UNDRAINED SHEAR STRENGTH OF PARTIALLY SATURATED SAND IN TRIAXIAL TESTS

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SUMMARY

The undrained shear strength of partially saturated sand is examined based on a series of undrained triaxial compression and extension tests on Toyoura sand. The effects of partial saturation, density, and triaxial compression/extension modes are discussed in detail. The conditions of "flow" and "no flow" are categorised in terms of contractive and dilative behaviours. The threshold values of the undrained shear strength ratio dividing "flow" and "no flow" are found to be independent of the B-value and triaxial compression/extension modes. The empirical relations of the B-value against the initial state ratio r_c and the undrained shear strength ratio S_{us}/p'_c are discussed in detail.

INTRODUCTION

There have been many research activities based on laboratory triaxial tests aimed at examining stability of saturated soil deposits against liquefaction and lateral flow during earthquakes. In estimating whether soil liquefaction would take place at a particular soil layer, the liquefaction potential, in other words, the factor of safety against liquefaction needs to be evaluated based on the liquefaction strength obtained from undrained cyclic triaxial tests on saturated soil specimens. On the other hand, in estimating whether lateral flow would take place as a result of liquefaction during earthquakes, the flow potential needs to be evaluated based on the undrained shear strength obtained from undrained triaxial tests on saturated soil specimens. It is known from velocity logging tests that the velocity of propagation of primary waves stays at around 500 to 1000 m/sec at soil layers located down to a depth of 5 metres below a groundwater level, which is lower than 1600 m/sec observed under fully saturated conditions (e.g., [1]). This implies that partially saturated soil layers containing some amount of minute air bubbles prevail beneath the ground water level. It also became known from laboratory triaxial tests that the liquefaction strength of soils changes depending upon the degree of partial saturation (e.g., [2]). However, the influence of partial saturation on the development of lateral flow has not been examined in detail in past studies, despite the fact that the most damaging lateral flow occurs at soil layers located within the depth of 5 metres beneath the ground water level. The present study is aimed at examining the influence of partial saturation on the undrained shear strength of sands.

BASIC MECHANICS OF PARTIALLY SATURATED SAND

Partial saturation

The terminology of "partial saturation" is used in the present study to make its distinctions from fully saturated and unsaturated conditions in sands. Figure 1 shows conceptually the re-examination of the transformation from unsaturated to fully saturated conditions, which was illustrated by Fredlund and Rahardjo [3]. When the soil is nearly saturated under atmospheric pressure, the state of soil would be determined at a point A along the hysteresis of matric suction and water content as shown in Figure 1. At this stage, the soil is under an unsaturated condition and there would be some continuous air phases within soil structures. The matric suction of $s_u = u_a - u_w$ therefore comes into effect due to the surface tension developed at the pore-air and water interfaces within soil structures. The surface tension tends to interact with soil structures, and hence affects the mobilisation of shear strength. When the soil undergoes some increase in the isotropic confining stress σ_3, the pore-air and pore-water pressures would also increase, where the rates of such increases are defined by the pore pressure parameters, $B_a = \Delta u_a/\Delta\sigma_3$ and $B_w = \Delta u_w/\Delta\sigma_3$. In the absence of soil aggregates, these parameters would be equal to 1. However, in the presence of soil aggregates, these parameters are lower than 1 due to the surface tension. The pore-water pressure tends to increase faster than the pore-air pressure in response to the increase in the isotropic confining stress, and the matric suction gradually reduces. The continuous air phases within soil structures consequently tend to diminish, and instead the occluded air bubbles would become predominant. The occluded air bubbles would not interact with soil structures; however, they would affect the compressibility of pore fluids.

