AN ECONOMIC SOLUTION FOR STABILISING A SLENDER ARCH ROOF

D. D. Spurr*

ABSTRACT

A special study was made to evaluate the seismic response of the ribs of four reinforced concrete arch aircraft hangars and to assess methods of strengthening the ribs. The study was based on a series of non linear dynamic analyses of the arch ribs, with particular consideration being given to the choice of earthquake records, to the effect of P-delta actions on the arch responses and to the validity of the behaviour predicted by the computer program. The analyses indicated that the arch ribs could become unstable as a result of the P-delta effect acting in combination with reduction in the rib stiffness due to concrete cracking. However, the study also shows that the arches can be stabilised very economically by using prestressing strand to provide cross bracing.

BACKGROUND

In 1981 the Ministry of Works and Development carried out an extensive evaluation of all aircraft hangars at RNZAF bases in New Zealand. One of the findings of this evaluation was that the four large reinforced concrete arch hangars at Ohakea and Whenuapai (two at each base) were deficient in terms of current code requirements for seismic loading, particularly for loading acting in the plane of the arch ribs (transverse direction). The reports on these hangars also noted some deterioration of the concrete, with a number of areas of spalling, especially from the interior surfaces.

All four hangars were built in the late 1930's and their basic structure is shown in Fig. 1. The main roofs are made up of three structurally independent sections, each consisting of four arch ribs at approximately 5.8 m spacing with a 100 mm-150 mm slab between ribs. The arches span 67 m and are hinged at each abutment.

The initial (1981) evaluation of the hangars was based on the equivalent static load procedure and indicated that the flexural strength of the arch ribs of the Ohakea hangars was less than half that required by the code [1]. Strengthening by bolting and epoxying steel plates to the top and bottom surfaces of the ribs was investigated. The rough order of cost for this work was assessed in November 1981 at \$470,000 (= \$640,000 June 1985) for the two Ohakea hangars and \$307,000 (= \$420,000 June 1985) for the two hangars at Whenuapai.

Subsequent re-evaluation of the arch behaviour during final design led to a decision to evaluate the arch using dynamic analysis methods.

Among the considerations leading to this decision was the recognition that, because of the geometry, lateral loading of the arch produces largest components of deflection in the vertical direction (Figure 2). This differs considerably from the situation for a normal building on which the equivalent static load procedure is based, where deflections are principally parallel to the direction of loading [2]. Also, the displacements under lateral loading are quite large and the arch geometry and vertical loading are such that geometric non linearity, and particularly P-delta effects can significantly influence the arch behaviour. Under these conditions, the code equivalent static load procedure was considered to be inapplicable.

The deflected shape taken up by the arches under lateral loading, together with the magnitude of the deflections, suggested that it may be possible to directly restrain the ribs with crossed tension strands anchored against the arch abutments (Figure 3). This alternative strengthening scheme was also evaluated using the dynamic analysis program and its performance compared with the plate strengthening scheme of the initial study. These analyses confirmed the effectiveness of the crossed strands and it was subsequently established that they would cost only a fraction (approximately 10%) of the plate strengthening.

This article covers aspects of the special study made to evaluate both the behaviour of the existing arch roofs and the effect of the proposed strengthening under the application of dynamic earthquake loading. For brevity, only the internal ribs are considered here.

COMPUTER MODEL

The arches in the transverse direction of the hangars were modelled as two-dimensional

* Ministry of Works and Development Head Office Structural Design, Wellington.

FIGURE 1: HANGAR DETAILS.

FIGURE 2: DEFLECTION OF ARCH RIBS UNDER LATERAL LOADING.

