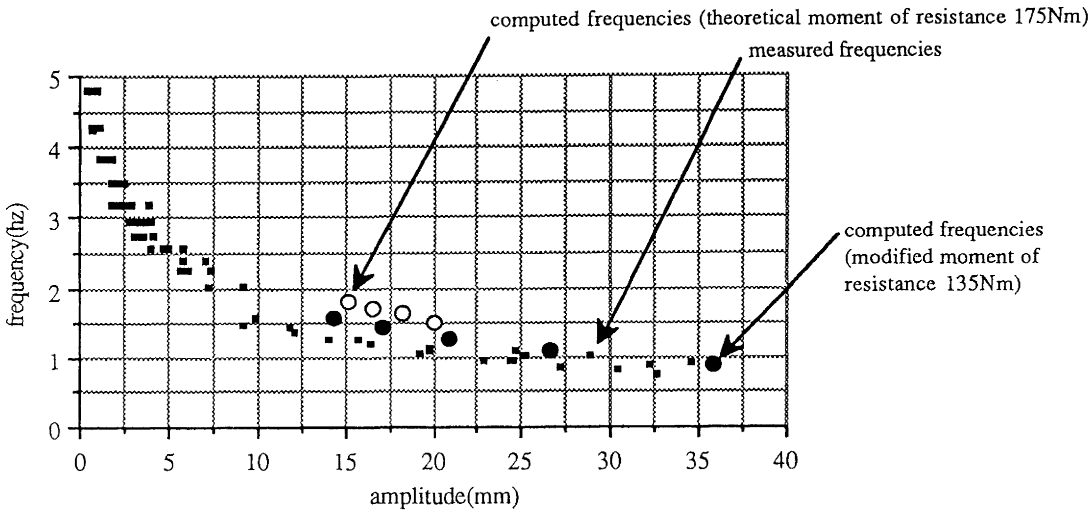

FIGURE 5 Computed and measured rocking frequencies

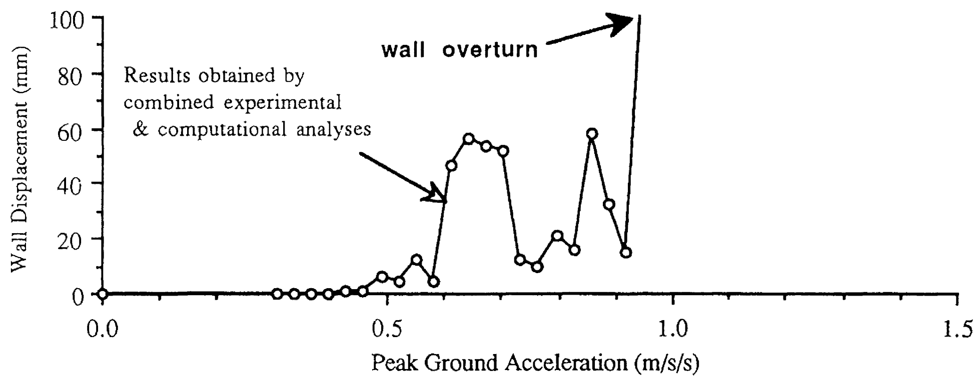

FIGURE 6 Time-history analyses results showing variation of maximum wall displacement with PGA

f) The moment of resistance of the hinge at incipient rocking is in theory the weight of the wall multiplied by half the wall thickness, which in this case equals 175 Nm. However, the dynamic test results showed that some reduction of this moment was needed to account for practical considerations such as inadequate bedding and non-uniform distribution of mass. The moment of resistance at the base of the wall was adjusted to 75% of the theoretical value in order that the computed rocking frequencies were more consistent with the measured frequencies (refer Figure 5).

g) P-δ effects have been included in the finite element analysis to allow for the decrease in resistance of the wall as the wall rotation increases.

h) The ground motion data used in the analysis was based on the El-Centro earthquake ground motion recorded in 1940 with a peak ground acceleration of 3.06 m/sec^2.

The response of the cantilever wall to the El Centro ground motion scaled to different peak ground accelerations (PGA) is shown in Figure 6. The wall displacement was very sensitive to changes in the PGA values but the relationship was not monotonic. At a PGA of 0.9 m/sec^2, the wall displacement became very large indicating overturning, which was consistent with the experimental results.

3. RESPONSE SPECTRUM METHOD BASED ON QUASI-STATIC LOAD

The response of the cantilever wall to the El Centro ground motion obtained by time-history analyses is compared in this section to that obtained by the response spectrum method. The response spectrum for the EL Centro record normalised with respect to the PGA for a damping ratio of 3% is shown in Figure 8. Assuming the mode shape is a straight line and the participation factor is equal to 1.5, the top of wall acceleration is 1.5 Sa_{norm} (PGA), where Sa_{norm} is the normalised response spectral value corresponding to the natural period of vibration as shown in Figure 7. For a triangularly distributed acceleration profile, the total inertia force is {(0.5) M 1.5 Sa_{norm} PGA} where M is the mass of the wall. With the centroid of the force acting at two-thirds of the wall height h, the overturing moment is {(0.5) M (1.5) Sa_{norm} PGA (0.67) h}. The PGA required to overturn the wall is then obtained by equating the overturning moment to the restoring moment which is equal to the product of the self weight of the wall and half of the wall thickness, t.

Hence $(0.5)\ M\ (1.5)\ Sa_{norm}\ PGA\ (0.67)\ h = M\ g\ (0.5t)$

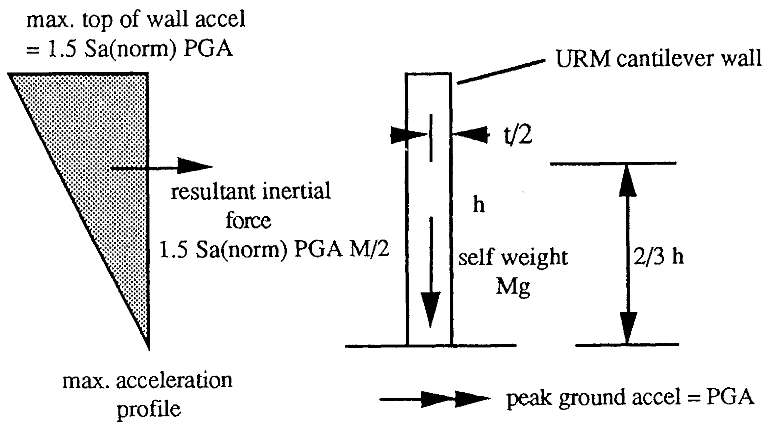

FIGURE 7 Quasi-static load on a URM cantilever wall

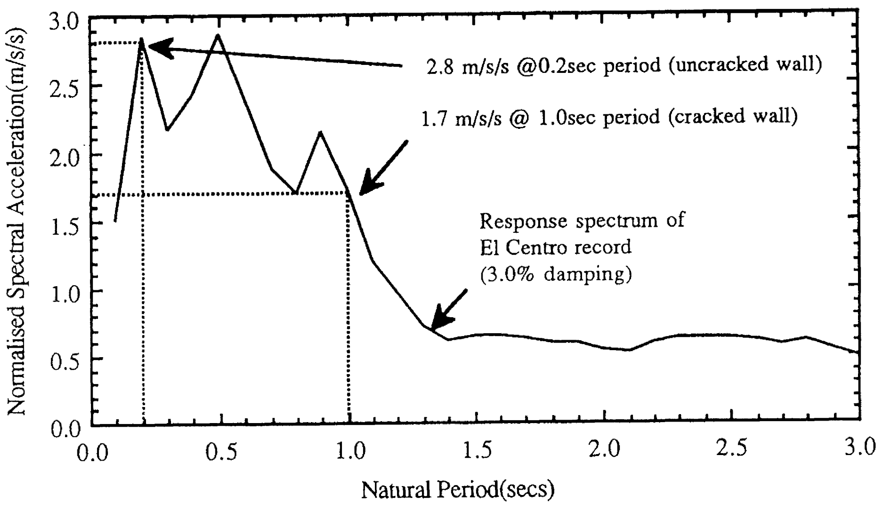

FIGURE 8 Normalised response spectrum of the El Centro record

$$=> \quad \frac{PGA}{g} = \frac{t/h}{Sa_{norm}} \qquad (1)$$

where g is the gravitational acceleration

Assuming a natural period of 0.2 seconds for a 1 m high 110 mm thick URM uncracked wall [4], the normalised spectral acceleration Sa_{norm} is 2.8 (refer Figure 8). The PGA required to fail the wall using Equation 1 is therefore 0.039g which is only 43% of the 0.09g obtained from the time-history analysis (see Table 1).