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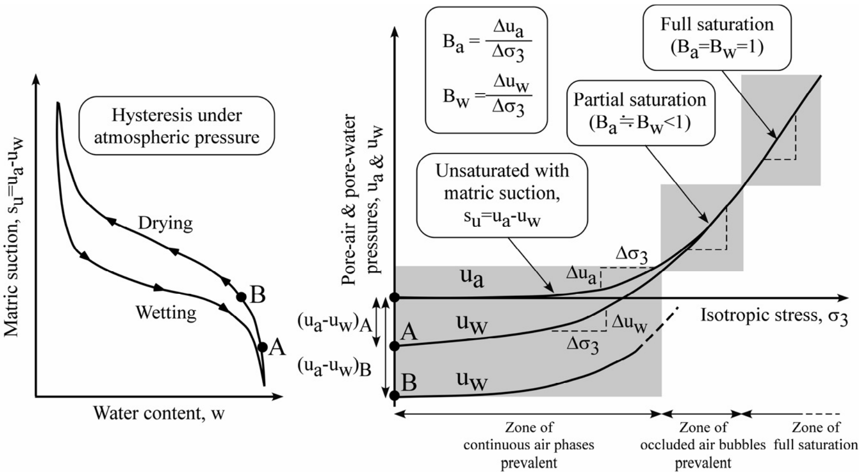

Figure 1:Conceptual interpretation of partial saturation in sands.

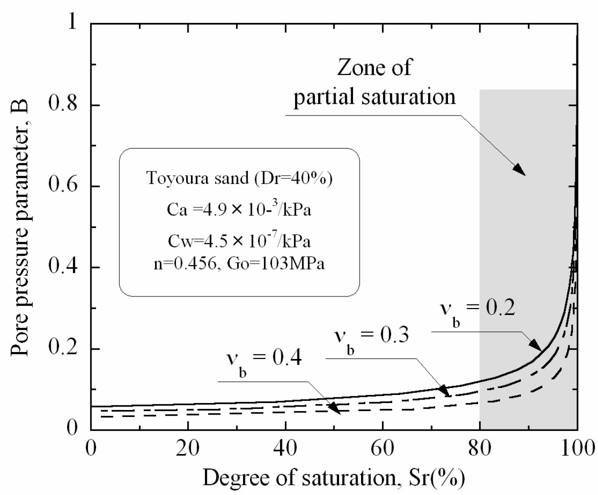

Figure 2: ***Degree of saturation in partially saturated sands.***

There would be some point at which the matric suction becomes negligible, leading to the two pore pressure parameters being equal as $B_a \approx B_w$. This assumption would be impractical for silts and clays, but would be fairly reasonable for sands. These two parameters would eventually become equal to 1 at full saturation, i.e., $B_a = B_w = 1$. The zone of "partial saturation" would therefore be characterised as follows. There would be almost no continuous air phases within soil structures, and the occluded air bubbles would be prevalent. Hence the effects of surface tension would be negligible. However, the presence of such a small volume of pore-air phase would affect the compressibility of pore fluids, and results in the pore pressure parameter B_w being lower than 1.

There are a couple of parameters that can define the state of saturation in soils, such as degree of saturation S_r and the pore pressure parameter B. The relation between S_r and B can be formulated as follows. The compressibility C_l of partially saturated sands can be derived by separately considering the influence of pore-air and pore-water phases, and therefore neglecting the effects of surface tension [3], as follows,

$$C_l = C_a(1-S_r)B_a + C_w S_r B_w \qquad (1)$$

where C_a and C_w are the compressibility of air and water phases. Herein, the assumption of $B_a = B_w$ (= B) would be reasonable for sands within the zone of partial saturation as discussed above. The following relation was separately derived by assuming the theory of wave propagation through a poro-elastic medium [2],

$$B = \frac{1}{1+\dfrac{nC_l}{C_b}} \quad \text{therefore} \quad C_l = \frac{C_b(1-B)}{nB} \qquad (2)$$

where n is the porosity, and C_b is the compressibility of soil skeleton and given as follows,

$$C_b = \frac{1}{K_b}, \quad K_b = \frac{2G_o(1+\nu_b)}{3(1-2\nu_b)} \qquad (3)$$

Herein, G_o is the initial shear modulus and ν_b is the skeleton Poisson's ratio. Therefore, by combining equations (1) to (3), the following relation between S_r and B can be obtained,

$$S_r = \frac{C_a - \dfrac{C_b(1-B)}{nB^2}}{C_a - C_w} = \frac{C_a - \dfrac{3}{2G_o n}\dfrac{1-2\nu_b}{1+\nu_b}\dfrac{1-B}{B^2}}{C_a - C_w} \qquad (4)$$

This relation is indicated in Figure 2, in which the properties for Toyoura sand obtained by Tsukamoto et al. [2] are assumed. It is found in Figure 2 that the B-value changes quite widely in the range from S_r = 80% to 100%. The pore pressure parameter B is therefore found to serve as a good parameter to evaluate the state of saturation for partially saturated sands.