**FIGURE 3: PROPOSED ALTERNATIVE STRENGTHENING
CROSSED PRESTRESSING STRANDS.**

FIGURE 4: DRAIN 2D STRUCTURE IDEALIZATION.

plane frames (Figure 4) and analysed using the DRAIN 2D dynamic analysis program [3]. Specific model and analysis details are as follows:

- Approximately 3½% critical viscous damping was assumed for the first 2-3 modes of response. This value is lower than is often used in this type of dynamic analysis (i.e. typically 5% critical damping) but was chosen because of the absence of significant non-structural components as compared with the situation for a normal building. (Note however, that the spectral displacements in Figure 5 are given for 5% damping as values for 3½% damping are not generally available).
- A bilinear approximation to the inelastic moment-curvature relationship for the arch segments was assumed for cases where yielding was modelled. In practice, where yielding did occur, the magnitude of plastic deformations was quite small and the form of the model was therefore probably not critical.
- Analyses were carried out for the arches with initial elastic properties ranging from the uncracked to the fully cracked state. The corresponding natural periods estimated for the arches ranged from approximately 2 seconds to 6+ seconds respectively.
- A range of different soil stiffness coefficients was evaluated. However, in all cases the side frame deflections were small compared to the arch deflections which were virtually unaffected for the range of soil stiffnesses considered.
- Two analyses were carried out with the arches subjected to the vertical component of the 1940 El Centro earthquake. The results from these indicated that this component had little effect on the overall arch response. Comparisons of the response spectrum for this motion with the vertical component spectra for other earthquakes indicated that similar results would hold for those cases also. However, in these evaluations it was only possible to consider identical seismic input at the two ends of the arches.

The effect of different (but related) vertical components of shaking at each end could not be evaluated. In all other analyses, only lateral seismic loading was applied.

- Geometric non linearities were modelled by allowance for P-delta effects only. Consideration of all large displacement effects would have been desirable for evaluating the behaviour of the arches at onset of collapse, but this capability was not available.
- One analysis of the arches was also carried out using STRUDL DYNAL [4], which is capable only of linear elastic dynamic analyses. The results were compared with a DRAIN 2D analysis in which the P-delta influence was not modelled.

SITE GEOLOGY AND SELECTION OF EARTHQUAKE RECORDS

Primary attention at this stage has been focussed on the Ohakea hangars which are scheduled for strengthening this year (1986). The site in the vicinity of these hangars is flat river terrace terrain, with a thin layer of loess overlying 4 m to 10 m of river gravels and sand. Several thousand metres of comparatively weak marine mudstone underlie these layers [5,6,7]. The south bank of the Rangitikei River is about 1 km north of the site. This area has been subjected to considerable recent seismic activity and is considered by some authorities to be one of the highest seismic risk parts of the country [8]. Two major faults are in close proximity to the site; the Rangitikei fault passing 1.5 km to the north and the Rauoterangi fault terminating 14 km to the SE. The NZ Geological Maps for the area [5,6] class both these faults as active, i.e. movement in the last 500,000 years. The Wellington (-Tuahine, -Mohaka) fault (Class I active) passes approximately 40 km to the SE.

Although several investigations of regional seismicity in New Zealand have been made recently [8-11], there is little firm data qualifying levels of seismicity appropriate to structures with natural

periods as long as those of the arches, i.e. 2 to 6+ seconds. The approach taken was therefore to use a selection of earthquakes with spectral intensities which were among the largest recorded over this period range. Those selected for the majority of analyses were the El Centro 1940 earthquake NS and EW components, the Parkfield 1966 Array No. 2 record, and the Holiday Inn (Orien Blvd) NS and EW records from the 1971 San Fernando earthquake. The spectral displacements for these records are shown in Figure 5, together with several other records exhibiting strong long period shaking. (The 1985 Mexican earthquake occurred after the analysis had been completed. Neither the records from this earthquake, nor the 1971 Pacoima Dam record were considered appropriate to the hangars because of the specific geological conditions at the sites where the recordings were made).

The five records used were all recorded at "alluvium" sites in zones of moderate to strong shaking intensity (up to MM9 assigned). For analysing the hangars, these records were scaled up by a factor of 1.3. This was done to account for the $R = 1.3$ status required by the code for Defence establishments [1].

