AN APPLICATION OF SPECTRAL TECHNIQUES TO INELASTIC SEISMIC RESPONSE PREDICTIONS

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SYNOPSIS

Techniques of seismic load prediction using response spectra have been in general used for many years and the normal mode-response spectrum approach forms the background to a majority of modern earthquake resistant design codes. It is widely appreciated that in the general case of multi-degree of freedom systems, extension of the approach to the prediction of structural response in the post-elastic range is invalid. Consequently analyses based on the system, using a time history of ground acceleration as the excitation, have been developed.

However, some structures may be modelled reasonably accurately by a single degree of freedom system and in these circumstances spectral techniques may be applied justifiably to post-elastic seismic response calculations.

Predictions of the post-elastic response of a viaduct pier, making use of earthquake response spectra, are described in this paper.

A simple method involving consideration of conservation of energy and a second approach based on the reserve energy technique are reviewed and the behaviour predicted by applying each to the proposed pier is compared with the response determined from a computer analysis involving numerical integration of the equations of motion of the system.

It is shown that in the case of the particular structure examined the results of the spectral technique considerations are of the same order as those obtained from more complex analyses, thereby supporting the contention that, at least for some structures, estimates of post-elastic seismic behaviour based on response spectra applications may be of value, particularly at the initial design stage when a full computer based analysis may not be warranted.

INTRODUCTION

As part of a feasibility study undertaken in conjunction with New Zealand Railways' engineers, the post-elastic seismic response of a proposed 200 ft. high reinforced concrete viaduct pier has been undertaken(1).

The dimensions of the structure, shown in Figure 1, resulted from the preliminary design at which stage it was fully appreciated that under lateral load essentially axial forces are induced in the reinforced concrete pier legs and that axially loaded concrete is capable of very little plastic action.

In order to provide the structure with sufficient plastic response capability to withstand major seismic loading New Zealand Railways' engineers proposed the inclusion of sacrificial steel pins in the pier legs, near their base. The object of the feasibility study was to establish whether pins can be selected of such a diameter that they will yield axially under strong motion earthquakes, and thereby protect the structure, without excessive plastic deformations occurring.

Before undertaking a complex computer based analysis involving numerical integration of the equations of motion of the system, using a digitised earthquake record as the excitation, it was decided to investigate the application of spectral techniques to the evaluation of the post-elastic response. This investigation is described in the following sections. The results obtained are presented subsequently and are compared with those obtained from computer analyses.

2.0 PREDICTIONS OF EARTHQUAKE RESPONSE USING SPECTRAL TECHNIQUES

There have been several attempts to modify elastic spectral response design methods to allow for plastic action. Two of these methods are outlined.

2.1 Simple Energy Approach (after Housner)

Housner(2) has suggested a simple method of equating the energies involved in a one degree of freedom yielding structure. The method is based on the Energy equation 1;

$$E_p = E_t - E_e \qquad \ldots 1$$

where E_p is the plastic energy

E_t is the total energy

E_e is the elastic energy

If E_t and E_e can be found, the plastic energy required to be absorbed in the structure can be determined.

Housner used $E_t = \frac{1}{2} \frac{W}{g} V_o^2$

where V_o = the maximum elastic spectral velocity,

W = structural weight,

and g = acceleration due to gravity

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and $E_e = \frac{1}{2} K X_y^2$

where K = stiffness

and X_y = the elastic limit deflection.

The method involves using equation 1 to find the plastic energy from which the plastic deflection is calculated.

This method has been modified, using the elasto-plastic response spectra originally presented by Blume(3) to give the following iterative method:

Step (i) Determine structural data; specifically the period, stiffness and mass of the system.

(ii) Calculate the yield load, F_y, and the displacement X_y at the onset of yield.

(iii) Compute $E_e = \frac{1}{2} F_y . X_y$

(iv) Assume a value of damping.

(v) Assume a value of μ, the ductility ratio, defined as the ratio of total displacement to maximum elastic displacement. (Initially take $\mu = 1$, i.e. assume elastic conditions).

(vi) Using the appropriate velocity response spectra, (Blume$^{(3)}$, page 12) obtain the maximum velocity response V_o' corresponding to the structure's period.

(vii) Compute $E_t = \frac{1}{2} \frac{W}{g} V_o'^2$

(viii) Calculate E_p from Equation 1.