Alternatively, if a natural period of 1.0 second was assumed based on measurements taken during rocking (refer Figure 3), Sa_{norm} would have equalled 1.7 and the PGA estimated to fail the wall would have equalled 0.065g which is about 70% of the time-history analysis.

The main shortcomings of the linear elastic response spectrum method are that the quasi-static load does not account for the rocking behaviour of a cantilever wall and further, the response of a rocking wall is strongly non-linear. For these reasons, the quasi-static load method will often underestimate the seismic resistance of a cantilever wall. An alternative method based on the equal energy principle to estimate the PGA required to overturn the wall is considered in the next section.

Table 1 PGA estimated to overturn a 1 m high 110 mm thick URM cantilever wall

Shaking table testing and time history analysis for rocking	Response spectrum method (elastic natural frequency of uncracked wall)	Response spectrum method (rocking frequency of cracked wall)
0.09g	0.039g	0.065g

4. THE EQUAL ENERGY METHOD

The equal energy approach is sometimes used to predict the seismic resistance for building frames [7] and for URM facade walls laterally supported at the floor and ceiling levels [8]. However, little research has been done to evaluate its accuracy [10].

In the following, closed form expressions for the seismic resistance of a URM cantilever wall are derived from first principles using the equal energy method and compared with the non-linear time-history analysis results. Problems inherent in the equal energy method are then identified and discussed.

4.1 Statics Of A Free-Standing Parapet Wall

The statics of a uniform cantilever wall free to rock about its base will be briefly described as an introduction to the equal energy method. Consider a cantilever wall subject to a horizontal force Pr applied at y metre above the base. The load versus displacement characteristics of the wall can be determined as the magnitude of P_r is increased and decreased by evaluating the equilibrium condition at each change of load.

Under self weight loading only the bearing stress at the base of the wall is uniformly distributed (see Figure 9a). As P_r is applied a bending moment is introduced resulting in a trapezoidal bearing stress distribution. At initial cracking, the stress is triangularly distributed, with the resultant reaction thrust acting through the centroid of the stress triangle (see Figure 9b). As P_r is increased, the reaction thrust moves further to the right, and partial cracking occurs as the effective cross-section of the wall is reduced (see Figure 9c). At incipient rocking the reaction thrust cannot move any further to the right (see Figure 9d).

Equating the disturbing moment to the restoring moment it can easily be shown that :

$$P_r = \frac{Mg.\frac{t}{2}}{y} \qquad (2)$$

where M=mass of wall; t=thickness of wall and g=acceleration due to gravity.

If the wall displacement is increased further, the restoring moment will be reduced due to P-δ effects (see Figure 9e). The value of P_r can be redefined by the expression :

$$P_r = \frac{Mg.\left[\frac{t}{2} - \frac{\Delta_r}{2}\right]}{y} \qquad (3)$$

where Δ_r is the reference displacement at the top of the wall.

Finally, at the limit of overturning $\Delta_r = t$ and $P_r = 0$ (see Figure 9f).

The complete load-displacement relationship is shown in Figure 10. It is assumed that the elastic deflections up to the point of incipient rocking are negligibly small (Stages A to D), whilst between Stages D and F, the wall rocks as a rigid body with the associated P-δ effects.

4.2 Potential Energy Of A Cantilever Wall Considering Non-linear Behaviour During Rocking

During rocking, there is a continuous exchange of kinetic energy (K.E.), potential energy (P.E.) and dissipated energy in the parapet wall (refer Figure 11). In an idealised system where damping can be neglected and the ground is stationary, the sum of the K.E. and the P.E. is constant at all times. When the rocking wall passes the vertical position, the K.E. and velocity are at a maximum. Conversely, when the wall reaches the maximum displacement position and about to reverse the direction of movement, the wall has maximum P.E. but has no K.E..

Thus, the maximum gain in P.E. is equal to the maximum K.E..

$$\text{Max. K.E.}_{\text{rocking}} = \text{Max. Gain in P.E.}_{\text{rocking}} \qquad (4)$$

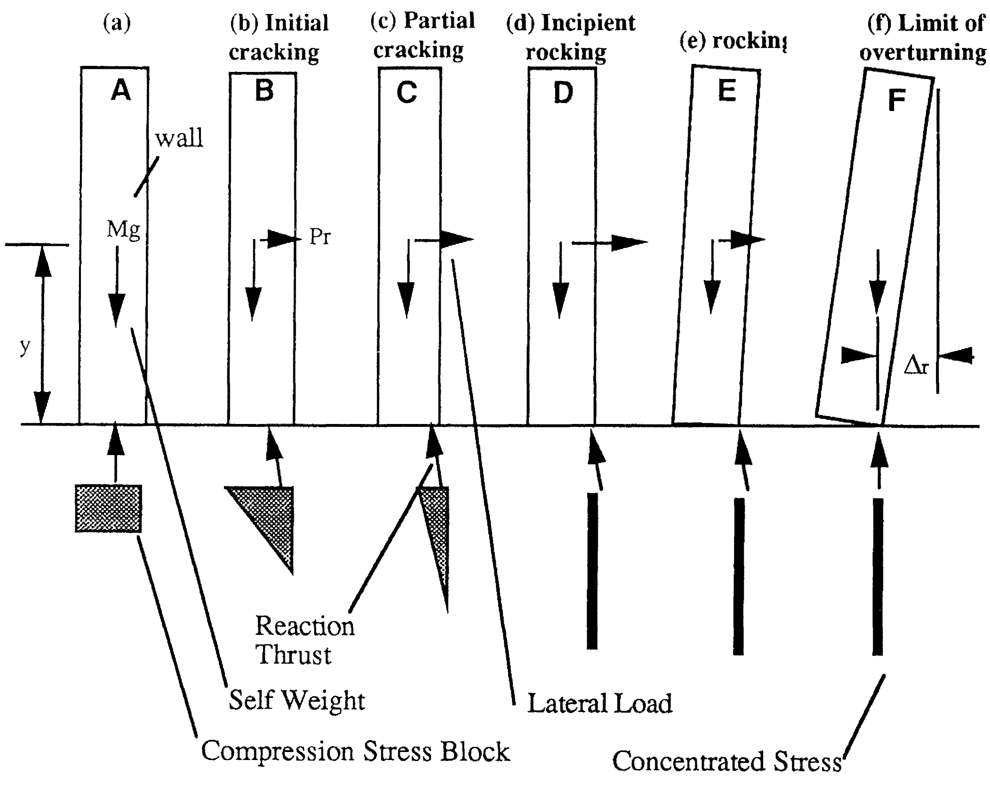

FIGURE 9 Action and reaction of a free-standing cantilever wall subject to horizontal load