While there is some argument about the applicability of Californian earthquake records to the New Zealand situation, there are still too few records available to convincingly justify use of smaller spectral intensities, or even to assume that larger intensities may not be applicable at the long periods of interest in this study. A number of recorded earthquakes exhibit magnitudes of spectral displacements similar to those of the records used and some of these are from zones of

**FIGURE 5: DISPLACEMENT RESPONSE SPECTRA
0 TO 6 SECOND PERIODS.**

comparatively moderate felt intensities. For example, the Bank of America NS component of the 1971 San Fernando earthquake (Figure 5) has spectral displacements in excess of 500 mm around the 3.4 second period range (5% damping), yet was recorded in a zone assessed as MM7 intensity shaking. Also, none of the three earthquakes used for analyses were particularly major events, with Richter magnitudes ranking from only 5.5 to 6.6.

ANALYSIS RESULTS

Results from the dynamic analyses of the arch ribs as existing and as strengthened are summarised in Tables 1 and 2 respectively. The values of maximum displacement given in Tables 1 and 2 were obtained directly from the DRAIN 2D output whereas the periods were estimated from the plotted displacement-time histories. In some cases the response "period" tended to be somewhat variable and no single value could be determined (Figure 6).

Existing Arches

Probably the most important features emerging from the analyses are the trends exhibited by the response periods with changes in the section properties. As indicated in Figure 7, the periods predicted for the unstrengthened arches begin to increase sharply as the arch sections approach the fully cracked state.

(a) EXISTING ARCH UNDER EL CENTRO 1940 ($E_{I_{cr}}$).

(b) STRENGTHENED ARCH UNDER HOLIDAY INN, NS ($E_{I_{cr}}$).

FIGURE 6. EXAMPLE DISPLACEMENT - TIME HISTORIES PREDICTED.

In some of the earlier analyses that were done, the cracked section rigidities were underestimated by 20% and the program actually predicted collapse of the arches ($T \rightarrow \infty$). In these cases the arches became "P Δ driven" and the displacements began increasing at an accelerating rate independently of the seismic excitation.

The point at which the arches are predicted to become unstable corresponds quite closely with the Euler buckling load estimated for an equivalent pin ended straight column*, i.e. column axial load and section properties equal to those at approximately the quarter points of the arches and length equal to the arch half length. This is shown in Figure 7 at the point where the column Euler load (P_{crit}) equals the maximum compression at the arch quarter point (node 14).

(* Note that although the arch is curved the axial compression induced by gravity loading approximately follows the arch centroid).

Stabilised Arches

The effectiveness of the crossed strands in improving the elastic stability of the arches is indicated in Figure 7 by the significant reduction in response period for these cases as compared with the existing arches. The use of two strands in each direction further improved the arch stability, but this was not considered necessary. Figure 7 also shows that the reduction in period for the plate strengthening was similar to that achieved with two strands in each direction.

As indicated in Tables 1 and 2, the predicted maximum displacements did not vary consistently with either arch stiffness or amount of strengthening provided, and were therefore not a direct indicator of elastic stability. For example, the maximum displacement for the case with twin strands in each direction (Table 2.10) was over 70% larger than that for the corresponding analysis with single cables. (Table 2.3). The reason for this is that the maximum displacements varied with arch period according to the corresponding spectral displacement (S_d) of the particular record used, generally being between 50% and 70% of S_d . Over the period range of interest (i.e. approximately 2 to 6 seconds) the pattern of S_d values differed considerably between records, though the maximum values for all records used were reasonably constant, e.g. generally between 400 mm and 500 mm for 5% damping (Figure 5). As discussed in the following sections, however, the maximum displacements are of relevance to the level of cracking and to the inelastic stability of the arches.