(ix) Obtain plastic deformation X_p using the relationship

$$X_p = \frac{E_p}{F_y}$$

(x) Obtain the new ductility ratio μ' using expression

$$\mu' = \frac{X_y + X_p}{X_y}$$

(xi) Compare μ' with μ (from step v) and recycle through steps (v) to (xi) using μ' in place of μ until convergence is achieved.

2.2 Reserve Energy Technique

This method was presented by Blume, Newmark and Corning$^{(3)}$ following development work by Blume$^{(4)}$ and enables the elastic response spectra, modal analysis method to be extended to consideration of inelastic response.

Tests reported by Veletsos and Newmark$^{(5)}$ prompted the suggestion that elasto-plastic spectra could be reasonably accurately determined from elastic spectra by scaling down the elastic spectra ordinates by a factor $\sqrt{2\mu - 1}$. This corresponds to equating the input energies of the elastic and elasto-plastic systems as may readily be seen from Figure 2

i.e. $(X_m - X_y) F_y + \frac{1}{2} F_y X_y = \frac{1}{2} F_{ee} . X_{ee}$

and putting the reduction factor $R = \frac{F_y}{F_{ee}}$

and $\mu = \frac{X_m}{X_y}$

it may be shown that $R = \frac{1}{\sqrt{2\mu - 1}}$

However Blume emphasised that the stiffness of a structure is likely to be a function of both the deflection and of damage sustained in an earthquake, hence the reserve energy technique attempts to allow for such changes and consequently appears of greater practical value than the simple application of a reduced elastic response spectra to the determination of inelastic seismic response.

Outline of the Technique

The technique is applicable to structures designed to a code so that it is possible to define a load-deflection curve.

The energy input to the structure is assumed to be equal to

$$\frac{W}{2g} . V_i^2$$

where V_i is the maximum velocity response which is dependent on the deflection X_i of the system. It is therefore necessary to estimate a deflection X_i on which to base the estimation of input energy.

The input energy may be compared with the available energy absorption capacity of the system and successive iterations of X_i will result in the determination of a particular value of this parameter for which the input and absorption energies are equal. This will define the value of the acceleration response spectra, α_i, corresponding to V_i. This value of α_i may be then compared with the seismic design coefficient C and, if the structure has zero safety margin, $\frac{C}{\alpha_i} = R_i$ the acceleration response spectra reduction factor.

The procedure may be summarised as follows:

Step (i) the load deflection curve of the structure is determined

(ii) a maximum value of deflection X_i is chosen

(iii) an assessment of the total energy capacity of the system is made. If deterioration is expected this is allowed for and the effective energy capacity is calculated.

(iv) the effective stiffness of the system is calculated, taking into account the results of step (iii)

(v) the effective period of the structure is determined using the effective stiffness value

(vi) the acceleration response spectrum ordinate corresponding to the effective period is read off an appropriate spectrum, usually assuming 5% critical damping

(vii) the input energy is calculated using the acceleration response spectra value as a convenient way of determining V_i

(viii) the response spectra reduction factor R_i is determined from considerations of the relation between input energy and energy capacity

(ix) the safety factor is determined

and (x) the steps (ii) to (ix) are recycled until a suitable safety margin corresponding to an acceptable deflection X_i is determined

Comment on Reserve Energy Technique

As formulated, the method is not a design, but an analysis technique. It requires the force deflection curve of the structure to be known before it can be applied. The method will give the expected maximum ductility, μ, for a given design subjected to a chosen earthquake. If this μ is too large, the structure has to be redesigned, probably incorporating more energy absorption, so that the final μ is acceptable.

The Reserve Energy Technique is an advance on designing to elasto-plastic response spectra, obtained by dividing the elastic response spectra by μ or by $\sqrt{2\mu - 1}$, but it is evidently of limited value now that direct integration computer analyses are readily available.

The simple single mass elasto-plastic analysis of the proposed viaduct leg constitutes a special, and particularly simple, application of the Reserve Energy Technique.

3.0 APPLICATION OF THE ENERGY BALANCING APPROACH

The Housner energy approach was used in the analysis of the proposed New Zealand Railways pier.