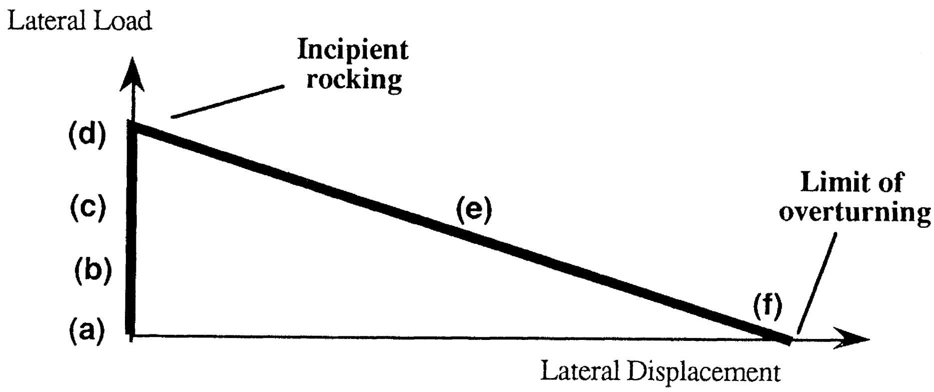

FIGURE 10 The load-displacement relationship

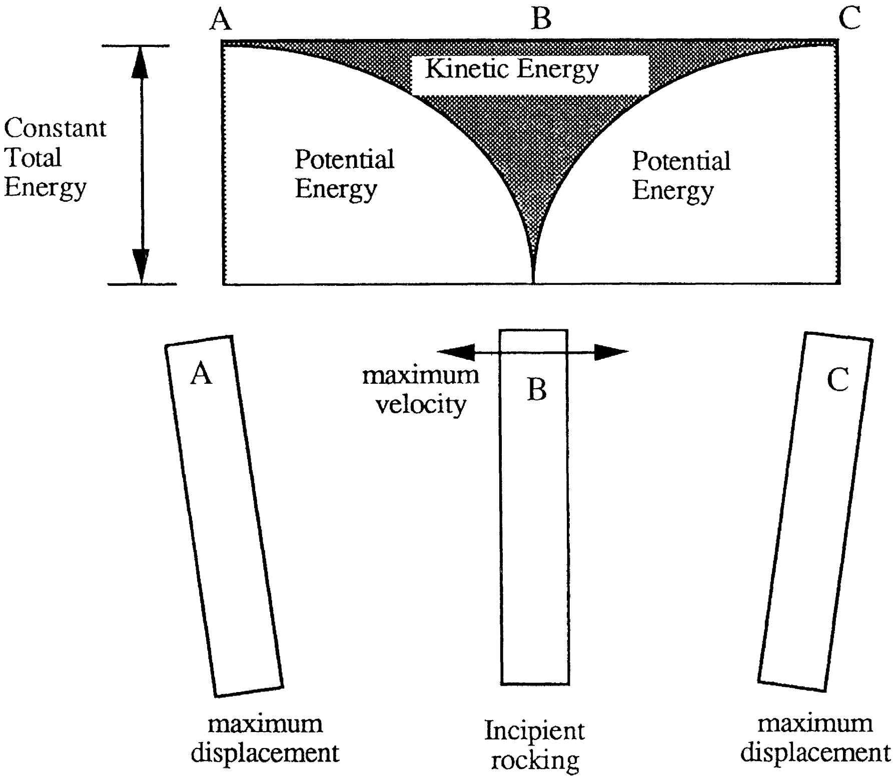

FIGURE 11 Energy exchange during rocking

In a system with a continuous distribution of mass such as a cantilever wall, it is necessary to consider the energy gained by individual particles which are uniformly distributed along the height of the wall to evaluate the total gain in P.E.. It is shown in appendix A that the maximum P.E. gained by the parapet wall during rocking is :-

$$E_r = \frac{1}{2}.\frac{M}{3}.\frac{3}{2}.\frac{gt^2}{h}.\frac{\Delta_r}{t}\left[2 - \frac{\Delta_r}{t}\right] \qquad (5)$$

where Δ_r is the maximum displacement.

At the limit of overturning $\Delta_r = t$ and $\Delta_r/t = 1$,

hence:

$$E_r = \frac{Mgt^2}{4h}$$

4.3 Potential Energy Of An Equivalent Linear Elastic Rocking Wall Model

In contrast to the non-linear model used in the previous section, consider an equivalent linear elastic model as shown in Figure 12.

This equivalent linear model is identical to the original non-linear model except that an elastic hinge of constant rotational stiffness is installed at the base of the wall, and P-δ effects are ignored.

The total P.E. gained by the equivalent model E_e is derived in appendix B in a similar manner to the non-linear model, and is given by :

$$E_e = \frac{1}{2}.\frac{M}{3}.\left[\frac{3}{2}\right]^2.\frac{Sa^2}{(2\pi f)^2} \qquad (6)$$

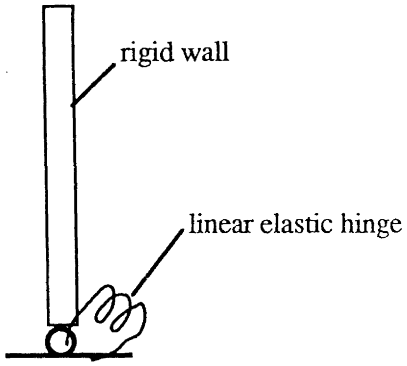

FIGURE 12 Equivalent linear elastic model

where f is the natural frequency of vibration and Sa is the corresponding spectral acceleration of the earthquake ground motion.

4.4 Application Of The "Equal Energy" Method

The "equal energy" method assumes that the maximum kinetic or potential energy developed in the linear and non-linear models are equal when subjected to identical base excitations. Hence, by equating the potential energy gained in the two systems :

$$\underset{(Eq.6)}{E_e} = \underset{(Eq.5)}{E_r}$$

$$=> \quad \frac{3}{2} \cdot \left[\frac{Sa}{2\pi f}\right]^2 = \frac{gt^2}{h} \cdot \frac{\Delta_r}{t}\left[2 - \frac{\Delta_r}{t}\right]$$

This equation can be re-written in the form :

Spectral Velocity $S_v = V_c\ \gamma$ (7a)

where S_v = Spectral response velocity

$$= \frac{Sa}{2\pi f} \quad (7b)$$

V_c = Minimum critical response velocity required to overturn wall

$$= \sqrt{\frac{2}{3} \cdot \frac{gt^2}{h}} \quad (7c)$$

γ = Ratio of actual to critical response velocity

$= S_v / V_c$

$$= \sqrt{\frac{\Delta_r}{t}\left[2 - \frac{\Delta_r}{t}\right]} \quad (7d)$$

The relationship between the ratios γ and Δ_r/t is shown in Figure 13.

Equation 7a can be re-written in terms of the peak ground acceleration (PGA):

$$PGA = \frac{2\pi f V_c \gamma}{Sa_{norm}} \quad (8)$$

The seismic resistance of a URM cantilever wall expressed in terms of the PGA is hence obtainable from a simple expression involving V_c, γ, f and Sa_{norm}.

V_c and γ are uniquely defined, whereas f and Sa_{norm} are directly dependent on the response displacement of the rocking wall. The selection of appropriate values for f and Sa_{norm} are considered in the following sections.

4.5 Results obtained from The Equal Energy Method

A comparison of the results obtained from the equal energy method using two alternative ways (Alternatives I,II) for defining the natural frequency of the wall is made with the results from the time history analyses in the following section. In addition, Appendix C considers a further two methods (Alternatives III,IV) using a constant rocking natural frequency and an idealised response spectrum which may have application for the design of cantilever walls subject to earthquake ground motions.