Arch Moments

Typical peak moment distributions predicted for the arch ribs are shown in Figure 8, together with approximate values for the arch flexural strengths at different sections. This shows that the moments predicted generally increased with increases in the modelled section rigidity,

TABLE 1 : DRAIN 2D ANALYSIS RESULTS FOR EXISTING ARCH

Earthquake Record (All records x 1.3)	Section Rigidity	Estimated Period (seconds)	Maximum Displacement (mm)	Spectral Displacement (mm)
<u>Arch modelled elastically</u>				
1 El Centro NS	EI_g	2.3	210	352
2 Parkfield	EI_g	2.3	300	496
3 Parkfield	$0.7 EI_g$	3.1	317	546
4 Parkfield	$0.4 EI_g$	6	210	413
5 El Centro NS	EI_{cr}^*	32	290	-
6 Parkfield	EI_{cr}^*	v.large	280	-
7 El Centro NS	EI_{cr}^o	collapsed	20 m >	-
8 Parkfield	EI_{cr}^o	collapsed	10 m >	-
<u>Bilinear Arch Response</u>				
9 El Centro NS	EI_{cr}	6-8	187	290 ?
10 El Centro EW	EI_{cr}	7	408 y	835 ?
11 Holiday Inn NS	EI_{cr}	?	184	?
<u>No Allowance for P-delta</u>				
12 El Centro NS	EI_g	1.9	150	215
13 ditto (STRUDYL DYNAL)	EI_g	1.99	156	
14 Parkfield	EI_{cr}^o	3.45	335	542

Notes to Tables 1 and 2

Section rigidities:

 EI_g = gross EI for steel and concrete EI_{cr}^o = initial estimate for EI for fully cracked section EI_{cr}^* = EI_{cr}^o except for end and central segments which were assumed uncracked. EI_{cr} = corrected value of EI for fully cracked section, 20% - 25% greater than EI_{cr}^o

> = displacement increasing at an accelerating rate at the end of analysis

y = critical sections just yielded. No yielding predicted in other cases where modelled.

TABLE 2 : DRAIN 2D ANALYSIS FOR STRENGTHENED ARCH

Earthquake Record (All records x 1.3)	Section Rigidity	Estimated Period (seconds)	Maximum Displacement (mm)	Spectral Displacement (mm)
<u>2-single strands - elastic response</u>				
1 El Centro NS	$0.7 EI_g$	2.3	246	352
2 El Centro NS	$0.4 EI_g$	3.0	236	399
3 El Centro NS	EI_{cr}^*	3.4	150	325
4 El Centro EW	EI_{cr}^*	4.2	330	530
5 Holiday Inn NS	EI_{cr}^*	4.4	460	780
<u>2-single strands - yielding modelled</u>				
6 El Centro NS	EI_{cr}	3.25	178	342
7 Parkfield	EI_{cr}	3.2 ?	368 y	546
8 Holiday Inn EW	EI_{cr}	3.3-3.8	298	465-676
9 Holiday Inn NS	EI_{cr}	4.2	322	660
<u>2-double strands</u>				
10 El Centro NS	EI_{cr}^*	2.75	267	420
<u>600 x 10 F_L fixed top and bottom of L arch rib</u>				
11 El Centro NS	EI_{cr}^o + plates	2.5	246	350

Drain 2D:

- Arch response.
- 2 pair 12.7mm dia. strands.
- Plate strengthening alternative.
- E_{cr}^0 Initial estimate of E_{cr} .
- E_{cr}^P $E_{cr}^0 + EI$ (steel plates).

**FIGURE 7: EFFECT OF EI AND STRENGTHENING
ON RESPONSE PERIOD.**

although the maximum displacements are also of significance. In particular, significant yielding was predicted only for analyses in which a value of 0.7 or 1.0 times the uncracked section rigidity was assumed. These results are however, misleading as the arches will crack and the section rigidities reduce significantly before yield is reached. The maximum displacements predicted in these analyses were not large enough to induce yielding in the arches.

