DISCUSSION

"A CONSIDERATION OF P-DELTA EFFECTS IN DUCTILE REINFORCED CONCRETE FRAMES" - T. PAULAY
BNZNSEE VOL. 11, No. 3, 1978

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Professor Paulay's thorough and helpful summary of recent P-delta literature is the basis of his proposal for a design procedure, one of two methods for unbraced frames which appear in the draft replacement for NZS 3101P. The other method, a copy of the moment augmentation procedure of ACI 318, deals only with elastic and creep deformation so is inappropriate and should be removed from the proposed standard.

Last year I suggested flexibility limitations for controlling P-delta so that its effects would be small enough to ignore. Simple controls for the purpose can be devised within the framework of our loadings code because our base shear spectra do not have coefficients reducing as period increases at low frequencies. My paper showed that suitable controls differed so little from those already applied for other purposes that they would be acceptable. Flexibility limitation has simplicity as a significant advantage over alternatives. If it has no faults not shared by alternatives then I believe it should be preferred.

Professor Paulay's principal objection, perhaps his sole objection, appears to be that he thinks it is too difficult to quantify drift. This, it seems to me, is equally an objection to his own proposal, as it must be to any other which seeks to account for or to limit effects generated by the interaction between gravity load and lateral deflection. I do not fully understand his reason for offering the generalisation, apparently bearing on this matter, that stiffening, to be effective, must be 'radical". Presumably this is meant to suggest that stiffness is not such an important parameter that P-delta will vary approximately reciprocally with it as a first order appraisal ignoring the acceleration response increment which might be produced by a stiffness increment would indicate. A Powell and Row numerical analysis result cites is said to have shown that stiffening which might have been expected to reduce P-delta by about 60% effected a reduction of only 40%. But Professor Paulay's method features the very reciprocity that the result appears to have been quoted to discredit. Again an objection to my proposal has equal force when offered against his. Moreover, I think the single not especially anomalous result is scarecely sufficient evidence for the generalisation.

The amplification factor in Professor Paulay's proposal has first order accuracy as have some others of the same general type that have been devised. But these others are intended, in the main, for elastic structures and contemplate much smaller values of stability coefficient. It is reasonable to ignore higher order terms by substituting

$$1 + Q_r$$

for

$$1 + Q_r + Q_r^2 + Q_r^3 + \ldots\ldots \frac{1}{1 - Q_r}$$

when the relative error, which is seen to be Q_r^2, is small as it is for ranges of stability coefficient like that of McGregor and Hage, .0475 to .2. Professor Paulay's range has a lower limit of .15. When there is such a convenient summation available as there is for the amplification factor's geometric series, there is no computational advantage to be had from the approximation and no adequate excuse for using it in an inappropriate range of stability coefficients.

In establishing the stability coefficient the paper accounts for the displacement factor set out in Table 10 of NZS 4203; but the accompanying warning that "for structures dissipating energy in a ductile flexural mode the separation requirement of this standard gives average damage protection to a class III building with 5% damping in seismic zone A at levels of motion up to one third El Centro N-S only" is unheeded. The warning is surely an acknowledgement by those who wrote the code that the provision it contains accounts very inadequately for drift and that that may or may not be tolerable for secondary damage control. A different view must be taken of unconservative draft provisions when the strength and stability of entire structures are at stake, especially as the warning suggests that a further multiplier for stability coefficient assessment might be as high as 3.

Some questions about precision of assessment will have to be resolved before the lower limit of Q_r that the paper advocates, 0.15, can be agreed upon. Compare, for example, the deliberately ignored 18% error that is involved in this with the difference between zone A and zone B earthquake simulating load, 17% of zone A load, and it becomes clear that either the limit Q_r will have to be revised downward or the basis of NZS 4203 will have to be reconsidered.

I find some difficulty in understanding why the factor 2 used to ovtain storey drift was derived from the secant between ground and floor 4 in Figure 7. It seems that a factor of at least 3, obtained from detrusion of the bottom storey, ought to have been considered. I am also uneasy about the wisdom of getting so potent a factor from one study result, especially when curves 1 and 2 show that the framing of the subject building was proportioned to have flexural beam rather than the more usual shear beam characteristics. We should also not overlook the possible influence of foundation deformation, which, in the example of Figure 7, is seen from curves 1 and 2 to have been unusually small.