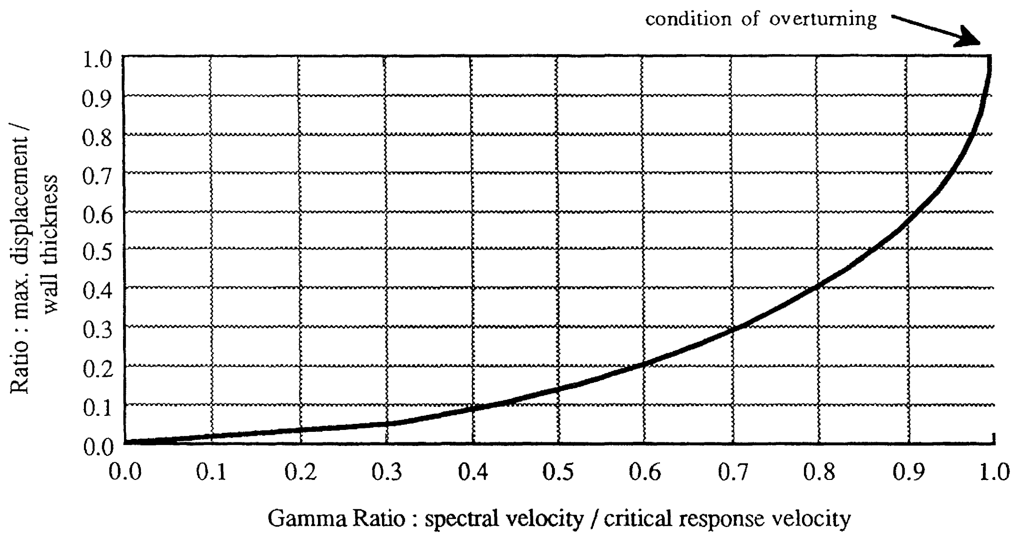

FIGURE 13 Relationship between the ratios g and Dr/t

4.5.1 Alternative I - Use of elastic natural frequency and actual response spectrum

The equal energy method has been applied to building frames and URM facade walls based on the natural frequency in the linear elastic response [7,8]. Extending this approach to the 1.0 m high URM cantilever wall and substituting the elastic natural frequency of 5 Hz [4],

V_c = 0.28m/sec^2 and Sa_{norm} = 2.8 (refer Figure 8) into Equation 8:

$$=> \quad PGA = \frac{2\pi(5)(0.28)\gamma}{2.8}$$

For $\gamma = 1$ (limit of overturning): PGA = 0.32g

Thus, assuming the elastic natural frequency as the rocking frequency in this application leads to an unconservative 3.5 fold overestimate of the seismic resistance of the wall.

4.5.2 Alternative II - Use of measured rocking frequencies and actual response spectrum

The natural frequency adopted for the purpose of Equation 8 in this alternative is based on the actual rocking frequencies measured in the shaking table tests, whilst Sanorm are derived from the El Centro response spectrum. For a given maximum wall displacement, the corresponding rocking frequency can be obtained directly from Figure 3 (the maximum displacement amplitude is reduced by a third to account for variation during the response). Results derived from the equal energy method are compared with results from the time-history analyses in Figure 14.

It can be seen that when the PGA is less than 0.9 m/sec^2 the slope of the peaks and troughs roughly match, suggesting that the equal energy method provides a reasonable approximation to the actual response of the wall. However, the method does not predict the sudden failure of the wall at 0.9m/sec2 but rather failure at a PGA approximately twice this level. A detailed comparison between the equal energy method and the time history analyses is considered for a PGA equal to 0.7 m/sec^2 and 1.1 m/sec^2 in the following section (Case A and Case B in Figure 14)

a) Case A : PGA = 0.7 m/sec^2

Consider a 1.0 m high, 110 mm thick cantilever wall with a maximum displacement of 40mm, it follows that V_c = 0.28 m/sec (Equation 7c), g = 0.77 (Equation 7d), f = 1.1 Hz (Figure 3) Sa_{norm} = 2.15 m/sec^2 (Figure 8) and therefore the PGA = 0.7 m/sec^2 (Equation 8) implying S_v = 0.22 m/sec and a top of wall velocity equal to 0.32 m/sec (based on a participation factor of 1.5).

From a time-history analysis of the El Centro ground motion scaled to 0.7m/sec2 PGA, the wall was found to reach a maximum velocity of 0.28 m/sec after 5.6 secs and a peak displacement of 52 mm shortly afterwards (refer Figure 15a). Both the velocity and the displacement were within 14% and 23% respectively of the equal energy method estimate. Although the rocking period varied considerably depending on the displacement amplitude, the 0.9 sec period measured just prior to the peak response coincided with the value used in applying Equation 8. Thus, in this case, the equal energy method has provided a reasonably accurate prediction of the wall's rocking response.

b) Case B: PGA = 1.1 m/sec^2

Consider the wall with a maximum displacement increased to 50 mm, it follows that g = 0.84 (Equation 7d), f = 0.9 Hz , Sa_{norm} = 1.19 (Figure 8) and therefore the PGA = 1.1 m/sec^2 (Equation 8) implying S_v = 0.23 m/sec spectral velocity and a top of wall velocity equal to 0.35m/sec.

A time-history analysis of the wall with PGA=1.1 m/sec^2 demonstrates that the wall would actually overturn after 4.8 seconds of the El Centro ground motion scaled to this PGA; as shown in Figure 15b. It is interesting to note that just prior to collapse, the wall had a rocking frequency of 0.9 Hz and a maximum top of wall velocity equal to 0.31 m/sec which was similar to the value predicted by the equal energy method. However, the wall collapsed as a direct result of an acceleration pulse applied to the wall already experiencing a large displacement.

5. DISCUSSION

Three different approaches for determining the seismic resistance of URM cantilever walls have been considered and evaluated in this paper using a simple 1.0 m high 110 mm thick wall as an example. The merits, shortcomings and scope for future developments in the methods are discussed herein.

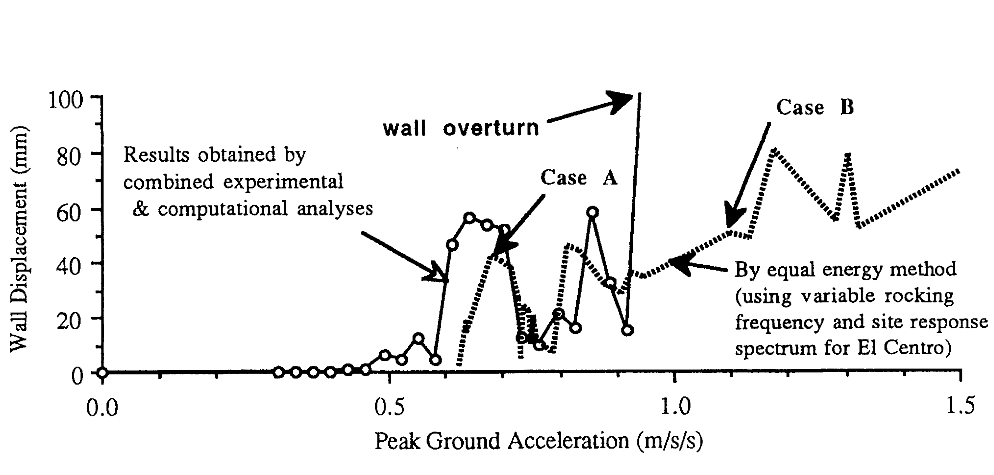

FIGURE 14 Comparison between time-history analyses results and the Equal Energy Method estimates (Alternative II)