Shear strength parameters

The mechanical behaviour of unsaturated soils is described by using two stress state variables, net normal stress $(\sigma - u_a)$ and matric suction $(u_a - u_w)$. In addition, the shear strength of unsaturated soils can be described with an extended Mohr-Coulomb failure envelope by adopting the matric suction $(u_a - u_w)$ as another axis in the stress diagram consisting of net normal stress $(\sigma - u_a)$ and shear stress τ [3]. The projections of different failure envelopes onto the usual stress diagram are known to produce different apparent cohesion intercepts c, depending on the matric suction, but with the same internal friction angle ϕ. On the contrary, as far as partially saturated sands are concerned, the apparent cohesion intercept would diminish due to negligible effects of matric suction. It would therefore be reasonable to assume that the shear strength parameters such as ϕ for partially saturated sands can be given

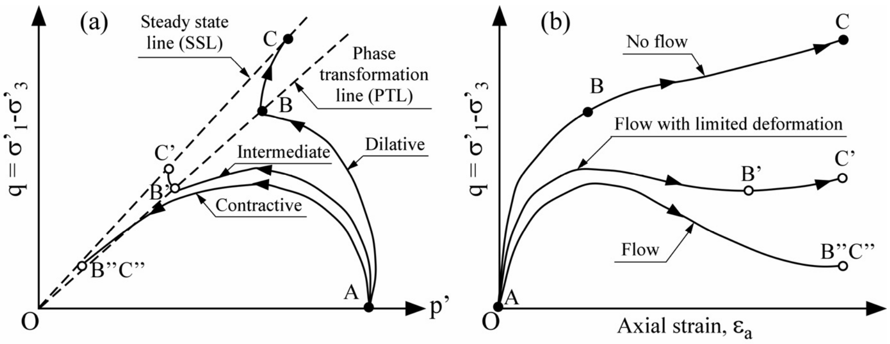

Figure 3: Definition of flow and no-flow in undrained triaxial tests.

uniquely and be considered as the same as those for fully saturated sands. This hypothesis would be supported by the above discussion, where the presence of occluded air bubbles prevalent in partially saturated sands would not affect the mobilisation of shear strength.

Pore pressure parameters

The pore pressure parameters A and B were introduced for triaxial tests to distinguish between the pore pressure increase due to the increments of isotropic confining stress and deviatoric stress [4], as follows,

$$\Delta u = B\{\Delta\sigma_3 + A\Delta(\sigma_1 - \sigma_3)\} \quad (5)$$

Rearranging the above equation leads to the following equation,

$$\Delta u = B(\Delta p + \frac{3A-1}{3}\Delta q) \quad (6)$$

where p = $(\sigma_1+2\sigma_3)/3$ and q = σ_1-σ_3. At full saturation satisfying B = 1, the pore pressure parameter A is the sole determinant of the pore pressure development. However, at partial saturation with B < 1, the two parameters A and B contribute to the pore pressure development. In the present study, the p-constant undrained triaxial tests are conducted to remove the effects of the first term in the equation (6), as follows,

$$\Delta u = C\Delta q \quad (7)$$

where C = B(3A-1)/3. Equation (7) is used below to characterise the undrained shear strength of partially saturated sands.

INTERPRETATION OF TEST RESULTS

Flow condition

The conditions of whether a particular soil deposit would cause lateral flow have long been studied within the framework of triaxial tests (e.g., [5], [6]). The same approach is adopted in the present study.