With no further explanations than are in the paper, it is hard to accept the author's invoking of more than the reliable strength as a means of offsetting P-delta. It seems to me that the structure cannot accommodate itself to a designer's thinking along these lines, being unable to differentiate between components of total force and moment all of which are, to some extent at least, self generated. Also, I think the equation of work represented by the triagles in Figure 6 is somewhat too facile an idea to offer much hope. As has been pointed out before (13) there are disturbing aspects to the energy

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dissipation characteristics of a P-delta affected system. For these reasons my interpretation has, in this comment, used $1 + Q_r$ rather than $1.1 + Q_r$ as the first order approximation to the magnifier, to be used with reliable rather than ideal strength in design. I think the author must give more developed support for his proposal than the paper contains to begin to make it convincing.

Finally, it seems to me that buildings at large are not of such a uniform nature that a generalisation to the effect that P-delta increments need not be considered for upper stories can possibly be justified.

ADDITIONAL REFERENCE

13. Discussion on Reference 4, Bull. N.Z.N.S.E.E., Vol. 11, No. 1, 1978.

T. PAULAY replies

The author is grateful for the detailed comments of Mr L. A. Andrews, many of which he finds himself in agreement with. Because the problem is one which, in the author's opinion, has so far eluded a satisfactory solution, his paper was submitted with the hope that, apart from outlining the major issues involved, it might solicit contributions from readers. It was disappointing that at this time of considerable activity in earthquake engineering in New Zealand, mainly in various committees of the Society and the Standards Association, only Mr Andrews has offered to contribute to the solution of these issues.

The summary of the literature on P-delta effect was intended to review the state of the art in current design procedures, rather than to form the basis for the proposals subsequently made. The author agrees with Mr Andrews that the moment augmentation procedure of the current ACI Building Code[7], of which no use was made in the paper, is irrelevant to seismic situations.

The flexibility control proposed by Mr Andrews in his paper[4] is indeed simple to apply. If it has no faults not shared by alternatives, then indeed it should be preferred. The author believes, however, that this is not the case because the proposed controls rely on stiffness, a property which has diminishing importance with the dominance of inelastic response. When a state of collapse due to excessive earthquake induced deformations is being approached, only the elasto-plastic dynamic response of the frame is likely to shed some light on the major factors involved.

The author concedes that his concern in the evaluation of P-delta effects is for the reliability in quantifying the drift. As Mr Andrews correctly pointed out, this is a weakness in both proposals, and indeed in any proposal which attempts to predict drift with the use of a chosen standard static load and stiffness properties. It was emphasized that a simple amplification of drifts obtained for an elastic structure, does not satisfactorily predict drifts that arise during major postelastic excursions during the predominantly inelastic response of a frame.

Mr Andrews felt that he did not fully understand the author's reasons for the generalization that stiffening, to be effective, must be "radical". He also stated that the single result mentioned[10] was scarcely sufficient evidence for such generalization. The results quoted were obtained with the use of five earthquake records with different spectral characteristics. In the author's view they offered credible evidence to the effect that, to obtain a reduction in displacement, a radical increase in stiffness is required.

When the response of a frame is predominantly elastic, that is when only limited yielding has occurred, the influence of stiffness can be expected to be significant, but even then a change in stiffness will produce different changes in the predominantly elastic response to different grount motions. During a very large earthquake, when extensive plastification of the frame has occurred, as examples in Fig. A show, the influence of the stiffness on the displacement response of the structure will be insignificant. It is at such critical instants of dynamic response that the consequences of P-delta effect should be examined.