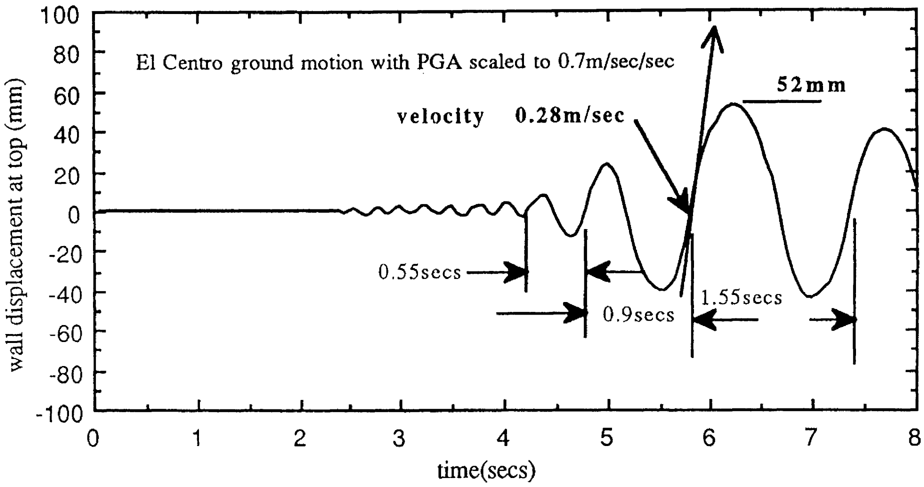

FIGURE 15a Rocking response time-history of 1.0m high parapet wall, PGA=0.7m/sec2 (Case A)

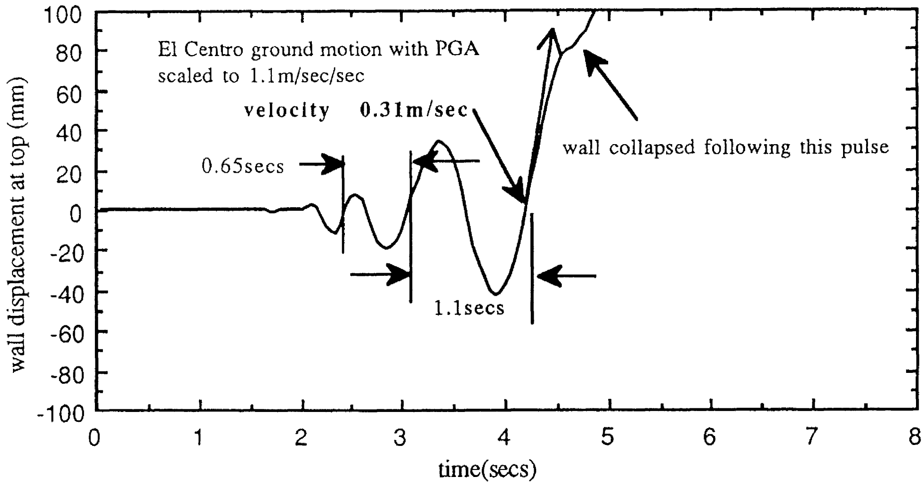

FIGURE 15b Rocking response time-history of 1.0m high parapet wall (Case B)

Non-linear time-history analyses provide a fairly accurate and reliable method for determining the response of a stable structures to a specified base excitation, provided that the actual mechanical properties of the structure are accurately represented in the finite element model. Shaking table tests of the URM cantilever wall demonstrated that the measured rocking natural frequencies and damping properties were different from those initially predicted, and that the overall response of the wall was very sensitive to variations in these properties. Hence, for time history analyses to be used with confidence to predict the response of URM walls either an experimental study to determine the actual dynamic properties or a sensitivity study to account for a range of properties is required. Further, a large number of representative records need to be used to account for variations in possible earthquake ground motions.

The response spectrum method based on the quasi-static load principle is commonly adopted for the analysis of building frames and other structures subjected to seismic loads. However, the non-linear rocking behaviour of the wall under time-dependent excitations cannot be accurately modelled by simply applying a sustained quasi-static load. Although a URM face loaded wall is conventionally treated as brittle, some "ductility" in the lateral load-displacement relationship exists. For example, when this method was applied to the URM cantilever wall based on the uncracked natural period, the estimated PGA required to overturn the wall was underestimated by a factor of 2. It is postulated that the wall's rocking behaviour could be incorporated into the method by means of a ductility response factor. A wide range of wall sizes would need to be analysed and tested using a probabilistic approach with an ensemble of representative ground motions to establish suitable ductility factors.

The equal energy method is sometimes used to predict the inelastic response of structures subject to dynamic loads. However, the main difficulty with the method when applied to URM cantilever walls is in the selection of a suitable natural frequency, since the natural frequency of a rocking wall is directly dependent on the displacement amplitude. In addition, the equal energy method assumes no energy input over the critical time period (i.e. after the maximum response velocity has been reached) and hence it does not take into account the effect of further pulses applied to a wall already experiencing a large displacement. A semi-empirical procedure using the equal energy method has been outlined in Appendix C for evaluating the seismic resistance of cantilever walls in the height range of 0.5 metre to 1.1 metre. The results suggest that a rocking frequency of 1.25 Hz provides a reasonable estimate for predicting failure of these cantilever walls. Different natural frequency values may have to be assumed for walls in other height ranges, and with different aspect ratios.

The following limitations should be noted when interpreting the results obtained from the analyses:

a) The earthquake data employed in the study were limited to the earthquake motion recorded at El Centro whose response spectrum matches reasonably well with the shape of the idealised design spectrum specified by building codes and standards in many countries including Australia [9].

b) The ground motion recorded in the free field was taken as motion applied to the base of the cantilever wall.

c) The rocking model examined in this paper has ignored the contribution of restraints such as wall returns, which provide additional resistance.

6. CONCLUSION

The main conclusions can be summarised as follows :-

(i) From shaking table tests and non-linear time-history analysis of a finite element model, a 1.0m high 110m thick wall was shown to be capable of withstanding the El Centro ground motion scaled to 0.09g PGA. However, significant tuning of the parameter used in the finite element model was required to match the measured dynamic characteristics of the rocking wall.

(ii) The quasi-static load method based on the use of an elastic response spectrum underestimated the PGA required to overturn the cantilever wall by a factor of about 2.0. This was due to the static inertial load not simulating the wall's non-linear rocking behaviour when subjected to a time-dependent base acceleration. A ductility response factor could be derived from probabilistic studies.

(iii) The equal energy approach has been used to derive a closed form expression for the PGA required to overturn an URM cantilever wall cracked at the base. The expression for the PGA is dependent on the critical response velocity V_c, the natural frequency of the wall, f ,and the normalised spectral acceleration, Sa_{norm}.

(iv) The equal energy method overestimated the PGA required to fail a URM cantilever wall by a factor of approximately 3.5 and 2 when the elastic uncracked natural frequency and the cracked natural rocking frequency of the wall were used respectively.

(v) The equal energy method assumes zero energy input over the critical time period and hence does not account for the effects of acceleration pulses applied to a wall already experiencing a large displacement.

(vi) By calibrating the equal energy method estimates against time-history analysis results, a 1.25 Hz natural frequency has been identified to be suitable for single skin cantilever walls in the height range of 0.5-1.1 m subject to the El Centro earthquake ground motion. Further probabilistic studies need to be undertaken to confirm this value and to extend the work for walls of other dimensions and properties.

(vii) This study has highlighted the inherent difficulties of accurately modelling and predicting the response of simple masonry cantilever walls using simplified analytical methods.