VERIFICATION OF RESULTS AND EVALUATION OF BEHAVIOUR

The results predicted in the preceding section indicate a potential for elastic instability of the arches and that either of the two strengthening proposals would significantly reduce this risk. Independent verification of these results was even more important than usual, particularly due to lack of experience in using the DRAIN 2D program under conditions where axial load

FIGURE 8: PREDICTED BENDING MOMENTS

effects had a dominant influence on the response. Therefore, in addition to normal checks on the results, a number of simple analyses and correlations with theory were made to check the performance of the programme. These included:

- Spectral displacements: Spot checks were carried out to confirm that the program accurately predicted published values of spectral displacement (S_d) for two earthquake records.
- Effect of P-delta: As indicated in Table 1 and Figure 7, the main effect of modelling P-delta in the analyses was to significantly increase the response periods of the arches. To check the validity of these results, the program was used to analyse an "equivalent" straight column and the predicted response periods for different values of EI were compared with the corresponding analytic solutions (Figure 9). As indicated, generally very good agreement was obtained between the predicted and analytic periods. There was also surprisingly close agreement between the column periods and those predicted for the unstrengthened arches at approximately equivalent values of EI. These results indicate that the program does adequately model the effect of the axial loads under the conditions applying in these analyses, at least prior to the point of instability being reached.
- Maximum displacement of arch: As noted previously, the maximum predicted displacements of the arches were typically 50% to 70% of the corresponding S_d values. This is very different from the situation for most buildings where

the maximum roof deflections (elastic response) would normally be about $1.3S_d$. An approximate model response evaluation was made. This demonstrated that the difference arises because of the components of arch displacement perpendicular to the direction of loading and confirmed that the relationships predicted for the arches were in the range expected. (Refer Appendix A). This comparison also highlighted the need to consider the applicability of the code procedure to special structures. In this instance, the seismic effects are in fact less than suggested by the code.

The magnitude of displacement is also of importance to the inelastic stability of the arches. Instability develops in this case when the P. Δ moment exceeds the maximum internal restoring moment, i.e. from Figure 10, instability develops when

$$P.\Delta \geq M_{dep}$$

and hence

$$\Delta_{crit} = M_{dep}/P$$

For the internal ribs this gives Δ_{crit} of approximately 600 mm in their unstrengthened state and approximately 1200 mm for the plate strengthening scheme. The strengthening proposal using prestressing strand cannot be assessed in the same manner. However, at 600 mm displacement (i.e. when $P.\Delta \approx M_{dep}$) the restoring force in the strand would be approximately 160 kN, which is about 10% of the weight of the arch and tributary area of the roof.

Comparison with the spectral displacements

in Figure 5 (and noting $\Delta_{\max} \approx 0.6S_d$) suggests that only the shaking recorded in the 1985 Mexican earthquake and possibly at Pacoima Dam would have been strong enough to have initiated inelastic instability in the unstrengthened structures. Consideration of an $R = 1.3$ factor and deterioration of the concrete etc. reduces the margin of safety, although the structures could still possibly be satisfactory from this point of view, providing it could be established that the effect of the vertical components of shaking (i.e. different at each end) was not significant. Either of the strengthening schemes proposed would, however, considerably decrease the risk of inelastic instability.

The load-extension characteristics of typical 12.7 mm diameter prestressing strand are given in Table 3. This indicates that over a length of 43 m, the strands will stretch 300 mm-400 mm before they start to become non linear (elastic limit) and a maximum of 1500 mm to 1900 mm before fracture. Only in one of the analyses carried out (1.3 x Holiday Inn NS, Table 2.5) were the prestressing strands stretched beyond their elastic limit, and then only by a comparatively small amount.