In Figure 3(a), the effective stress paths are indicated in terms of *p'* and $q = \sigma'_1 - \sigma'_3$. The point A indicates that the specimens are isotropically consolidated. In case of dense specimens, the effective stress path follows along the points B and C, where the shear stress continues to increase. The points B and C correspond to the state of phase transformation and steady state, erspectively. This is called "dilative" behaviour, and categorised as "no flow". The undrained shear strength is herein defined as the shear stress mobilised at the state of phase transformation. On the other hand, in case of loose specimens, the effective stress path follows along the points B'' and C'', where the shear stress experiences peak and then begins to reduce. The points B'' and C'' correspond to the quasi-steady state and steady state, respectively. This is called "contractive" behaviour and categorised as "flow". In this case, the quasi-steady state and steady state coincide, and the undrained shear strength is defined at this largely deformed state. Medium dense specimens show intermediate behaviour along the quasi-steady state B' and the steady state C'. Since the phase transformation takes place at the transition from contractive to dilative behaviour at point B', at which the shear stress is smaller than that at point C', the undrained shear strength is defined at the quasi-steady state B'. This intermediate behaviour is categorised as "flow with limited deformation". The condition of "flow" implies that the corresponding layers of soils would be susceptible to ceaseless large deformation upon perturbation such as earthquakes. On the other hand, the condition of "no flow" implies that the soil layers would not lead to such large deformation, though would involve instability. The undrained shear strength is therefore defined as follows,

$$\frac{S_{us}}{p'_c} = \frac{q_s}{2}\cos\phi_s\frac{1}{p'_c} = \frac{M_s\cos\phi_s}{2}\frac{1}{r_c} \quad \text{(TC \& TE)} \quad (8)$$

where the subscripts "c" and "s" denote the stress states at consolidation and phase transformation, respectively, $M_s = q_s/p'_s$, and the initial state ratio is defined as $r_c = p'_c/p'_s$. In case of saturated sand, the initial state ratio r_c was found to serve as a good parameter in characterizing the effects of soil density on the response of the excess pore water pressure (Ishihara, 1993). In case of partially saturated sand, r_c would represent the effects of soil density as well as initial state of partial saturation. As far as the isotropically consolidated p-constant undrained triaxial tests are concerned, since the pore pressure parameter C_s defined at states of phase transformation can be given as $C_s = \Delta u_s/q_s$, the initial state ratio r_c is, by definition, related to C_s, as follows,

$$r_c = 1 + M_s C_s \quad (9)$$

Since M_s is a material constant, equation (9) implies that the dependency of r_c on the relative density D_r and initial B-value can be replaced with that of C_s. The undrained shear strength ratio can then be expressed as function of the parameter C_s as follows,

$$\frac{S_{us}}{p'_c} = \frac{M_s\cos\phi_s}{2}\frac{1}{1+M_s C_s} \quad \text{(TC \& TE)} \quad (10)$$

Herein, the parameter C_s is defined at states of phase transformation, and therefore is in definition given as $C_s = B_s(3A_s-1)/3$. It is to note here that during undrained triaxial tests, the A-value would change from 0 to A_s and the B-value would change from B_o to B_s, until the states of phase transformation are achieved. However, since the parameter C_s would change in accordance with the initial state of partial saturation, the value of C_s can be strongly correlated with the initial B_o-value defined at consolidation. In particular, since it would be reasonable to assume $C_s = 0$ at $B_o = 0$, the initial state ratio would also be equal to $r_c = 1$ at $B_o = 0$, according to the equation (9). Therefore, the undrained shear strength ratio would also become equal to $S_{us}/p'_c = M_s \cos\phi_s/2$ at $B_o = 0$, according to the equation (10). It is to note here that the above equations (9) and (10) are valid under the testing condition of a constant mean principal stress, p.

Testing details

Toyoura sand was used in the present study, which is a clean fine sand containing no fines (i.e., less than 0.075 mm). The physical properties of this sand are as follows, mean particle diameter of D_{50} = 0.17 mm, specific gravity of G_s = 2.657, maximum void ratio of e_{max} = 0.973, and minimum void ratio of e_{min} = 0.607.