7. ACKNOWLEDGEMENTS

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8. NOTATION

D_r	:	Top of wall displacement
f	:	natural frequency
g	:	gravitational acceleration
g	:	Ratio of critical response velocity to spectral response velocity
h	:	Wall height
M	:	Wall mass
PGA	:	Peak ground acceleration
S_a	:	Spectral response acceleration
Sa_{norm}	:	Spectral response acceleration normalised with respect to PGA
S_v	:	Spectral response velocity
t	:	Wall thickness
URM	:	Unreinforced masonry
V_c	:	Critical response velocity required to overturn wall

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APPENDIX A1

DERIVATION OF Equation 5:

$$E_r = \frac{1}{2}\cdot\frac{M}{3}\cdot\frac{3}{2}\cdot\frac{gt^2}{h}\cdot\frac{\Delta_r}{t}\left[2-\frac{\Delta_r}{t}\right] \qquad (5)$$

Consider a wall rocking under its own inertial force and displaced by a very small amount dr . Assuming a linear variation of acceleration from zero at the base to ar at the top of the wall, the P.E. gained by each particle is :

$dE_{particle}$ = inertia force x displacement

$$= m.\left[\frac{y}{h}a_r\right].\left[\frac{y}{h}.\delta_r\right].dy \qquad (A1)$$

where m = mass per unit length
y = height of individual particle
h = height of wall
a_r = acceleration at top of wall
δ_r = incremental displacement at top of wall

The energy gained by the wall as a whole can be obtained by evaluating the integral :

$$dE_{wall} = \int_0^h m\left[\frac{y}{h}a_r\right]\left[\frac{y}{h}\delta_r\right]dy \qquad (A2)$$

where m= mass per unit length along the height

$$=> \quad dE_{wall} = M\ a_r\ \delta_r\ /\ 3 \qquad (A3)$$

At incipient rocking, let $a_r = a_o$. It is shown in Appendix A2 that:

$$a_o = \frac{3}{2}\frac{gt}{h} \qquad (A4)$$

According to the linear relationship between a_r and d_r (see Figure 2 and Equation 2) :

$$a_r = a_o\ (1 - d_r/t) \qquad (A5)$$

Let E_r be the total gain in P.E. as the displacement at the top of the wall reaches the maximum value of D_r. By substituting Equation A5 into Equation A3 and integrating with respect to $d(d_r)$ (letting $d(d_r) = \delta_r$) :

$$E_r = \frac{Ma_0}{3}\int_0^{\Delta_r}\left[1-\frac{d_r}{t}\right]d(d_r)$$

$$=> \quad = \frac{1}{2}\frac{Ma_0}{3}\Delta_r\left[2-\frac{\Delta_r}{t}\right]$$

$$=> \quad = \frac{1}{2}\cdot\frac{M}{3}\cdot\frac{3}{2}\cdot\frac{gt^2}{h}\frac{\Delta_r}{t}\left[2-\frac{\Delta_r}{t}\right] \qquad (A6)$$

At the limit of overturning, $\Delta_r = t$ and E_r is given by :

$$E_r = \frac{1}{2}\cdot\frac{M}{3}\cdot\frac{3}{2}\cdot\frac{gt^2}{h} \qquad \text{Q.E.D.(A7)}$$

APPENDIX A2

DERIVATION OF Equation A4: $a_0 = \frac{3}{2}\cdot\frac{gt^2}{h}$

Consider a parapet wall at incipient rocking :

Applied overturning moment due to inertia = Restoring moment due to self weight

$$\int_0^h m\left[a_0\frac{y}{h}\right]y\ .\ dy = M\ g\ t/2$$

$$=> \quad \frac{ma_o}{h}\left[\frac{y^3}{3}\right]_0^h = M\ g\ t/2$$

$$=> \quad \frac{ma_0 h^3}{3h} = M\ g\ t/2$$

$$=> \quad a0 = 3/2\ .\ gt/h \qquad \text{Q.E.D.(A6)}$$

APPENDIX B1

DERIVATION OF Equation 6:

$$E_e = \frac{1}{2}\cdot\frac{M}{3}\cdot\left[\frac{3}{2}\right]^2\cdot\frac{Sa^2}{(2\pi f)^2}$$

According to Equation A3, it can be shown that the energy gained by the wall :

$$dE_{wall} = (M\ a_e\ d_e\ )\ /\ 3 \qquad (B1)$$

where a_e and d_e are the acceleration and displacement respectively at the top of the wall in the elastic response.

For simple harmonic motion :

$$a_e = (2pf)^2\ d_e \qquad (B2)$$

Let E_e be the total gain in P.E. in the elastic system as d_e reaches the maximum value of Δe. By substituting Equation B2 into Equation B1 and integrating with respect to d_e:

$$E_e = \frac{M}{3}\int_0^{\Delta_e}(2\pi f)^2 d_e d(d_e)$$

$$=> \quad = \frac{1}{2}\frac{M}{3}(2\pi f)^2\Delta_e^2$$

$$=> \quad = \frac{1}{2}\cdot\frac{M}{3}\cdot\frac{a_{e0}^2}{(2\pi f)^2} \qquad (B3)$$

where a_{eo} is the maximum acceleration when $d_e = \Delta_e$.

If the wall is considered as a generalised single-degree-of-freedom system, it is shown in Appendix B2 that :

$$a_{eo} = 3/2\ S_a \qquad (B4)$$

Thus, the total P.E. gained by the elastic system is obtained by the expression:

$$E_e = \frac{1}{2}\cdot\frac{M}{3}\cdot\left[\frac{3}{2}\right]^2\cdot\frac{Sa^2}{(2\pi f)^2} \qquad \text{Q.E.D. (B5)}$$

APPENDIX B2

DERIVATION OF Equation B4: $a_{eo} = 3/2\ S_a$

Consider the dynamic equilibrium of a generalised single-degree-of-freedom system as shown in Figure 4, taking moments about the base:

$$\int_0^h m\left[a\frac{y}{h}\right]y.dy + K_\theta.\theta = -\int_0^h m a_g y dy$$

where a is the acceleration at the top of the wall relative to the ground and a_g is the ground acceleration.

$$=> \quad \frac{ma}{h}\left[\frac{y^3}{3}\right]_0^h + K_\theta\frac{\Delta_e}{h} = -ma_g\left[\frac{y^2}{2}\right]_0^h$$

$$=> \quad \frac{mah^3}{3h} + K_\theta\frac{\Delta_e}{h} = -Ma_g\frac{h}{2}$$

$$=> \quad Ma + \left[\frac{3K_\theta}{h^2}\right]\Delta_e = -\frac{3}{2}Ma_g$$

$$=> \quad a + \omega^2\Delta_e = -\frac{3}{2}a_g$$

$$=> \quad a_{eo} = +3/2\ S_a \qquad \text{Q.E.D.}$$

APPENDIX C: FURTHER STUDIES USING THE EQUAL ENERGY METHOD

A further 2 methods (Alternatives III,IV) are considered in this appendix based on a constant rocking natural frequency and idealised response spectra which have potential application for design purposes.

C.1 Alternative III - Use of constant rocking frequency and idealised response spectrum

The equal energy method may be simplified by using a smoothed response spectrum as shown in Figure C1 (based on site factor of 1.0) [9] together with a constant rocking frequency (i.e. independent of the displacement amplitude). The wall displacement versus PGA relationship for a range of rocking frequencies can be represented by a series of curves as shown in Figure C2. The advantage of this modification is that individual peaks and troughs in the curves have been removed. The appropriate equivalent rocking frequency in Equation 8 can then be selected from the curve which closely matches the results of the time-history analyses.

However, from the curves shown in Figure C2, the equivalent rocking frequency would have to be much less than 0.5 Hz for the results to match. In contrast, the actual rocking frequency was found from dynamic tests to be around 1.0 Hz for a 1.0m high wall. The poor correlation can be partly attributed to the use of a smoothed response spectrum which was characterised by a gradual drop in the spectral value with increasing natural period. Although the response of a linear elastic system to earthquake loading gradually reduces as the natural period increases in the medium or long period range, this is not necessarily the case in a rocking system in which the natural period changes during the response depending on the displacement amplitude. In the following section, the use of a response spectrum which is constant for all natural periods is investigated.