To illustrate this aspect of the discussion further, the example of the 12 storey frame, shown in fig. A, will be used. The deflected shapes, shown for selected instants of the Pacoima Dam earthquake record, were obtained with the use of the elasto-plastic analysis programme developed by Sharpe[15] and these were also confirmed by another analysis[10]. In this analysis P-delta effects were not considered. The envelopes for the corresponding storey drifts for this as well as the El Centro earthquake records are presented in fig. B(b). Subsequently Drs P.J. Moss and A.J. Carr of the University of Canterbury found that if, in using the Pacoima Dam record, allowance is made in the analysis for P-delta effects, the maximum drift in the second storey increased from 80 mm to about 130 mm. A 25% increase in stiffness reduced this maximum drift by 8% but it did not affect the drift elsewhere in the structure. A 25% increase of strength resulted in significant reduction of drift throughout the structure with the reduction of the critical drift being 26%.

Demand for inelastic deformations in the 12 storey example frame was much less during the El Centro excitation and, as expected, the consideration of P-delta effect did not affect significantly the response of the structure. The envelope for drifts remained essentially non-critical as shown in fig. B(b), with some reduction in the drift of the lower storeys when P-delta contributions were considered.

Fig. C shows the result of a recent study[16] for a one storey frame, where among other aspects the maximum drift-lateral load carrying capacity relationship was studied. The idealized frame was assumed to possess strengths corresponding with a base shear coefficient from 5% to 40%, while its flexibility, in terms of lateral yield displacement per storey height, was assumed to be 1/400 or 1/200 or 1/133. It is seen that in frames with a base shear capacity of more than 20% of the building weight, only moderate inelastic deformations, corresponding with a displacement ductility factor of less than 2, occurred. It may also be seen that the drift was roughly proportional to the flexibility F_s. However, for warker frames, designed with abase shear coefficient of less than 0.15, in which inelastic drift becomes dominant, the flexibility F_s has had little bearing on the response. If anything, a reverse trend is apparent, that is the most flexible frame has the smallest total drift and displacement ductility

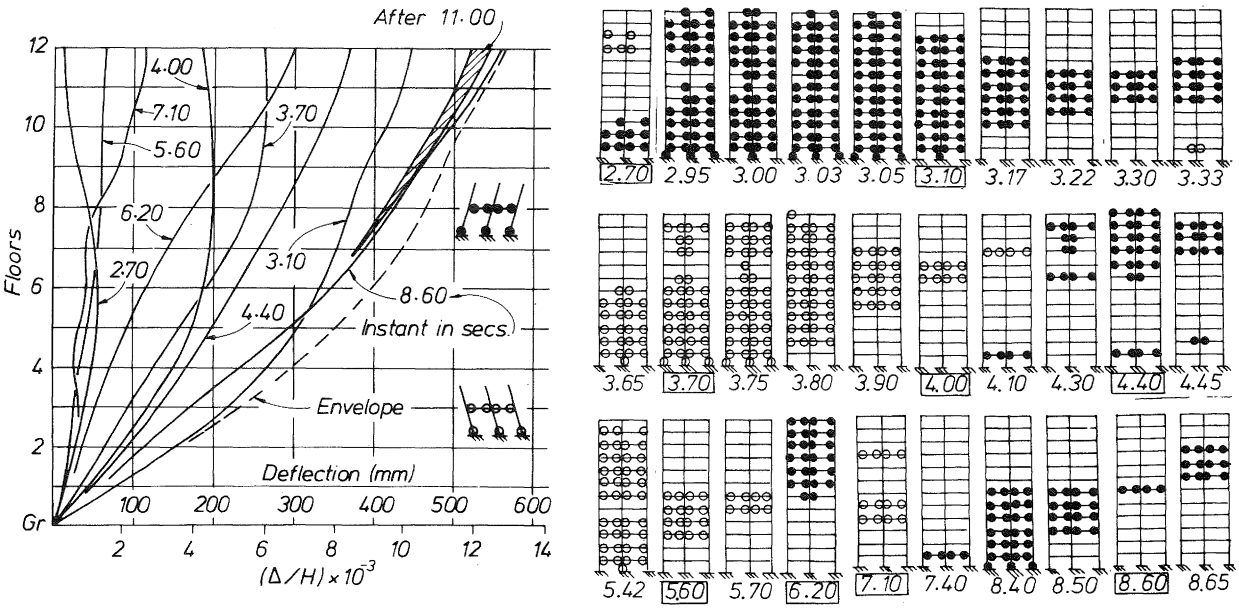

Fig. A - Deflection and Hinge Formation in a 12 Storey Frame during Instants of the 1971 Pacoima Dam Earthquake (14).