Using the steps outlined in Section 2.1,

Taking the weight = 2000 kip
Pin diameter = 10 in.
Stiffness = 90 kip/in
Lateral Yield Force F_y = 250 kip
Damping = 0.00% critical
Period = 1.5 sec.

Pin yield level F_y' = 2500 kip (see Fig. 3)

and from the geometry of the structure $X_p = 5\ X_p'$ where X_p' is the axial deformation in the pin.

Then $X_y = \frac{F_y}{K_e} = \frac{250}{90} = 2.78$ in.

and $E_e = \frac{1}{2} \times 2.78 \times 250 = 350$ kip in.

The trial and error ductility factor determination may be undertaken as shown in Table A.

Thus maximum Displacement $= 2.08 + 2.78$
$= 4.86$ in.

Permanent Set = 0.42 in.

4.0 APPLICATION OF RESERVE ENERGY APPROACH

The Reserve Energy Technique was applied to the New Zealand Railways pier as follows, using the steps presented in section 2.2

Taking the weight = 2000 kip
Pin Diameter = 10 in.
Stiffness = 90 kip/in.
Lateral Yield Force = 250 kip
Damping = 5% critical
Period = 1.5 sec.

and Seismic Design Coefficient C = 0.12

Then $X_y = 2.78$ in and, referring to Figure 4

$F_d = 240$ kip. $X_d = 2.66$ in

Area D' = 27,000 lb.ft. and Area D" = 29,000 lb.ft.

$R_i = \sqrt{\frac{D'}{U_i}}$ (see reference 3)

Tabulating the computations the values as shown in table B may be derived.

Thus maximum displacement of pier top = 14 ins. and permanent set in pin

$= \frac{14 - 2.78}{5.0} = 2.25$ ins.

5.0 COMPARISON OF RESULTS

The prediction of the inelastic seismic response of the pier leg made using the spectrum based methods are compared with the results of a computer analysis, in which the equations of motion of the system are solved using a numerical integration techniques, as shown in Table C.

The first method provides a satisfactory correlation with the computer analysis result for the maximum displacement and the Reserve Energy approach gives a reasonable estimate of the permanent set in the pins.

The differences can be explained in terms of the pseudostatic nature of the spectrum based analyses. In each case a single value is selected from the response envelope, a velocity in the first method and an acceleration in the second. Only in the computer analysis is allowance made for repeated excursions into the plastic range.

6.0 CLOSURE

For the particular structure examined it is evident that a useful indication of the maximum inelastic seismic response can be obtained from simple spectral techniques.

REFERENCES

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3. Blume, J. A., Newmark, N. M. and Corning, L. H. "Design of Multistory Reinforced Concrete Buildings for Earthquake Motions" Portland Cement Association, Chicago, U.S.A., 1961.

4. Blume, J. A., "A Reserve Energy Technique for the Earthquake Design and Rating of Structures in the Inelastic Range" Proceedings 2nd W.C.E.E., Tokyo, 1960.

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TABLE A

μ	V_o'	E_t	$E_p = E_t - E_e$	$X_p' = \frac{F_p}{F_y'}$	$X_p = 5 \cdot X_p'$	$\mu = \frac{X_y + X_p}{X_y}$
1.00	60.0	9300.0	8950.0	3.58	17.90	7.4
1.25	33.0	2810.0	2450.0	0.98	4.90	2.7
1.80	22.5	1310.0	960.0	0.38	1.90	1.68
1.70	24.0	1520.0	1170.0	0.47	2.35	1.85
1.75	23.3	1410.0	1060.0	0.42	2.08	1.75

Note: The value of V_o' is factored by 1.5 relative to the El Centro 1940 N/S record to represent a very large earthquake.

TABLE B

C	X_i in.	U_i x 10^4 lb. ft.	R_i	α_i	$\alpha_t \cdot R_i$
0.12	5.0	7.50	0.57	0.40	0.23
0.12	8.0	13.78	0.42	0.40	0.17
0.12	10.0	17.90	0.37	0.40	0.15
0.12	12.0	22.11	0.33	0.40	0.13
0.12	14.0	26.20	0.30	0.40	0.12

Note: The value of α_i is factored by 1.5 relative to Blume's value for the El Centro 1940 N/S record to represent a very large earthquake.