C.2 Alternative IV - Use of constant rocking frequency and flat response spectrum

This alternative investigates the adoption of a constant Sanorm equal to 2.5 for all natural periods in Equation 8. Figure C3 shows the response predicted by the equal energy method for a narrow range of wall rocking frequencies. A reasonable match between the equal energy method and the actual wall behaviour was obtained with a wall rocking frequency of 1.35 Hz, which was within the 1.0 to 1.5 Hz range measured experimentally. Clearly, a flat or constant response spectrum is considered more appropriate than the period dependent spectrum for the purpose of Equation 8.

Similar analyses have been carried out for walls ranging in height between 0.5-1.1 m. By comparing the time-history analyses results with the equal energy method estimates (refer Figure C4a-f), the equivalent rocking frequency for each wall was found as shown in Figure C5. Importantly, the rocking frequency was not very sensitive to a variation in the height of the wall and a natural frequency of 1.25 Hz could be safely adopted for these walls.

By substituting f=1.25 Hz and Sa_{norm}=2.5 into Equation 8, a semi-empirical expression can be obtained to predict the PGA required to overturn the cantilever wall:

$$PGA = \frac{2\pi(1.25)V_c\,1.0}{2.5} = 3.14V_c$$

$$=> \quad = 3.14\sqrt{\frac{2}{3}.\frac{gt^2}{h}}$$

$$=> \quad = 2.56\sqrt{\frac{gt^2}{h}} \qquad \text{(C1)}$$

It should be noted that in this study only the response of single skin brick walls up to a height of 1.1 m subject to the El Centro ground motion have been considered. Further studies are planned to investigate other walls of different dimensions subject to a range of earthquake ground motions.

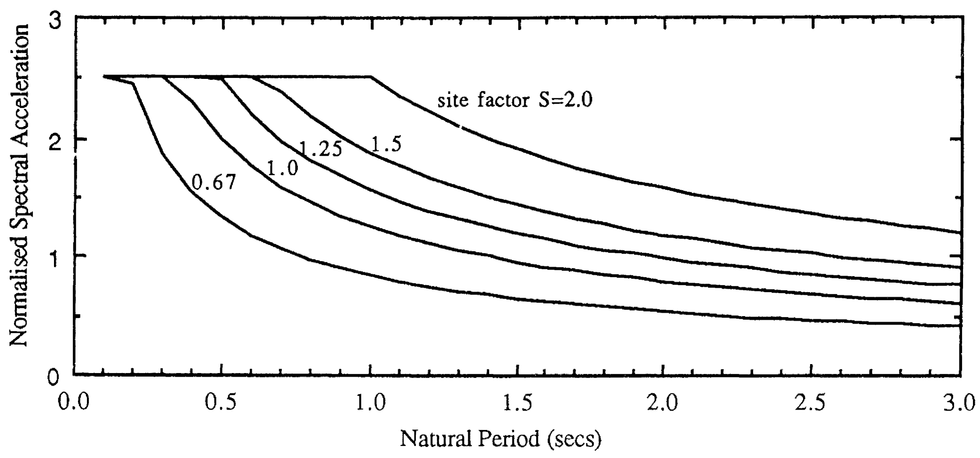

FIGURE C1 Normalised Smoothed Response Spectra (from Australian Earthquake Loading Standard AS1170.4 [9])

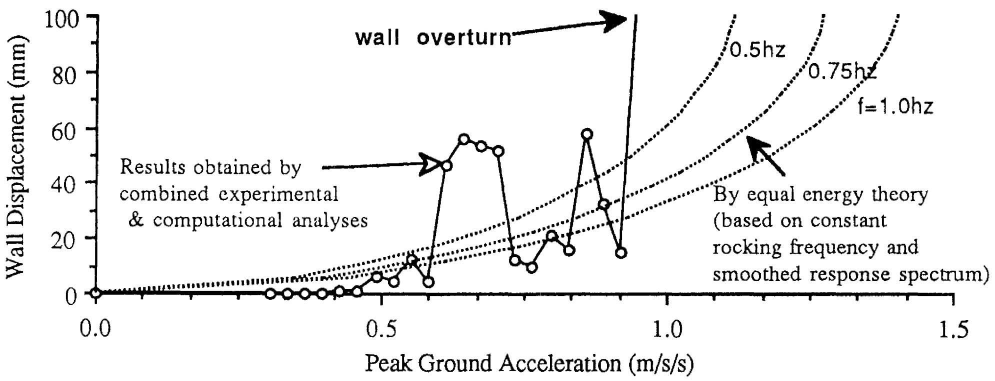

FIGURE C2 Comparison between time-history analyses and equal energy method estimates

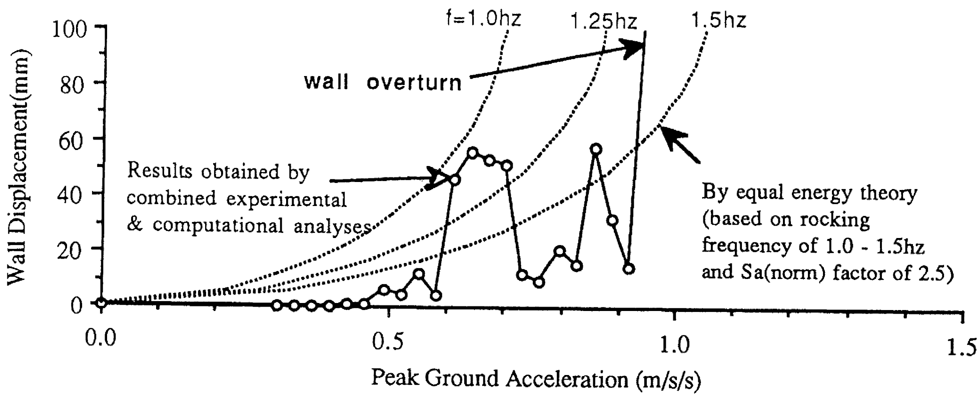

FIGURE C3 Comparison between time-history analyses and equal energy method estimates