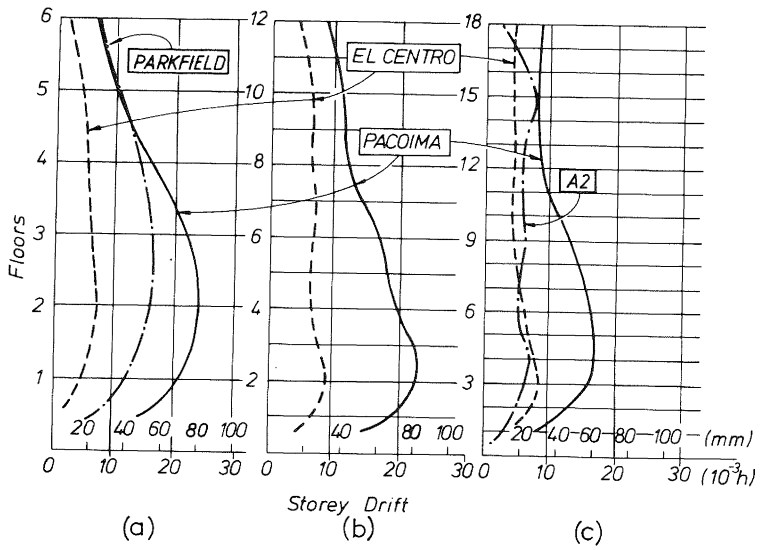

Fig. B - Envelopes of Storey Drifts computed for different Earthquake Records for a (a) six (b) twelve and (c) eighteen storey frame (14).

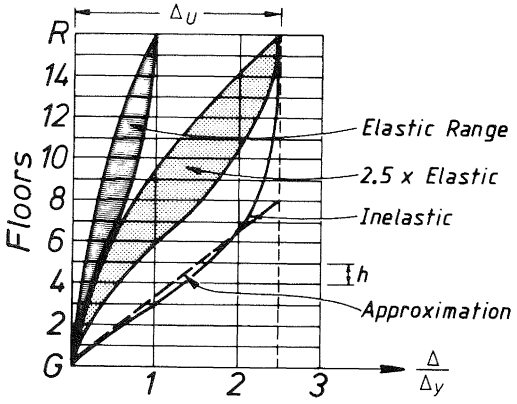

Fig. D - Comparison of a Range of deflected shapes of Elastic and Inelastic Frames.

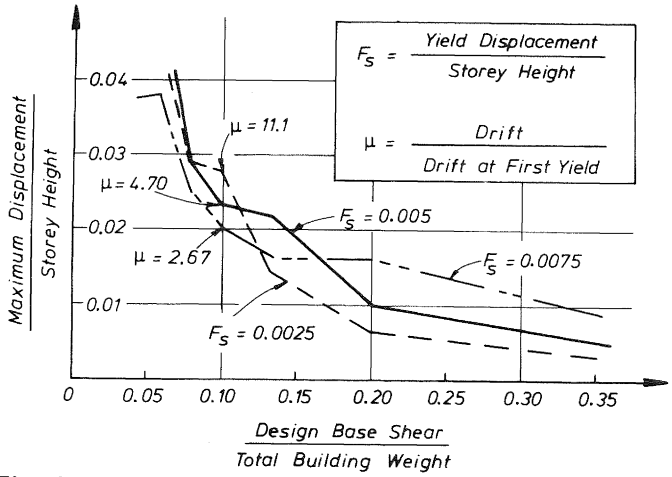

Fig. C - The Effect of Storey Flexibility on the Maximum Drift for a one-storey Frame with different Lateral Load Carrying Capacities, subjected to the Taft Earthquake (16)

demand. This is only an example and the results should not be taken as necessarily representative for other earthquake records or for multistorey frames. However, the example is considered to be valid enough to indicate that if a remedy is to be established for reducing drift in multistorey frames, with base shear coefficients[5] typically in the range of 0.05 to 0.10, this is more likely to be found in increasing strength than in reducing flexibility.