The triaxial specimens with the relative density of D_r = 20% were prepared by the method of wet tamping (W.T.), and those with D_r = 40% and 60% were prepared by the method of air pluviation (A.P.). The values of the relative density given above correspond to those measured after consolidation. Herein, the loose specimens can only be prepared by the method of wet tamping, while it becomes difficult to prepare the dense specimens by wet tamping. They were then isotropically consolidated to an effective confining stress of $\sigma'_c = p'_c$ = 98 kPa, and the initial B_o-value was measured. Herein, the initial B_o-value indicative of the degree of partial saturation was controlled by the application of the back pressure. Upon acquiring the designed B_o-value, the undrained triaxial compression (TC) or extension (TE) tests were conducted, in which the mean principal stress of $p = (\sigma_a+2\sigma_h)/3$ was kept constant, where σ_a is the axial stress and σ_h is the horizontal stress. Herein, the major and minor principal stresses were defined as $\sigma_1 = \sigma_a$ and $\sigma_3 = \sigma_h$ under triaxial compression, while they were defined as $\sigma_1 = \sigma_h$ and $\sigma_3 = \sigma_a$ under triaxial extension. The preference of the use of the p-constant tests in unsaturated soils was discussed above, and the details are given by Tsukamoto et al. [2].

TRIAXIAL TEST RESULTS

General observations

The inclusion of minute air bubbles within the voids of partially saturated soils brings about the fairly noticeable compressibility, and hence affects the mechanical behaviour of soils. In particular, the pore pressure response during undrained loading is directly inflicted by the compressibility of air-water mixture within the voids.

Typical results of the undrained triaxial compression tests are shown in Figure 4. In Figure 4(a), the change in excess pore pressures is plotted against the axial strain, ε_a, for the triaxial compression test with D_r = 40%. Positive pore pressures are observed at the initial stage of axial straining, which is termed as a "u>0" zone. The negative pore pressures are seen to develop after this stage, which is termed as a "u<0" zone. It would be noticed that the compressibility of air-water mixture within the voids appears to inhibit the development of the pore pressures, since the magnitudes of the pore pressures reduce as the B_o-value decreases, regardless of whether the excess pore pressures are positive or negative. It would imply that the resistance of soils to shear shows different characteristics in the "u>0" zone and "u<0" zone, since it is closely linked to

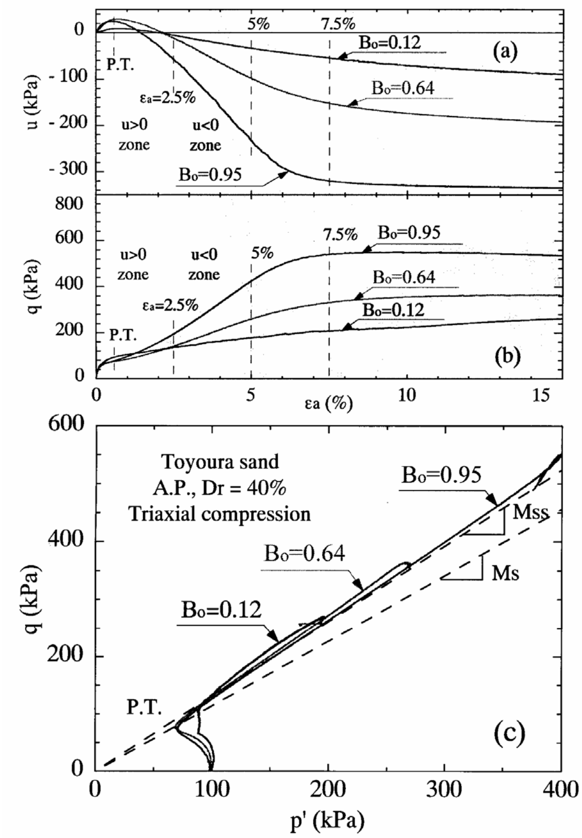

Figure 4: Triaxial test results (Toyoura sand, D_r=40%, Triaxial compression).