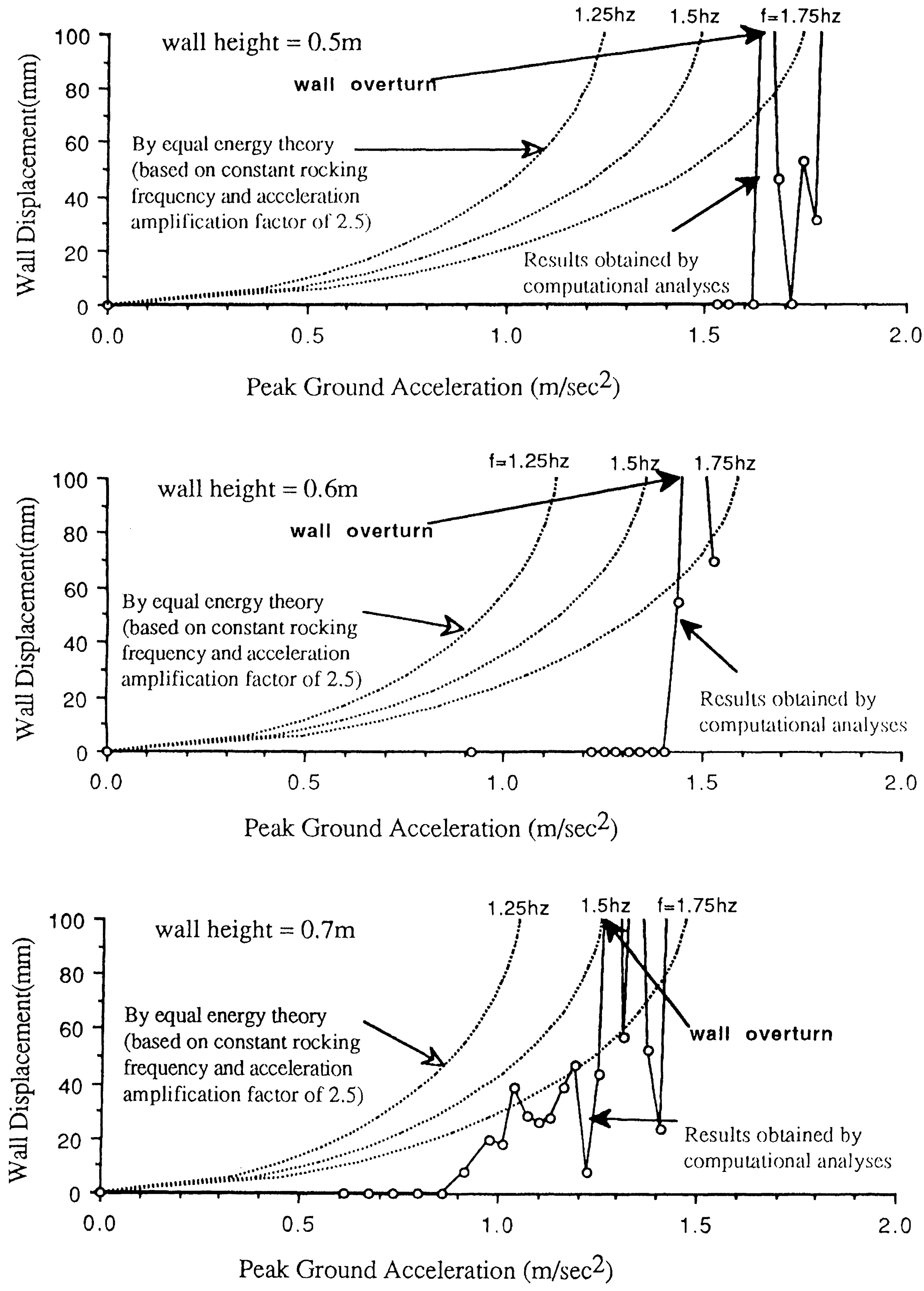

FIGURE C4a-c Matching time-history analysis results with equal energy method estimates

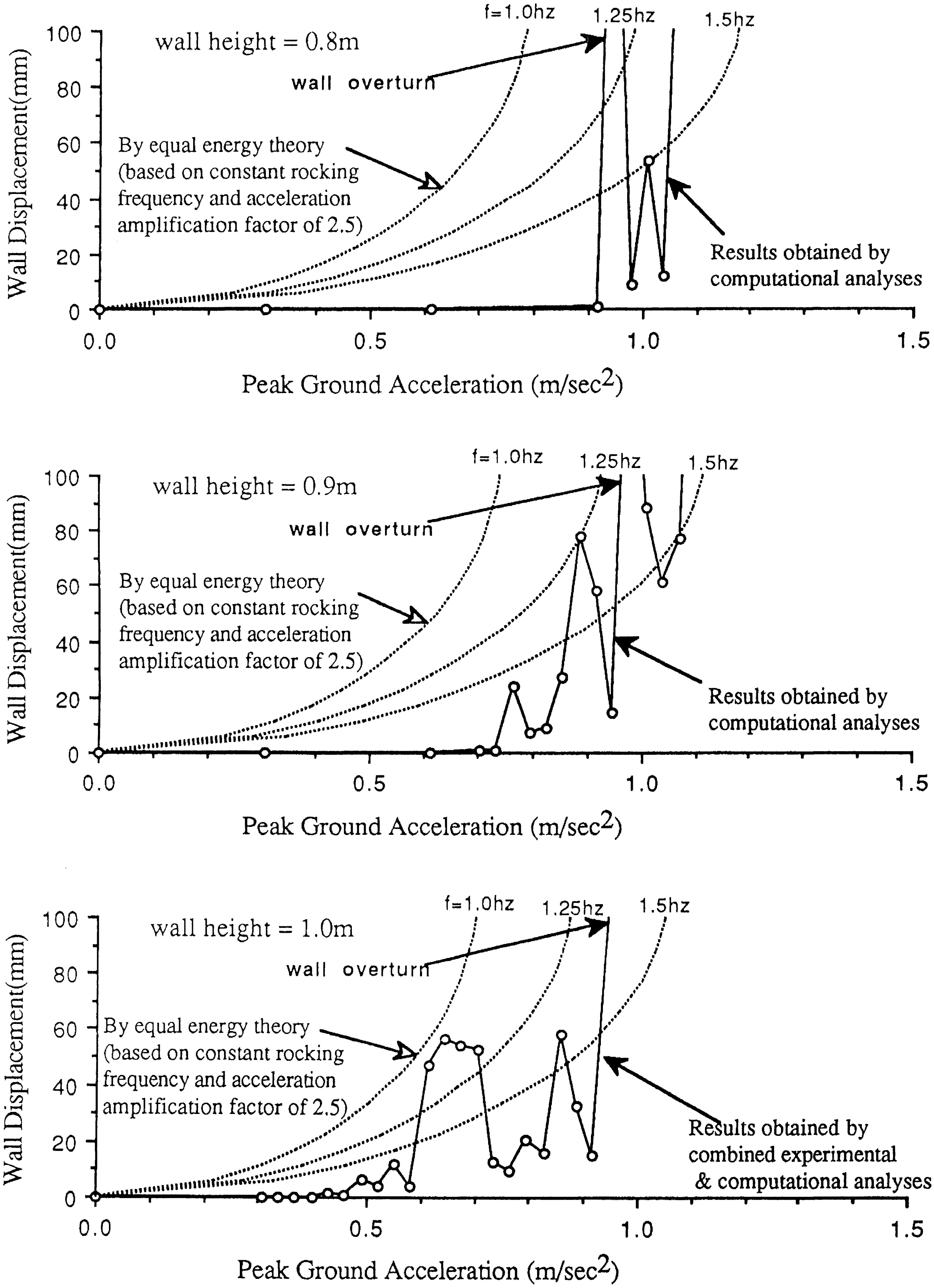

FIGURE C4d-f Matching time-history analysis results with equal energy method estimates

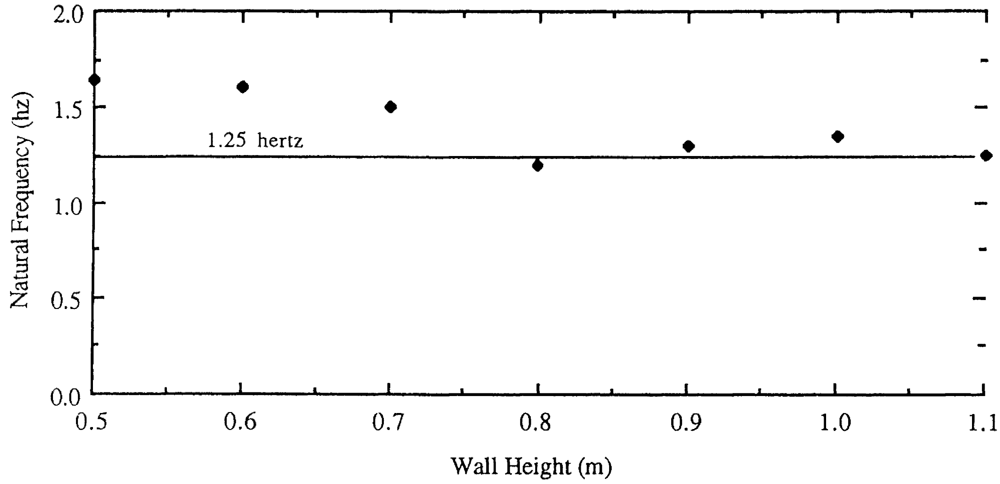

FIGURE C5 The equivalent rocking frequencies for walls 0.5-1.1 m high and 110 mm thick