One aspect not modelled in the analyses was the varying extent of crack opening. The model used assumed linear elastic response up to yield. However, the real behaviour is more complex, with the extent of crack opening and hence effective section rigidity varying in time with the magnitude of curvature. The effect of this on the arch stability and response period depends on the maximum displacements imposed, although to what extent is not known. Without a more accurate model of section behaviour, little can be done other than to assume the fully cracked section rigidity (EI_{cr}) as a worst case.

Although the computed values of EI_{cr} were 20%-25% larger than the predicted critical values at instability, it was considered that this provided insufficient margin to cover deficiencies such as construction defects, concrete degradation and spalling, model limitations of the program and failure to model different vertical excitations at each end of the arches.

FIGURE 9: COMPARISON BETWEEN DRAIN 2D AND THEORETICAL PERIODS FOR AN EQUIVALENT STRAIGHT COLUMN.

Arch becomes unstable at $P \cdot \Delta = M_{dep}$
i.e. zero residual moment capacity at (a)

FIGURE 10: INELASTIC INSTABILITY DUE TO P-Δ

TABLE 3 : PROPERTIES OF 12.7 MM DIAMETER PRESTRESSING STRAND
(Ex Shinko Wire Co. Ltd)

Type of Strand	80% of load at 1% extn (kN) *	Breaking Load (kN)	Minimum Extn (BSS)	Minimum elongation of 43 m long cable (mm)
Grade 270	125	184	3.5%	1500 mm
Compact	145	209	4.0%	1720 mm
Galvanised	102	170	4.5%	1930 mm

* Approximate load at which the load-extension relationship becomes non linear.

SUMMARY AND DISCUSSION

Evaluation of the behaviour of the arches indicated that the greatest threat to the hangars in an earthquake is probably from elastic instability. The risk in this case is dependent on the extent of cracking and does not necessarily require very large displacements to be imposed. Extensive cracking develops at smaller displacements than are required to reach yield. Although the computed stiffness for the arches in a cracked state was roughly 25% larger than the critical value, this was not considered an adequate margin to cover possible deficiencies or unknowns with respect to the arch geometry, the applied loading or the analysis. Even small reductions in the stiffness below that computed would cause large increases in the response period.

Safety margins on strength (i.e. probable/dependable) of up to 1.65 are required in the NZ Concrete Design Code [12] but, as is generally the case with other material codes, no margin on stiffness is considered. This is appropriate when failure is a result of inadequate strength. In the case of the concrete hangars, however, the potential failure is due to instability which is stiffness related. In view of the consequences of instability developing, an adequate margin against this type of failure was considered essential.

Both strengthening options considered were effective in stabilising the arches. However, the proposed alternative scheme with cross bracing provided by prestressing strand was costed at only \$50,000 for the two hangars, which is less than one tenth the cost of the steel plate strengthening scheme (June 1985 ROC values). This figure should also be compared with the value of contents of the two hangars which is in the order of \$100m.

Consideration of the extension characteristics indicates that the strands should remain elastic in most conceivable "design events" and that displacements several times larger than those imposed in the present analyses would be required before the strands fractured.

This study illustrates a design situation

where the adequacy of a structure to resist loading would have been very difficult to evaluate without recourse to and confidence in dynamic analyses of the structural behaviour. This applies particularly to evaluation of the effectiveness of the prestressing strands in stabilising the arches. However, the importance of independent verification of results in this type of analysis must not be overlooked.

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REFERENCES

1. NZS 4203: "Code of Practice for General Structural Design and Design Loadings for Buildings", SANZ, 1st Ed. 1976.
2. Clough, R.W. and Penzien, J., "Dynamics of Structures", Intl. Students Edition, McGraw-Hill Kogakusha Ltd, Tokyo.
3. Kanaan, A. and Powell, G.H., "General Purpose Computer Program for Inelastic Dynamic Response of Plane Structures", EERC 73-6, University of California, Berkeley, California, 1973.
4. STRUDL User Manual, McDonnell Douglas, Saint Luis, Mo., April 1985.
5. Lensen, G.J., "Geological Map of NZ 1: 250,000, Sheet 11, Dannevirke, 1st Ed. 1962, DSIR, Wellington, NZ.
6. Mansar, G., New Zealand Geological Survey, DSIR, Lower Hutt, NZ, Personal Communication.
7. Smith, W.D. and Berryman, K.R., "Revised Estimates of Earthquake Hazard in New Zealand", Bull. NZNSEE, Vol. 16, No. 4, 1983, pp.259-272.
8. Peek, R., Berrill, J.B. and Davis, R.O., "A Seismicity Model for New Zealand", Bull. NZNSEE, Vol. 13, No. 4, 1980, pp.355-364.