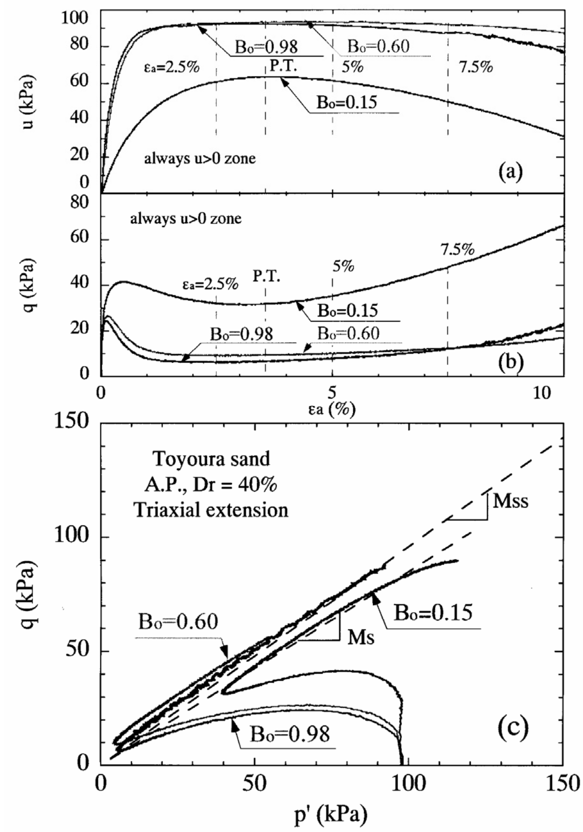

Figure 5: Triaxial test results (Toyoura sand, D_r=40%, Triaxial extension).

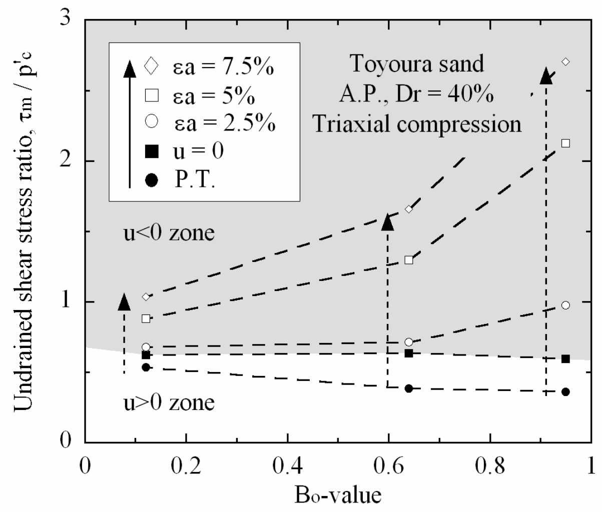

Figure 6: ***Plots of initial B_o–value against undrained shear stress ratio τ_m/p'_c at various levels of axial strain (Toyoura sand, D_r=40%, Triaxial compression).***

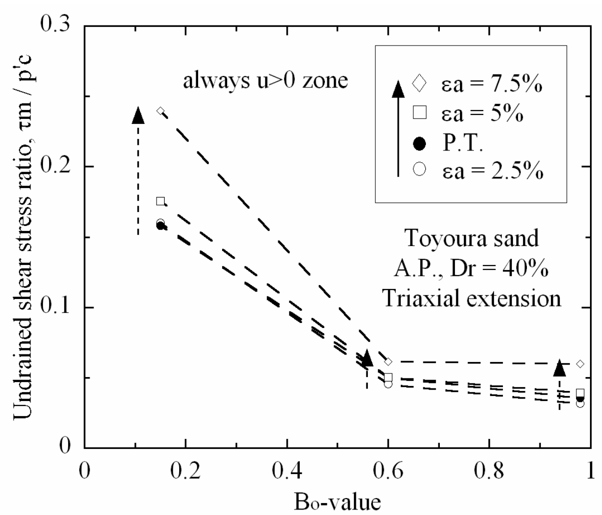

Figure 7: ***Plots of initial B_o–value against undrained shear stress ratio τ_m/p'_c at various levels of axial strain (Toyoura sand, D_r=40%, Triaxial extension).***