TABLE C

Response (inches)	Energy Balancing	Reserve Energy	Computer[(1)] Analysis
Maximum Lateral Displacement of Pier	4.86	14	6
Permanent Set in Pin	0.42	2.25	1.48

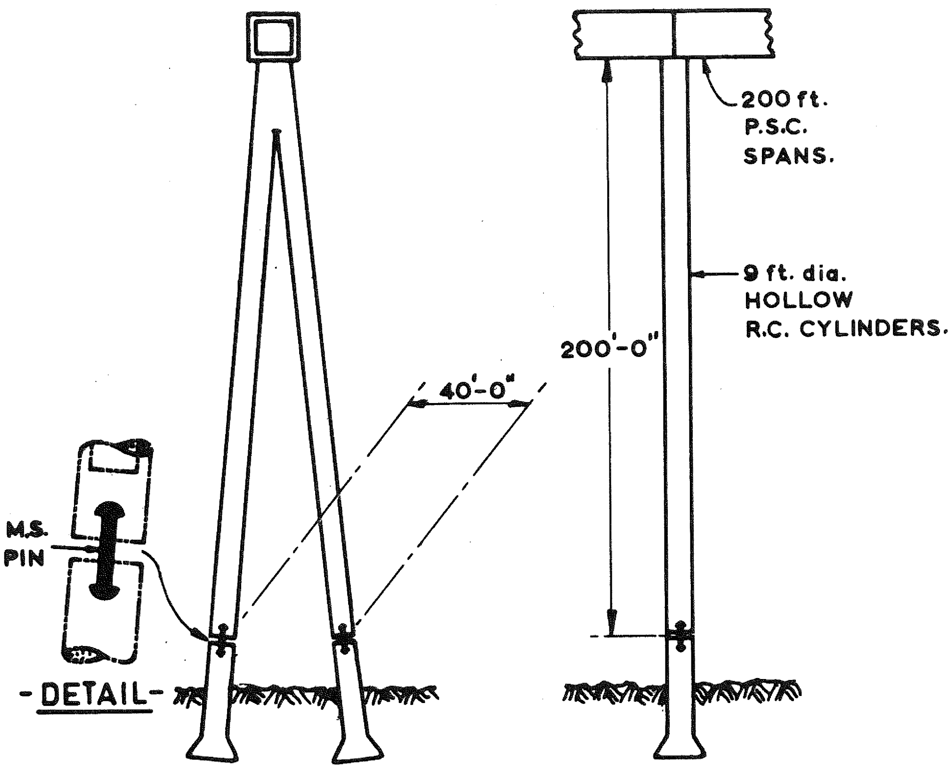
M.S. PIN
- DETAIL -
40'-0"
200'-0"
200 ft. P.S.C. SPANS.
9 ft. dia. HOLLOW R.C. CYLINDERS.

Figure 1

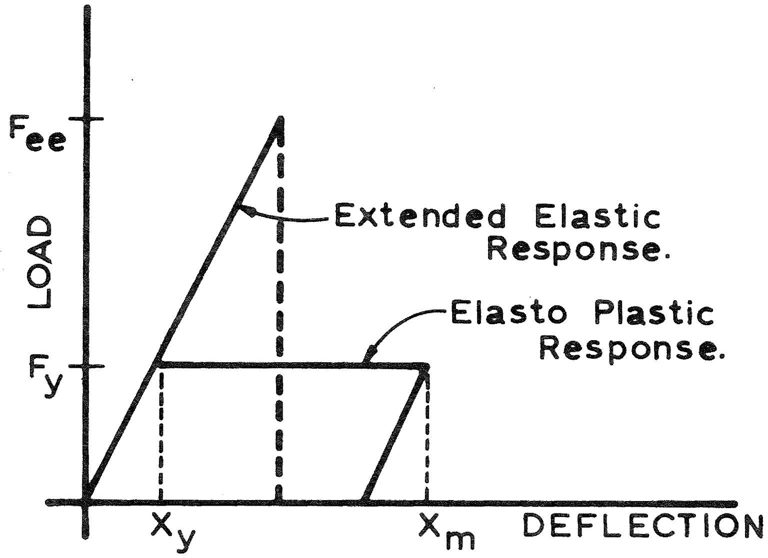
F_{ee}
LOAD
F_y
Extended Elastic Response.
Elasto Plastic Response.
X_y
X_m
DEFLECTION

Figure 2