In a similar study of Powell and Row[10], commented on by Mr Andrews, the stiffness of a prototype 10 storey frame was increased by 180%. This necessitated an increase of the strength of the structure by 13%. The combined effect of radical stiffness and modest strength increase resulted in only 40% reduction in drift for the frame in which the ductility demands for the selected earthquake records were moderate.

The author agrees with another point raised by Mr Andrews, that for elastic structures the amplification of drift should take into account the effect of P-delta actions on increasing further drift, in the traditional form of a series. Because of the critical primary drift, the subject of this discussion, is mainly due to inelastic deformations, the precepts of the classical amplification are invalidated. In a prefectly plastic mechanism, such as a set of completely hinging beams in several adjacent floors, the lateral load, irrespective of whether it is accelerations and P-delta generated, cannot be increased. Therefore P-delta moments cannot be used to predict further drifts generated by them. Moreover in a perfectly elastic-plastic mechanism the stiffness vanishes and hence no relationship exists between the drift and its causes. Therefore the higher order terms of Q_r are inappropriate in this situation. The stability coefficient used in the paper was for the purposes of amplifying strengths and not drifts. It should be noted, however, that very significant reduction in inelastic displacements, such as drifts, can be expected if the structure posses some post yielding stiffness[11]. This is typically 1% to 10% of that associated with the initial linear elastic response.

While writing the paper the warning of NZS 4203[5] with respect to its drift magnification was indeed disregarded. The background to the recommendations, whereby 2.5 times the elastic drift due to the specified lateral static loading [5] should not exceed 0.01 times the storey height, could not be located. Instead more confidence was placed in more recently developed inelastic dynamic time history analyses techniques [10,11,15], which indicated that drifts in relatively flexible multistorey frames did not exceed the limit of 0.01 times storey height during the full El Centro 1940 N-S earthquake record. Typical drift envelopes[14] for 6, 12 and 18 storey frames for Class III buildings located in Zone A, are presented in fig. B.

It was suggested in the paper that an acceptable estimate for the drift in the lower storeys of a frame, during a critical instant of its inelastic response, may be made by a suitable magnification of the average slope. It was pointed out that the deflected shapes during elastic or inelastic response may be very different and fig. 7 was intended to illustrate this point. It showed typical curves rather than a specific case, which seems to have been Mr Andrews' interpretation. To clarify this aspect fig. D is presented in which, instead of one deflected shape, a band of deflected shapes for elastic frames are shown. To illustrate further how inelastic drifts tend to increase particularly in the lower rather than in the upper half of a frame, the computed deflections of a 12 storey frame at various instants of the Pacoima Dam ground motions are presented in fig. A. To simplify the procedure for design office use, in cases when increase of strength to compensate for P-delta loading might be necessary, it was proposed that only the lower half of a frame, where both P and delta assume maximum values, should be considered. In this the fact that in upper storeys the gravity load demands are likely to lead to strength in excess of that required for the specified lateral loading, was also considered. In a more recent study[16] it was claimed that P-delta effects in the upper half of a frame were never critical. The author reaffirms his view that in the upper half of frames without unusual features, the P-delta effects, in terms of the inelastic response of the structure to the largest expected earthquake, are not likely to be critical design quantities.

The author does not disagree with the suggestion of Mr Andrews, that the storey shear capacity, accommodating also the estimated P-delta demands, as given by

$$\Sigma M_i > \Sigma M_e/\phi + Q_r \Sigma M_e = \Sigma M_e (1.1 + Q_R) \qquad (5)$$

should perhaps be recast, so that the estimated P-delta load demand is also met by dependable rather than ideal strength of the beams, in this form

$$\Sigma M_i > \Sigma M_e/\phi + Q_R \Sigma M_e/\phi = 1.1 \Sigma M_e (1 + Q_R) \qquad (5a)$$

Taking a rather serious case when $Q_R = 0.25$, Eq. (5a), as proposed by Mr Andrews, will require strengths 1.9% in excess of that given by the author's Eq. (5). It was felt that in view of the crudeness in assumptions, especially in estimating drift, differences of this order do not deserve attention. Also Eq. (5) reflects recent proposals[13], whereby strength reduction factors should not need to be used (i.e. $\phi = 1.0$) whenever loading on a member is derived from conditions corresponding with the largest anticipated inelastic deformations i.e. when capacity design procedures are relevant.