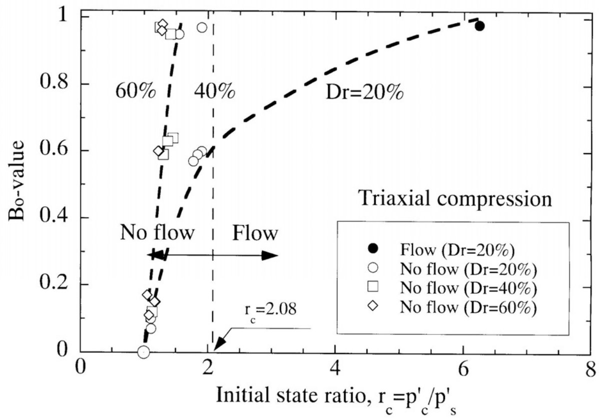

Figure 8: ***Plots of initial B_o–value against initial state ratio r_c (Toyoura sand, Triaxial compression).***

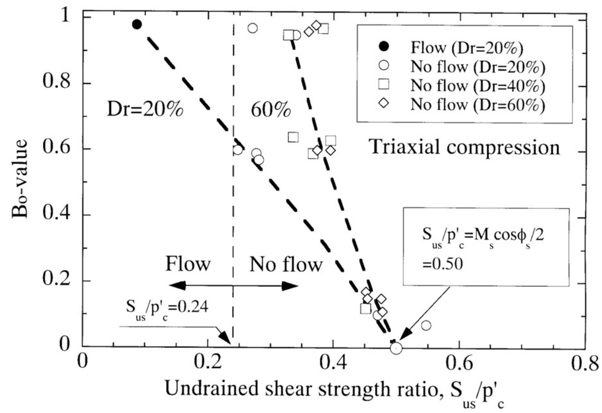

Figure 9: ***Plots of initial B_o–value against undrained shear strength ratio S_{us}/p'_c (Toyoura sand, Triaxial compression).***

the development of the excess pore pressure, as shown in Figure 4(b). This aspect is discussed in detail in Figures 6 and 7. The effective stress paths are shown in Figure 4(c), in which it is found that the steady state envelope and the phase transformation envelope can be determined uniquely regardless of the B_o-value. The value of $M_s = q_s/p'_s = f(\phi_s)$ defined at states of phase transformation can therefore be given as a unique value regardless of the B_o-value. It would support the hypothesis described above that the shear strength parameters such as the internal friction angle ϕ be not affected by states of partial saturation. In Figure 5, the results of the undrained triaxial extension tests with D_r = 40% are also shown. It is seen that there remains a positive excess pore pressure up to an axial strain of ε_a = 10%.

The influence of the B_o-value on the development of shear resistance under triaxial compression is summarised in Figure 6, where the values of the undrained shear stress ratio, $\tau_m/p'_c = q_m/(2p'_c) = (\sigma_1-\sigma_3)/(2p'_c)$, observed at various levels of axial strain are plotted against the B_o-value. It would be clearly noticed that at the strain levels located in the "u>0" zone, the undrained shear stress ratio decreases as the B_o-value increases, while at the strain levels within the "u<0" zone, the undrained shear stress ratio increases as the B_o-value increases. These contradictory behaviours are closely relatedto the difference between the pore pressure responses in the "u>0" zone and "u<0" zone.

In Figure 7, the same diagram is produced for the test results under triaxial extension. Since positive pore pressure is observed up to an axial strain of ε_a = 10%, it is seen that the undrained shear stress ratio decreases as the B_o-value increases.

Initial state ratio and undrained shear strength ratio

From a number of tests conducted with different values of relative density D_r and B_o-values, each data was categorised into "flow" (contractive or intermediate behaviours) and "no flow" (dilative behaviour), and the values of the initial state ratio r_c and the undrained shear strength ratio S_{us}/p'_c were deduced. The plots of the B_o-values against the values of r_c and S_{us}/p'_c thus deduced from the test results under triaxial compression are shown in Figures 8 and 9. The border between "flow" and "no flow" in Figure 9 is found to be independent of the B_o-value, and given by the threshold value of S_{us}/p'_c = 0.24. This threshold value is found to be the same as that derived in the previous study on saturated Toyoura sand [7]. The corresponding threshold value of r_c can be