10. Mulholland, W.M., "Estimation of Design Earthquake Motion for New Zealand", Report No. 82/9, Dept. of Civil Engineering, University of Canterbury, NZ, 1982.
11. Berrill, J.B., "Seismic Hazard Analysis and Design Loads", Bull. of the NZNSEE, Vol. 18, No. 2, June 1985.
12. NZS 3101:1982, New Zealand Standard Code of Practice for Design of Concrete Structures, Parts 1 and 2, Standards Association of New Zealand, Wellington, 1982.

APPENDIX A : RELATIONSHIP BETWEEN MAXIMUM AND SPECTRAL DISPLACEMENTS : MODAL ANALYSIS EVALUATION

In a modal analysis, the maximum displacements of an elastic multi-degree of freedom structure subject to earthquake loading are given by [2]:

$$V_i = \alpha_i \psi_i S_d \quad (A.1)$$

where i denotes the i th mode, V is the vector of maximum modal displacements, ψ is the mode shape vector, S_d is the spectral displacement of the earthquake corresponding to the period of the i th mode, and α is the modal participation factor given by

$$\alpha_i = \frac{\sum_j m_j \psi_{ij} r_j}{\left(\sum_j m_j \psi_{ij}^2 \right)} \quad (A.2)$$

where m_j is the mass associated with the j th degree of freedom and r is a direction vector whose components equal 1 for displacements parallel to the loading and 0 for displacements perpendicular to the loading.

For say a cantilever structure such as shown in Figure A.1, $r = (1,1,1,1,1)$ since all displacements considered are parallel to the direction of loading. Therefore, for the mode shape shown in Figure A.1,

$$\alpha_1 = \frac{2.343}{1.686} = 1.39$$

i.e. the maximum displacement at the top of the cantilever is $1.39 S_d$.

A coarse approximation of the arches is shown in Figure A.2. For simplicity equal masses have been assumed at each node and the first node shape has been approximated to the maximum displacement computed for an analysis of the arch subjected to the NS x components of the 1977 Vrancea earthquake. The vectors ψ and r are shown in Figure A.2. Following the same steps as for the cantilever leads to

	ψ_i	$V_i = \alpha_i \psi_i S_d$
	1.00	1.39 S_d
	0.69	0.96 S_d
	0.41	0.57 S_d
	0.19	0.26 S_d
	0.05	0.07 S_d
	0.0	0.0

FIGURE A.1: EXAMPLE FIRST MODE SHAPE FOR CANTILEVER.

Note: r given for horizontal components of earthquake loading.

FIGURE A.2: APPROXIMATE FIRST MODE SHAPE FOR HANGER ARCHES.

$$\alpha_1 = 444/229376 = 0.0019357$$

This results in maximum displacements of $V_{1,max} = \alpha_1 \psi_{1,max} S_d$

$$= 0.0019357 \times 311 S_d = 0.60 S_d \text{ in the vertical direction}$$

$$\text{and } = 0.0019357 \times 172 S_d = 0.33 S_d \text{ in the horizontal direction.}$$

These agree well with the output from the DRAIN 2D analyses. The difference as compared with the cantilever case is due to the large displacements perpendicular to the direction of loading. In equation A.2, these contribute to the denominator, but not to the numerator, thereby reducing the value of α .