It seems appropriate to recall the two questions which were posed at the beginning of the paper:

(1) Are secondary moments due to P-delta effects critical in seismic design, and if so, when are they critical?

(2) Is the remedy to be found in increased lateral stiffness or in added strength to compensate for loss in lateral load carrying capacity and in energy dissipation?

In attempting to answer these questions the following points are made.

(a) Continuing studies by Drs A.J. Carr and P.J. Moss at the University of Canterbury and one recent work in Canada[16] indicate that P-delta effects have negligible influence on the elasto-plastic response of frames provided that inelastic excursions are small. For example Class III frames [5] located in

Zone A and designed in accordance with recently developed capacity design procedures[17], were found to be unaffected by P-delta actions during the El Centro 1940 N-S excitation. Only moderate plastic hinge rotations in all beams were observed during the analyses[14].

(b) When inelastic deformations become large, which was the case when the prototype frames referred to in fig. A were analyzed for the Pacoima Dam earthquake record, the drift response was increased by more that 50% when P-delta contributions were also considered. Corresponding increases occurred in the ductility demands in plastic hinges of beams and columns. However, there was no change in the pattern of hinge formations throughout the frame and, what is more important, with the exception of the ground floor level column hinges did not develop anywhere. The extremely large plastic hinge rotations, associated with drifts 50% in excess of those seen in fig. B(b), were at the limits of the capabilities of members detailed in accordance with current proposals[17]. Studies by Montgomery[15] showed signs of instability in 5 and 10 storey frames, designed with a base shear coefficient of less than 0.058, during the El Centro earthquake record.

(c) In the assessment of P-delta contribution to the "crawling" type of collapse, illustrated in fig. 4 of the paper, the fact should be considered that many large pulses, causing large inelastic drifts, are involved. The causative earthquake must therefore be both intense and of long duration.

(d) P-delta effects are more likely to be critical in tall frames because of the large mass i.e. large P. These frames have a relatively long fundamental period of vibration, and hence for an earthquake of given duration they will experience lesser numbers of inelastic excursions involving critical storey drifts.

(e) Present loading provisions in New Zealand[5] are known to be rather severe for long period structures, which thus inherently have larger reserve strengths to accommodate also P-delta load demands.

(f) In view of the above points the author would offer the following answers to the previous two questions:

P-delta effects are not likely to be critical for structures in Zone A and designed to NZS 4203 requirements. This is likely to be the case also for frames in Zones B and C. However, further studies of the response of such frames designed to correspondingly lower strength levels, need to be carried out before recommendations for the disregard of P-delta effects in the design can be made.

Should the study of the response of frames to selected ground motions, considered to be representative of the most severe earthquake to be expected in Zones B and C during the probable life of the structure, show that the contribution of P-delta effect could lead to unstable behaviour or excessive ductility demands, then this effect should be considered. The appropriate step to be taken in this case would appear to be the adoption of increased strength requirements rather than flexibility restrictions more severe than those existing[5]. One approach to the estimation of increased strength, that might be required, is that suggested by Eq. (5). Whether this approach or other similar methods, all of which stipulate added strength [2,3,7] rather than stiffness, will give adequate protection again undesirable P-delta effects is still to be confirmed.

(g) Through further studies the approach to the consideration of P-delta effects should be unified because the problem is not unique to reinforced concrete. The findings should then be incorporated in the New Zealand loadings code[5] so that, if it proves to be necessary, steel and reinforced concrete frames can be treated in a similar manner.

ADDITIONAL REFERENCES

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17. DZ 3101 : Part 1 and Part 2, Draft New Zealand Standard Code of Practice for the Design of Concrete Structures, Standards Association of New Zealand, 1978.