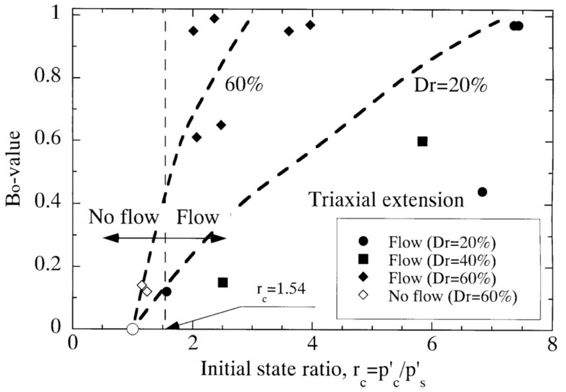

Figure 10: ***Plots of initial B_o–value against initial state ratio r_c (Toyoura sand, Triaxial extension).***

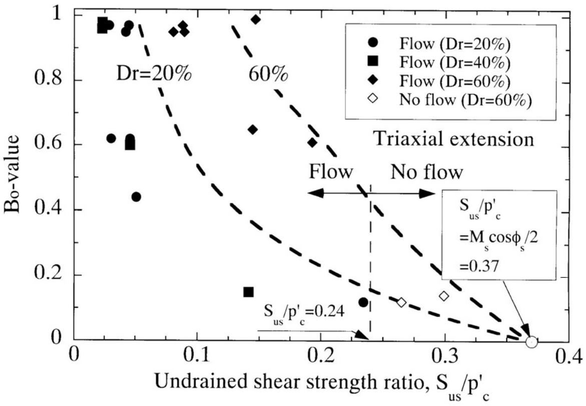

Figure 11: ***Plots of initial B_o–value against undrained shear strength ratio S_{us}/p'_c (Toyoura sand, Triaxial extension).***

calculated by using the equation (8), and is shown in Figure 8. There would be uniquely fixed points at (r_c, B) = (1, 0) and (S_{us}/p'_c, B) = ($M_s\cos\phi_s/2$, 0), as derived from the formulated equations (9) and (10), which are also plotted in Figures 8 and 9.

The zones of the data for the tests conducted with the same relative density D_r are also indicated in Figures 8 and 9. In case of the specimens with D_r = 20%, it would be susceptible to "flow" when the B_o-value is larger than about 0.6. On the other hand, in case of the specimens with D_r = 60%, it would not be susceptible to "flow" regardless of the B_o-value. It is therefore found that the range of the B_o-value categorised into "flow" is dependent on the relative density D_r and is larger as the relative density D_r is lower. The same set of the diagrams is shown in Figures 10 and 11 for the tests under triaxial extension. It is found that the threshold values of S_{us}/p'_c under triaxial compression and extension are the same. However, the corresponding threshold values of r_c under triaxial compression and extension are different, which is expected from the equation (8), since the values of M_s and ϕ_s are different under triaxial compression and extension.

IMPLICATION TO PRACTICAL APPLICATIONS

The occurrences of soil liquefaction and associated flow deformation during earthquakes are evaluated based either on the penetration resistance of field penetration tests or on the triaxial tests using undisturbed saturated specimens. In case of field penetration tests, the relative density is estimated and the other state parameters such as the degree of saturation are neglected. In case of triaxial tests, the undisturbed specimens are usually saturated and subjected to triaxial tests. The effects of the degree of saturation are therefore not considered. However, the outcome of the present study indicates that the degree of partial saturation has some noticeable influence on the undrained shear strength, and therefore suggests that it would be worthwhile to take into account the effects of partial saturation.

CONCLUSIONS

The definition of partial saturation in soils was discussed in detail, and the preference of using the pore pressure parameter B in representing the state of partial saturation was highlighted. The undrained shear strength of partially saturated sand was examined within the framework of a steady state concept, based on the series of isotropically consolidated undrained triaxial compression and extension tests on Toyoura sand. The conditions of "flow" and "no flow" were examined in terms of contractive and dilative behaviours, and the undrained shear strength defined at states of phase transformation was discussed. The threshold value of the undrained shear strength ratio dividing "flow" and "no flow" was found to be independent of the initial B_o-value and triaxial compression/extension modes. The range of the B_o-value categorised into "flow" was found to be dependent on the relative density D_r and is larger as the relative density D_r is lower. The formulated relations of the B_o-value against r_c and S_{us}/p'_c were compared with the test results, and it was found that these relations could be empirically calibrated.

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